PHY/VI/09 (R) **Student's Copy**

2 0 1 8

(6th Semester)

PHYSICS

NINTH PAPER

(Method of Mathematical Physics—II)

(Revised)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. In the differential equation

$$(1 \quad x)^2 \frac{d^2y}{dx^2} \quad 2x\frac{dy}{dx} \quad l(l \quad 1)y \quad 0$$

(a) $x \quad 0$ is an ordinary point ( )

(b) $x \quad\; 1$ are regular singular points ( )

(c) Both *(a)* and *(b)* are correct ( )

(d) $x \quad\; 1$ are irregular singular points ( )

2. The displacement function for a string of length l fixed at both ends with zero initial velocity is given by

$$u(x,\ t) \qquad {}_{n \;\; 1}\ C_n \cos\frac{n \;\; ct}{l} \quad D_n \sin\frac{n \;\; ct}{l} \quad \sin\frac{n}{l}x,$$

then

(a) $D_n \quad 0$ ( )

(b) $C_n \quad 0$ ( )

(c) $C_n \quad D_n \quad 0$ ( )

(d) $C_n \quad D_n \quad 0$ ( )

3. The value of the integral $_0\{J\ _2(x)\quad J_2(x)\}\,dx$ is

(a) 0 ( )   *(b)* 2 ( )

(c) 2 ( )   *(d)* 1 ( )

4. The value of $P_3(x)$ is

(a) $\frac{1}{2}(3x^2\quad 1)$ ( )   *(b)* $3x$ ( )

(c) $\frac{1}{2}(5x^3\quad 3x)$ ( )   *(d)* $\frac{1}{2}(5x^3\quad 1)$ ( )

5. In the half-range series of the function $f(x)$ in the interval $(0,\quad)$, we get sine series if

(a) $f(x)$ is even in the interval $(\quad,\quad)$ ( )

(b) $f(x)$ is odd in the interval $(\quad,\quad)$ ( )

(c) $f(x)$ is even in the interval $(0,\quad)$ ( )

(d) $f(x)$ is odd in the interval $(0,\quad)$ ( )

6. The Fourier transform of $\frac{df}{dt}$ i.e., F.T. $\frac{df}{dt}$ is

(a) $\frac{}{\sqrt{2}}\quad f(t)e^{\ i\ \ t}\,dt$ ( )

(b) $\sqrt{\frac{}{2}}\quad f(t)e^{i\ \ t}dt$ ( )

(c) $\frac{i}{\sqrt{2}}\quad f(t)e^{\ i\ \ t}dt$ ( )

(d) $\frac{1}{i\ \sqrt{2}}\quad f(t)e^{i\ \ t}dt$ ( )

7. If $f(s)$ is the Laplace transform of $F(t)$, then $\mathscr{L}\ ^{1}[f(as)]$ is

(a) $\frac{1}{a}F\ \ \frac{t}{a}$ ( )

(b) $\frac{1}{a}F\ \ \frac{a}{t}$ ( )

(c) $aF\ \ \frac{t}{a}$ ( )

(d) $aF\ \ \frac{a}{t}$ ( )

8. $\mathscr{L}\ \ \frac{dF(t)}{dt}$ is

(a) $sF(t)\quad F(0)$ ( )

(b) $\mathscr{L}[F(t)]\quad sF(0)$ ( )

(c) $s\mathscr{L}[F(t)]\quad F(0)$ ( )

(d) $s\mathscr{L}\ \ \frac{dF(t)}{dt}\quad F(0)$ ( )

9. The final value of I in the DO statement, DO 10 I 1, 5, 2 is

(a) 1 ( )

(b) 3 ( )

(c) 2 ( )

(d) 5 ( )

10. If I 3, J 8 and K 4, the value of N in the following statement

$$N\quad 3\quad J/I\quad K\quad 4/J$$

is

(a) $\frac{1}{2}$ ( )

(b) $\frac{3}{2}$ ( )

(c) 1 ( )

(d) 0 ( )

SECTION—B

(*Marks* : 15)

Answer the following questions : 3×5=15

1. Find the differential equation for which the solution is $y \quad c_1 e^x \quad c_2 e^{\ x} \quad 3$.

2. Using the recursion relation

$$\frac{d}{dx}[x^n J_n(x)] \quad x^n J_{n \ 1}(x)$$

show that $\ _0^x x^n J_{n \ 1}(x) dx \quad x^n J_n(x)$.

3. If $g(\ \ )$ is the Fourier transform of $f(t)$, then show that the Fourier transform of $f(t)\cos at$ is given by $\frac{1}{2}[g(\quad a) \quad g(\quad a)]$.

4. If $f(s)$ is the Laplace transform of $F(t)$, then show that $\mathscr{L} \ ^1[f(s \quad a)] \quad e^{at}F(t)$.

5. A candidate passes an examination if he secures 40 marks or more. Write a program segment to display PASS or FAIL according to the marks obtained.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

1. *(a)* Solve the differential equation

$$\frac{d^2y}{dx^2} \quad \frac{1}{2x}\frac{dy}{dx} \quad \frac{1}{4x}y \quad 0$$

about $x \quad 0$ by power series method. 6

(b) Solve the differential equation of heat flow $\frac{}{t} \quad h^2 \frac{^2}{x^2}$ under the boundary conditions $(0,\ t) \quad (l,\ t) \quad 0,\ t \quad 0$ and $(x,\ 0) \quad x,\ 0 \quad x \quad l$; where l is the length of the bar. 4

OR

2. *(a)* Write down the Laplace equation in two-dimensional polar coordinates and solve it by the method of separation of variables. 5

(b) Solve the following differential equation $(1 \quad x^2)D^2 \quad xD \quad y \quad 0$; where $D \quad \frac{dy}{dx}$ by power series method. 5

3. *(a)* Show that

$$P_n(x) \quad \frac{1}{2^n(n!)}\frac{d^n}{dx^n}(x^2 \quad 1)^n$$ 5

(b) Prove that $J_{\frac{1}{2}}(x) \quad \sqrt{\frac{2}{x}}\sin x$. 5

OR

4. *(a)* Prove that ${}_0^a J_n(\quad r)J_n(\quad r)rdr \quad 0$; where and are different roots of $J_n(\quad a) \quad 0$. 6

(b) Show that $H_n(x)$ is the coefficient of z^n in the expansion of $e^{x^2 \quad (z \quad x)^2}$. 4

5. *(a)* Obtain the Fourier series of a function $f(x) \quad x^2; \qquad x \qquad$. Hence show that $_{n \quad 1}\frac{1}{n^2} \quad \frac{2}{6}$. 5+1

(b) Express the function

$$f(x) \quad \begin{matrix} 1 & \text{for} & |x| \quad 1 \\ 0 & \text{for} & |x| \quad 1 \end{matrix}$$

as a Fourier integral and hence evaluate $_0 \frac{\sin \quad \cos \quad x}{} d$. 3+1

OR

6. *(a)* Find the Fourier sine transform of $\frac{e^{\quad ax}}{x}$. 4

(b) Find the Fourier sine and cosine transforms of $f(t) \quad e^{\quad pt}, p \quad 0$. Hence evaluate $_0 \frac{\cos \quad t}{p^2 \quad {}^2} d$. 5+1

7. *(a)* Find the Laplace transform of $F(t) = t^2 e^t \sin 4t$. 4

(b) Obtain the Laplace transform of half-wave rectifier wave function

$$F(t) = \begin{cases} \sin \omega t & ; \; 0 \le t \le \frac{\pi}{\omega} \\ 0 & ; \; \frac{\pi}{\omega} \le t \le \frac{2\pi}{\omega} \end{cases}$$

with period $T = \frac{2\pi}{\omega}$. 6

OR

8. *(a)* Find the inverse Laplace transforms of $f(s) = \frac{1}{s(s+2)^3}$. 5

(b) Using Laplace transform, evaluate the integral $\int_0^\infty \cos x^2 dx$. 5

9. *(a)* Write down the general form of the DO loop and explain it. 1+2

(b) Write a FORTRAN program to read x and n and evaluate the sum of the series $1 + x + x^2 + \cdots + x^n$. 4

(c) Write a DO loop to evaluate and print the values of quadratic polynomial $y = x^2 - 10x + 11$ for the values $x = 0{\cdot}0$ to $1{\cdot}0$ in steps of $0{\cdot}2$. 3

OR

10. *(a)* Write a FORTRAN program to read t and print x, y, z, if $x = 8t^2 + t^3 - 4$, $y = \sin t + \cos 2t$, $z = e^{3t} + 5$. 3

(b) Write a program segment to evaluate the function

$$f(x) = \begin{cases} x^2 + \sin 2x & ; \; x < 3 \\ 10{\cdot}5 & ; \; x = 3 \\ x^3 + \cos 3x & ; \; x > 3 \end{cases}$$

3

(c) What do you mean by FORMAT specification? Explain different types of FORMAT specifications. 1+3

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