2018

(6th Semester)

PHYSICS

NINTH PAPER

(Method of Mathematical Physics—II)

(Pre-Revised)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. The value of (1) is

(a) ( ) *(b)* 1 ( )

(c) $\sqrt{\ }$ ( ) *(d)* 0 ( )

2. The value of $(z, 1)$ is

(a) $\frac{1}{z}$ ( )

(b) $\frac{1}{z \quad 1}$ ( )

(c) $\frac{1}{z(z \quad 1)}$ ( )

(d) $\frac{z}{z \quad 1}$ ( )

3. For an arbitrary well-behaved function, the value of

$$f(x) \quad (x \quad a)\,dx$$

is

(a) 0 ( )

(b) a ( )

(c) $f(x)$ ( )

(d) $f(a)$ ( )

4. Which of the following represents Fourier transform?

(a) $g(\ ) \quad {}_0 f(t)e^{\ i\ t}dt$ ( )

(b) $g(\ ) \quad {}_0 f(t)e^{\ t}dt$ ( )

(c) $g(\ ) \quad {}_0 f(t)\,t\,J_n(\ t)\,dt$ ( )

(d) $g(\ ) \quad {}_0 f(t)\,t^{\ 1}dt$ ( )

5. The inverse Laplace transform of $\dfrac{1}{s(s^2 \quad 1)}$ is

(a) $(1 \quad \cos t)$ ( )

(b) $(1 \quad \cos t)$ ( )

(c) $(1 \quad \sin t)$ ( )

(d) $(1 \quad \sin t)$ ( )

6. The Laplace transform of (t) is

(a) 1 ( )

(b) 0 ( )

(c) $\sqrt{2}$ ( )

(d) $\dfrac{1}{\sqrt{2}}$ ( )

7. In the group $G \quad \{E, A, A^2\}$, the element conjugate to A^2 is

(a) E ( )

(b) A ( )

(c) A^2 ( )

(d) A^2 ( )

8. The group of order 4

(a) is always a cyclic group ( )

(b) is never a cyclic group ( )

(c) may or may not be a cyclic group ( )

(d) does not contain identity element ( )

9. A number or a string of FORTRAN characters is called

(a) constant ( )

(b) character set ( )

(c) expression ( )

(d) variable ( )

10. In FORTRAN, _____ statement is used to transfer data from input device to the main memory of the computer.

(a) WRITE ( )

(b) READ ( )

(c) FORMAT ( )

(d) END ( )

SECTION—B

(*Marks* : 15)

Answer the following questions : 3×5=15

1. Prove that $(n \quad 1) \quad n \quad (n)$.

2. If $f(t) \quad a_1 f_1(t) \quad a_2 f_2(t) \quad \cdots$, then show that the Fourier transform of $f(t)$ is given by $g(\ ) \quad a_1 g_1(\ ) \quad a_2 g_2(\ ) \quad \cdots$ where $g_1(\ )$, $g_2(\ ), \cdots$ are Fourier transforms of $f_1(t)$, $f_2(t) \cdots$ and a_1, $a_2, \cdots$ are constants.

3. If $f(s)$ is the Laplace transform of $F(t)$, then show that the Laplace transform of $F(at)$ is $\frac{1}{a} f \left(\frac{s}{a} \right)$.

4. Generate a group starting from an element A subject only to the condition $A^n \quad E$, such that n is the smallest positive number satisfying the condition.

5. Write a FORTRAN program to convert Centigrade to Fahrenheit.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

1. *(a)* Show that the gamma function can be expressed as

$$(n) \quad \frac{1}{n} \int_0 e^{y^{\frac{1}{n}}} dy$$

and hence deduce the value of $\frac{1}{2}$. 3+1

(b) Show that

$$\int_0^{/2} \sin^p \cos^q d \quad \frac{\frac{p \quad 1}{2} \quad \frac{q \quad 1}{2}}{2 \quad \frac{p \quad q \quad 2}{2}}$$ 3

(c) Show that the error function satisfies the relation $erf_c(x) \quad erf_c(\quad x) \quad 2$. 3

OR

(a) Show that

$$\int_0 \frac{y^{m\ 1}}{(1\quad y)^{m\ n}}\,dy \qquad \int_0^1 \frac{y^{m\ 1}\quad y^{n\ 1}}{(1\quad y)^{m\ n}}\,dy \qquad (m,\ n)$$ 5

(b) Show that

$$\int_0^1 x^{m\ 1}(1\quad x^a)^n\,dx \quad \frac{1}{a}\frac{\frac{m}{a}\ n!}{\frac{m}{a}\quad n\quad 1}$$ 3

(c) Show that for factorial function

$$(\ )_n \quad \frac{(\quad n)}{(\ )}$$ 2

2. (a) Obtain Fourier series expansion for a half-wave rectifier in which current is given by

$$I \qquad \begin{matrix} I_0 \sin\ t; & 0 \quad t \quad T/2 \\ 0 \quad\ ; & T/2 \quad t \quad T \end{matrix}$$ 7

(b) If $g(\ )$ is the Fourier transform of $f(t)$, then show that the Fourier transform of $f(at)$ is $\frac{1}{a}g\ \frac{}{a}$. 3

OR

(a) Find the Fourier transform of the slit function $f(x)$ defined as

$$f(x) \qquad \begin{matrix} 1/\ \ ; & |x| \\ 0\ \ ; & |x| \end{matrix}$$ 3

(b) Find the Fourier sine transform of $f(t) \quad e^{\ pt}$, $p \quad 0$. Hence evaluate

$$\int_0 \frac{\sin\ t}{p^2 \quad 2}\,d$$ 3+1

(c) Find the Fourier cosine transform of a function $f(x)$ which is unity for $0 \quad x \quad a$ and zero for $x \quad a$. 3

3. *(a)* Find Laplace transform of t^n, n 1. What will be the Laplace transform of $\sqrt{t}$? 2+1

(b) Find the Laplace transform of sawtooth wave function

$$F(t) \quad \frac{at}{T} \text{ for } 0 \quad t \quad T \text{ and } F(t \quad T) \quad F(t)$$ 3

(c) Find the inverse Laplace transform of

$$\frac{s^2 \quad 2s \quad 3}{s(s \quad 3)(s \quad 2)}$$ 4

OR

(a) Find the inverse Laplace transform of $\frac{1}{(s \quad 1)(s^2 \quad 1)}$. 5

(b) Using Laplace transform, evaluate the integral

$$\int_0 t^2 e^{\ t} \sin t \, dt$$ 3

(c) Find the Laplace transform of the function

$$F(t) \quad \frac{e^{at} \quad 1}{a}$$ 2

4. *(a)* Prove that the reciprocal of a product of two or more elements of a group is equal to the product of the reciprocals in reverse order. 3

(b) What do you mean by conjugate elements? State and prove the properties of conjugate elements. 1+6

OR

(a) Show that the three cube roots of unity form an Abelian group under multiplication. 3

(b) Generate a group from two elements A and B subject only to the relations $A^2 \quad B^3 \quad (AB)^2 \quad E$. 3

(c) Show that the four matrices

$$E \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}, \quad A \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}, \quad B \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \text{ and } \quad C \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}$$

form a group under matrix multiplication. 4

5. *(a)* State and explain FORMAT specifications. 3

(b) Write a FORTRAN DO loop to print the negative number between 1 and 100 and their squares and cubes. 5

(c) Suppose A 2 5, B 3 5, J 5 and K 10. What will be the value of J after the following program segment is executed? 2

```
      IF ( 2*K .LE. 3*J ) GO TO 50
      J = J + 1
      GO TO 60
50    J = K
60    J = J + K
```

OR

(a) Explain the 'arithmetic IF' statement with an example. 3

(b) Write a FORTRAN DO loop to read 100 numbers and print all the numbers less than 40. 4

(c) Write a program segment to evaluate the function

$$f(x) \quad \begin{cases} x^2 \quad \sin 2x & ; \quad x \quad 3 \\ 10 \quad 5 & ; \quad x \quad 3 \\ x^3 \quad \cos 3x & ; \quad x \quad 3 \end{cases}$$ 3

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