2025

(NEP—2020)

(5th Semester)

PHYSICS (MAJOR2)

(Classical and Statistical Mechanics)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. If the Lagrangian does not contain a certain coordinate, then the corresponding conserved quantity is

(a) kinetic energy ( )

(b) velocity ( )

(c) angular momentum ( )

(d) force ( )

2. The particle constraints to move only on the surface of a sphere is an example of

(a) holonomic constraint ( )

(b) non-holonomic constraint ( )

(c) There is no constraint ( )

(d) All of the above ( )

3. The Hamiltonian is a function of

(a) generalized coordinates and generalized velocities ( )

(b) generalized coordiantes and generalized momenta ( )

(c) generalized velocities and generalized momenta ( )

(d) constants ( )

4. For a conservative system, Hamiltonian (H) is given by

(a) $H = T + V$ ( )

(b) $H = T - V$ ( )

(c) $H = 2T - V$ ( )

(d) $H = 2T + V$ ( )

(where T = kinetic energy, V = potential energy).

5. The Hamiltonian of simple pendulum is given by

(a) $H = \frac{1}{2}\frac{p_\theta^2}{ml^2} + mgl(1 - \cos\theta)$ ( )

(b) $H = \frac{1}{2}\frac{p_\theta^2}{ml^2} + mgl(1 + \cos\theta)$ ( )

(c) $H = \frac{p_\theta^2}{ml^2} + mgl(1 + \cos\theta)$ ( )

(d) $H = \frac{1}{2}\frac{p_\theta^2}{ml^2} - mgl(1 + \cos\theta)$ ( )

6. In equilibrium state, the probability and entropy of the system are

(a) both minimum ( )

(b) minimum and maximum ( )

(c) maximum and minimum ( )

(d) both maximum ( )

7. In Grand Canonical Ensemble, the Grand potential is expressed as

(a) $\Omega = \frac{p}{V}$ ( )

(b) $\Omega = -\frac{p}{V}$ ( )

(c) $\Omega = pV$ ( )

(d) $\Omega = -pV$ ( )

8. The area of phase cell is

(a) h ( )

(b) h^2 ( )

(c) h^3 ( )

(d) $\frac{h}{2}$ ( )

9. In quantum statistics, the occupation index is given by

(a) $\frac{n_i}{g_i} \approx 1$ ( )

(b) $\frac{n_i}{g_i} \ll 1$ ( )

(c) $\frac{n_i}{g_i} \gg 1$ ( )

(d) $\frac{n_i}{g_i} \approx 0$ ( )

10. Bose-Einstein statistics is obeyed by

(a) protons ( )

(b) electrons ( )

(c) neutrons ( )

(d) photons ( )

(SECTION : B—SHORT ANSWERS)

(*Marks* : 25)

Answer *five* questions, taking at least *one* from each Unit : 5×5=25

UNIT—I

1. Show that the total energy in central force motion is given by $\frac{1}{2}m\dot{r}^2 + \frac{1}{2}\frac{J^2}{mr^2} + U$, where m is mass of the particle, J is angular momentum and U is potential energy of the particle.

2. Obtain Lagrangian for simple pendulum and hence solve for the time period of oscillation.

UNIT—II

3. Obtaint the time period for one dimensional harmonic oscillator using Hamiltonian.

4. Write down the statement of Hamilton's variational principle. What do you mean by 'action'? Hence obtain the dimension.

UNIT—III

5. Explain the condition of thermal equilibrium between two systems and relate with β-parameter.

6. State the postulate of equal a priori probability. Explain how this postulate leads to concept of a cell in a compartment.

UNIT—IV

7. Explain the application of phase space in 1-dimensional Harmonic oscillator showing that energy is quantized.

8. Describe in brief the differences between classical and quantum statistics.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 40)

Answer *four* questions, taking *one* from each Unit : $10 \times 4 = 40$

UNIT—I

1. *(a)* Show that the equation of orbit in central force is given by $\frac{d^2u}{d\theta^2} + u = -\frac{m}{j^2u^2}F\left(\frac{1}{u}\right)$, where the symbols have their usual meaning. 6

(b) A particle of mass m under the action of a force $r = r_0e^{\theta}$ describes a spiral orbit. Obtain the force responsible for this orbit. 4

2. *(a)* Obtain Lagrange's equation of motion from d'Alembert's principle. 6

(b) What do you mean by the principle of virtual work? Obtain the condition for static equilibrium. 4

UNIT—II

3. *(a)* Using Hamiltonian method, show that in central force motion, radial equation is given by $m\ddot{r} - mr\dot{\theta}^2 = \frac{K}{r^2}$. Also, show that angular momentum is conserved. 7

(b) Show that Hamiltonian represents the total energy of the system. 3

4. *(a)* What is Hamiltonian? Show that Hamiltonian is given by $H = \Sigma\dot{q}_i p_j - L$. 4

(b) Obtain Hamilton's canonical equation of motions. 6

UNIT—III

5. *(a)* What is an ensemble? Explain the different kinds of ensemble. 4

(b) Describe in detail the different thermodynamic quantities in Grand Canonical Ensemble. 6

6. *(a)* Define the functions $\phi(E)$ and $\Omega(E)$. Hence, show that from the relation between $\phi(E)$ and $\Omega(E)$ for a single particle, $\Omega(E)$ is directly proportional to $\sqrt{E}$. 4

(b) Explain in detail the probability distribution in canonical ensemble and show that the distribution function (ρ) is given by $\rho(E) = Ae^{-\frac{E}{\tau}}$, where $A = Ce^{\sigma_r(E_t)}$, C = constant, E = energy of the entire system and $\tau = kT$. 6

UNIT—IV

7. *(a)* Show and explain in detail how indistinguishability of particles affects the distribution of particles in B-E and F-D statistics. 3

(b) Write the postulates of Bose-Einstein statistics. Prove that in B-E statistics, the total number of particles in the i-th compartment is $n_i = \frac{g_i}{e^{\alpha+\beta E_i} - 1}$, where g_i is the number of eigenstates, i.e., degeneracy. 7

8. *(a)* What is Fermi energy? Obtain the expression for calculation of Fermi energy in free electrons. 4

(b) Show that according to Fermi-Dirac statistics, the total number of particles in the i-th compartment is given by $n_i = \frac{g_i}{e^{\alpha+\beta E_i} + 1}$, where the symbols have their usual meaning. 6

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