PHY100 (MAJOR) **Student's Copy**

2025

(NEP—2020)

(1st Semester)

PHYSICS (MAJOR)

(Mathematical Physics—I)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. The scalar triple product $\vec{A}\cdot(\vec{B}\times\vec{C})$ represents

(a) the area of the triangle formed by $\vec{B}$ and $\vec{C}$ ( )

(b) the volume of the parallelepiped formed by $\vec{A}$, $\vec{B}$ and $\vec{C}$ ( )

(c) the area of the parallelogram formed by $\vec{B}$ and $\vec{C}$ ( )

(d) the magnitude of $\vec{A}$ ( )

2. If $\nabla \times \vec{F} = 0$ everywhere in a region, then the vector field is

(a) solenoidal ( )

(b) uniform ( )

(c) divergence-free ( )

(d) irrotational ( )

3. A line integral $\int_C \vec{F} \cdot d\vec{r}$ physically represents

(a) flux ( )

(b) work done by a force field along a path ( )

(c) rotation of a vector field ( )

(d) volume of a region ( )

4. In orthogonal curvilinear coordinates (u_1, u_2, u_3), the unit vector $\hat{e}_i$ along coordinate u_i is given by

(a) $\hat{e}_i = \dfrac{\partial \vec{r}}{\partial u_i}$ ( )

(b) $\hat{e}_i = h_i \dfrac{\partial \vec{r}}{\partial u_i}$ ( )

(c) $\hat{e}_i = \dfrac{1}{h_i} \dfrac{\partial \vec{r}}{\partial u_i}$ ( )

(d) $\hat{e}_i = \dfrac{\partial u_i}{\partial \vec{r}}$ ( )

5. If the divergence of a vector field $\vec{A}$ is zero $(\nabla \cdot \vec{A} = 0)$, then the field is said to be

(a) solenoidal ( )

(b) irrotational ( )

(c) conservative ( )

(d) divergent ( )

6. If A is a square matrix, then $A + A^T$ is

(a) symmetric ( )

(b) skew-symmetric ( )

(c) zero ( )

(d) identity ( )

7. $A^{\Theta} A = I$, the square matrix A is called

(a) Hermitian ( )

(b) unitary ( )

(c) orthogonal ( )

(d) symmetric ( )

8. For a Hermitian matrix, the diagonal elements are always

(a) real ( )

(b) complex ( )

(c) zero ( )

(d) equal to their conjugate but not necessarily real ( )

9. The value of $\Gamma(0)$ is

(a) 0 ( )

(b) 1 ( )

(c) 2 ( )

(d) ∞ ( )

10. A function is called odd if

(a) $f(t) = f(-t)$ ( )

(b) $f(t)$ has only cosine terms in its Fourier series ( )

(c) $f(t)$ has only sine terms in its Fourier series ( )

(d) $f(t) = 0$ ( )

(SECTION : B—SHORT ANSWERS)

(*Marks* : 15)

Answer *five* questions, taking at least *one* from each Unit : 3×5=15

UNIT—I

1. Find a unit vector perpendicular to the plane of $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$. Also, find the sine of the angle between $\vec{a}$ and $\vec{b}$.

2. Show that the divergence of curl of $\vec{F}$ is zero.

UNIT—II

3. Convert the rectangular coordinates $(x, y, z) = (2, 2, 4)$ to cylindrical coordinates (ρ, ϕ, z).

4. If u_i, where $i = 1, 2, 3$, are orthogonal curvilinear coordinates, then prove that $|\vec{\nabla} u_i| = \frac{1}{h_i}$.

UNIT—III

5. If A and B are Hermitian, then show that $AB + BA$ is Hermitian and $AB - BA$ is skew-Hermitian.

6. Prove that $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

UNIT—IV

7. Show that $\Gamma(n+1) = n!$

8. Obtain the complex form of Fourier series for the function $f(x) = x$ in $-\pi < x < \pi$.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer *five* questions, taking at least *one* from each Unit : 10×5=50

UNIT—I

1\. (a) Using vector method, show that the points $A(2, -1, 3)$, $B(4, 3, 1)$ and $C(3, 1, 2)$ are collinear. 5

(b) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, 1)$ in the direction of $2\hat{i} + \hat{j} + 2\hat{k}$. 5

2\. (a) Show that the vector $\frac{\vec{r}}{r^3}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $r = |\vec{r}|$, is both solenoidal and irrotational. 5

(b) State Green's theorem. Using Green's theorem, evaluate the integral

$$\oint_C [x^2 dx + (x + y^2)dy]$$

where C is the closed curve given by $y = 0$, $y = x$ and $y^2 = 2 - x$ in the first quadrant, oriented counterclockwise. 1+4=5

UNIT—II

3\. (a) Show that the cylindrical coordinate system is orthogonal. 6

(b) Express the divergence of $\vec{F}$, where $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$ in terms of spherical polar coordinate system. 4

4\. (a) Show that in orthogonal curvilinear coordinates (u_1, u_2, u_3), the gradient of a scalar field f is

$$\nabla f = \frac{1}{h_1}\frac{\partial f}{\partial u_1}\hat{e}_1 + \frac{1}{h_2}\frac{\partial f}{\partial u_2}\hat{e}_2 + \frac{1}{h_3}\frac{\partial f}{\partial u_3}\hat{e}_3$$

where h_1, h_2 and h_3 are the scale factors. 6

(b) Verify that curl(grad ϕ) = 0 for any orthogonal system. 4

UNIT—III

5. (a) Find the inverse of the matrix

$$A = \begin{bmatrix} 3 & -10 & -1 \\ -2 & 8 & 2 \\ 2 & -4 & -2 \end{bmatrix}$$ 6

(b) Show that the product of two orthogonal matrices is also orthogonal. 4

6. (a) Determine the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$ 6

(b) Diagonalize the matrix

$$A = \begin{bmatrix} \frac{3}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{3}{2} & 0 \\ 0 & 0 & 3 \end{bmatrix}$$ 4

UNIT—IV

7. (a) Show that $\beta(m, n) = \dfrac{\Gamma(m) \cdot \Gamma(n)}{\Gamma(m+n)}$. 5

(b) Prove that $\int_0^{\pi/2} \dfrac{d\theta}{\sqrt{\sin\theta}} \times \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta = \pi$. 5

8. (a) Find the Fourier series in $(-\pi, \pi)$ to represent the function $f(x) = x - x^2$ and deduce that

$$\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$$ 4+2=6

(b) Obtain the complex form of Fourier series for the function $f(x) = e^{-x}$ in $-1 < x < 1$. 4

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