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(CBCS)

(6th Semester)

PHYSICS

NINTH PAPER

(Quantum Mechanics)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Put a Tick (✓) mark against the correct answer in the brackets provided : 1×10=10

1. The value of the de Broglie wavelength of an electron having kinetic energy of 9 eV is nearly

(a) 4 Å ( )

(b) 1·36 Å ( )

(c) 0·4 Å ( )

(d) 13·6 Å ( )

2. Which of the following conditions cannot be satisfied by a well-behaved wave function ?

(a) must be finite for all values of x, y and z ( )

(b) must be single-valued at each point (x, y, z) ( )

(c) must be continuous for all regions ( )

(d) must be a real function of x, y, z, t ( )

3. The quantum mechanical tunnelling provides explanation for the following physical phenomena, *except*

(a) the emission of alpha particles from a radioactive nucleus ( )

(b) the motion of electrons inside an atom ( )

(c) the electrical breakdown of insulators ( )

(d) the switching action of a tunnel diode ( )

4. If the operator $\hat{A} \quad \frac{d^2}{dx^2}$ operates on the eigenfunction $(x) \quad \sin 2x$, the eigenvalue is

(a) 1 ( ) *(b)* 2 ( )

(c) 4 ( ) *(d)* 4 ( )

5. The total number of energy levels (or degeneracy) for the nth state of hydrogen atom is

(a) n ( ) *(b)* $n \quad 1$ ( )

(c) n^2 ( ) *(d)* $n^2 \quad 1$ ( )

6. The momentum eigenvalue for a particle trapped in cubical box of side a in the ground state (1, 1, 1) is

(a) $\frac{3 \quad \hbar}{a}$ ( ) *(b)* $\frac{\sqrt{3} \quad \hbar}{a}$ ( )

(c) $\frac{6 \quad \hbar}{a}$ ( ) *(d)* $\frac{\sqrt{6} \quad \hbar}{a}$ ( )

7. The Bohr magneton is defined as the magnetic dipole moment associated with an atom due to

(a) orbital motion of an electron in the first stationary orbit ( )

(b) orbital motion of an electron in the first excited state ( )

(c) orbital motion of an electron in presence of magnetic field ( )

(d) electron spin ( )

8. Eigenvectors of $_x \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix}$ are

(a) $\begin{matrix} 1 \\ 0 \end{matrix}, \begin{matrix} 0 \\ 1 \end{matrix}$ ( )

(b) $\begin{matrix} 1 \\ i \end{matrix}, \begin{matrix} 1 \\ i \end{matrix}$ ( )

(c) $\begin{matrix} 1 \\ 1 \end{matrix}, \begin{matrix} 1 \\ 1 \end{matrix}$ ( )

(d) $\begin{matrix} 1 \\ 0 \end{matrix}, \begin{matrix} 1 \\ 1 \end{matrix}$ ( )

9. If $|\ \rangle$ and $|\ \rangle$ are the vectors in linear vector spaces and a, b are arbitrary complex numbers, then $\langle a \ | b \ \rangle$ is equal to

(a) $ab\langle \ | \ \rangle$ ( )

(b) $a \ b\langle \ | \ \rangle$ ( )

(c) $a \ b \ \langle \ | \ \rangle$ ( )

(d) $ab \ \langle \ | \ \rangle$ ( )

10. If $|\ _m\rangle$ and $|\ _n\rangle$ be two eigenvectors having eigenvalues $_m$ and $_n$ corresponding to the operator $\hat{}$, then

(a) $\langle \ _m | \ _n \rangle \ 0$ ( )

(b) $\langle \ _m | \ _n \rangle \ 1$ ( )

(c) $\langle \ _m | \ _n \rangle \ 1$ ( )

(d) $\langle \ _m | \ _n \rangle \ 0$ ( )

SECTION—B

(*Marks* : 15)

Write short answers to the following questions : 3×5=15

1. Explain how classical theories could not explain the blackbody radiation spectra and how this problem was resolved by Max Planck.

OR

2. Obtain the expression for de Broglie wavelength for a charged particle accelerated under a potential difference V.

3. Show that the momentum operator $\hat{p}_x \quad i\hbar\frac{d}{dx}$ is a Hermitian operator.

OR

4. Show that the total energy operator is given by $\hat{E} \quad i\hbar\frac{}{t}$.

5. What is the significance of zero-point energy (E_0) in linear harmonic oscillator? Express energy eigenvalues (E_n) in terms of E_0.

OR

6. Show that the product of two Hermitian operators is Hermitian if and only if they commute.

7. Explain the concept of electron spin as introduced by Uhlenbeck and Goudsmit.

OR

8. Explain Stern-Gerlach experiment in brief.

9. Show that the three vectors $a \quad (1, 2, 3)$, $b \quad (3, \quad 1, 1)$ and $c \quad (1, 1, \quad 2)$ in R^3 space are linearly independent.

OR

10. What do you mean by 'norm' of a vector? What is a 'orthonormal basis set'?

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

1. *(a)* Describe photoelectric effect. Explain the limitations of classical wave theory to explain it. Also deduce Einstein's photoelectric equation. 2+2+2=6

(b) What do you mean by 'group wave'? Find the relation between 'group velocity' and 'phase velocity'. Hence show that for a non-dispersive medium, group velocity and phase velocity are equal. 1+2+1=4

OR

2. *(a)* Describe Davisson-Germer experiment on electron diffraction. What inferences you draw from this experiment? 5+1=6

(b) State and explain Heisenberg's uncertainty principle. Express uncertainty relation in terms of energy and time. 2+2=4

3. *(a)* Obtain Schrödinger's one-dimensional time-independent wave equation. 4

(b) A particle of energy E is incident on a step potential of height V_0. Show that for $E \quad V_0$, there is a certain probability of being reflected as well as being transmitted and $R \quad T \quad 1$, where R is reflection coefficient and T is transmission coefficient. 6

OR

4. Find the normalized wave function for a particle moving in a one-dimensional rigid box. Also find the energy eigenvalues and show that the maximum probability of finding the particle is at $a / 2$, where a is the size of the box. 6+2+2=10

5. *(a)* Show that $[\hat{p}_x, \hat{x}] \quad i\hbar$, where $\hat{x}$ and $\hat{p}$ are position and momentum operators respectively. What is the physical significance of this relation? 3+1=4

(b) Show that the eigenvalues of Hermitian operator are real. 4

(c) If $\hat{A}$ and $\hat{B}$ are Hermitian operators, show that $[\hat{A}, \hat{B}]$ is skew-Hermitian operator. 2

OR

6. *(a)* A free particle of mass m moves in a three-dimensional rectangular potential box of sides a, b and c parallel to x-, y- and z-axes respectively. Derive an expression for its normalized wave function and show that the energy eigenvalue for its ground state is given by

$$E_{1,\,1,\,1} \quad \frac{^2\hbar^2}{2m} \quad \frac{1}{a^2} \quad \frac{1}{b^2} \quad \frac{1}{c^2}$$

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(b) If the potential box is cubical each of side a, then show that the first excited state is three-fold degenerate. Find the energy eigenvalue for this state. 2+1=3

7. (a) Starting from the Cartesian components of linear momentum operators, find the Cartesian components of angular momentum operator. 3

(b) Show that *(i)* $[L_y, L_z] \quad i\hbar L_x$ and *(ii)* $[L^2, L_x] \quad 0$, where the symbols have their usual meanings. 3+2=5

(c) Suppose we measure the magnitude of angular momentum of a system and find the value $L^2 \quad 6\hbar^2$. How many orientations of $\vec{L}$ are there with respect to z-axis? What are the corresponding values of L_z? 2

OR

8. (a) How are the Cartesian components of spin operators $(\hat{S}_x, \hat{S}_y, \hat{S}_z)$ related to their respective Pauli spin operators $(\hat{\ }_x, \hat{\ }_y, \hat{\ }_z)$? Write their corresponding eigenvalues. 2

(b) Show that—

(i) $[\ _x, \ _y] \quad 2i \ _z$;

(ii) $\ _z \ _x \quad \ _x \ _z \quad 0.$ 2+2=4

(c) Taking the value of Pauli spin matrix as

$$_z \quad \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array}$$

find the values of matrices $\ _x$ and $\ _y$. 4

9. (a) What do you mean by a set of n-vectors $\vec{a}_1, \vec{a}_2, \cdots, \vec{a}_r$ is linearly independent? 2

(b) Check whether the following sets of vectors form the basis set in R^3 vector space : 4

$$(2,\ 0,\ \quad 2),\ (0,\ 2,\ 0),\ (2,\ 0,\ 2)$$

(c) Use the Gram-Schmidt process to transform the basis vectors $u_1 \quad (1, 1, 1)$, $u_2 \quad (\; 1, 1, 0)$ and $u_3 \quad (1, 2, 1)$ into an orthogonal basis (v_1, v_2, v_3). 4

OR

10. *(a)* What are the conditions to be satisfied by a set of *n*-vectors to form the basis set in an *n*-dimensional vector space? 2

(b) Consider the following four elements from the vector space of real 2   2 matrices :

$$|1\rangle \quad \begin{matrix} 1 & 0 \\ 0 & 0 \end{matrix} \qquad |2\rangle \quad \begin{matrix} 0 & 1 \\ 0 & 0 \end{matrix} \qquad |3\rangle \quad \begin{matrix} 0 & 0 \\ 1 & 0 \end{matrix} \qquad |4\rangle \quad \begin{matrix} 0 & 0 \\ 0 & 1 \end{matrix}$$

Show whether they form a basis set or not. 4

(c) If $| \quad_m\rangle \quad 2|u_1\rangle \quad 3|u_2\rangle \quad i|u_3\rangle$ and $| \quad_n\rangle \quad 3|u_1\rangle \quad i|u_2\rangle \quad 4|u_3\rangle$, find $| \quad_m\rangle$, $| \quad_n\rangle$, $\langle \quad_m| \quad_n\rangle$ and $\langle \quad_n| \quad_m\rangle$. 4

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