PHY/V/CC/09 **Student's Copy**

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(CBCS)

(5th Semester)

PHYSICS

FIFTH PAPER

(Mathematical Physics—II)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Put a Tick (✓) mark against the correct answer in the brackets provided : 1×10=10

1. If $z \quad e^{i}$, then cos is given by

(a) $\frac{1}{2i} \; z \quad \frac{1}{z}$ ( )

(b) $\frac{1}{2} \; z \quad \frac{1}{z}$ ( )

(c) $\frac{1}{2i} \; z \quad \frac{1}{z}$ ( )

(d) $\frac{1}{2} \; z \quad \frac{1}{z}$ ( )

2. The function $f(z) \quad \dfrac{e^{z}}{z^{2} \quad 4}$ has

(a) two simple poles at $z \quad 2i$ and at $z \quad 2i$ ( )

(b) two simple poles at $z \quad 2$ and at $z \quad 2$ ( )

(c) a simple pole at $z \quad 2$ and a pole of order 2 at $z \quad 2$ ( )

(d) a simple pole at $z \quad 2i$ and a pole of order 2 at $z \quad 2i$ ( )

3. The differential equation $\dfrac{d^{2}y}{dx^{2}} \quad \dfrac{dy}{dx}^{2} \quad y \quad 0$ has

(a) order = 1, degree = 1 ( )

(b) order = 1, degree = 2 ( )

(c) order = 2, degree = 1 ( )

(d) order = 2, degree = 2 ( )

4. The differential equation for which the solution is $y \quad c_1 e^{x} \quad c_2 e^{\ x} \quad 3$, is

(a) $d^{2}y / dx^{2} \quad y \quad 3$ ( )

(b) $d^{2}y / dx^{2} \quad y \quad 3$ ( )

(c) $d^{2}y / dx^{2} \quad y$ ( )

(d) $d^{2}y / dx^{2} \quad y \quad 6$ ( )

5. Which of the following values of the Hermite polynomial is correct?

(a) $H_0(x) \quad x$ ( )

(b) $H_1(x) \quad 2x^2$ ( )

(c) $H_n(\ x) \quad (\ 1)^n H_n(x)$ ( )

(d) $H_{2n}(0) \quad 0$ ( )

6. Which of the following values of the Legendre polynomial is not correct?

(a) $P_n(1) \quad 1$ ( )

(b) $P_n(\ 1) \quad (\ 1)^n$ ( )

(c) $P_n(\ x) \quad (\ 1)^n P_n(x)$ ( )

(d) $P_{2n}(\ x) \quad P_{2n}(x)$ ( )

7. If $g(k)$ be the Fourier transform of $f(x)$, then Fourier transform of $f(ax)$ is

(a) $\frac{1}{a} g \ \frac{k}{a}$ ( )

(b) $a g \ \frac{k}{a}$ ( )

(c) $\frac{1}{a} g(ak)$ ( )

(d) $g \ \frac{k}{a}$ ( )

8. For an odd function, the Fourier series can be expressed as

(a) $f(x) \quad a_0 \quad \underset{n \quad 1}{} a_n \cos nx$ ( )

(b) $f(x) \quad \underset{n \quad 1}{} a_n \cos nx$ ( )

(c) $f(x) \quad \underset{n \quad 1}{} b_n \sin nx$ ( )

(d) None of the above ( )

9. If $F(s)$ is the Laplace transform of $f(t)$, then the Laplace transform of $t^2 f(t)$ is

(a) $\frac{dF}{ds}$ ( )

(b) $\frac{d^2F}{ds^2}$ ( )

(c) $\quad \frac{d^2F}{ds^2}$ ( )

(d) $s\frac{dF}{ds}$ ( )

10. $L^{\ 1} \frac{1}{(s \quad 1)^3}$ is equal to

(a) $e^{\ t} \frac{t^2}{2!}$ ( )

(b) $e^{t} \frac{t^2}{2!}$ ( )

(c) $e^{\ t} \frac{t}{2!}$ ( )

(d) $e^{2} \frac{t^2}{2!}$ ( )

SECTION—B

(*Marks* : 15)

Write short answers to the following questions : 3×5=15

1. Show that $f(z) = z^2$, where $z = x + iy$ is analytic function and satisfies the Cauchy-Riemann conditions.

OR

2. Show that

$$\oint_C \frac{dz}{z} = \begin{cases} 0 & \text{, if } C \text{ does not enclose the origin} \\ 2\pi i & \text{, if } C \text{ encloses the origin} \end{cases}$$

3. Show that the solution of the differential equation $(1 + x^2)y'' + 2xy' = 0$ is $y = a\tan^{-1} x + b$, where a and b are constants.

OR

4. Show that the solution of the differential equation $y'' + 4y - 4b = 0$ is $y = a\sin 2x + b$, where a and b are constants.

5. For the Bessel function $J_n(x)$, show that $J_{-n}(x) = (-1)^n J_n(x)$, $n = 0, 1, 2, \cdots$.

OR

6. Prove that for the Legendre function $P_n(1) = 1$.

7. If $f(x)$ is an even function, then show that the Fourier series can be written as

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx$$

OR

8. Find the infinite Fourier sine transform of $f(x) = \dfrac{e^{-ax}}{x}$.

9. Show that Laplace transform of the function $(\sin at \quad at \cos at)$ is $\dfrac{2as^2}{(s^2 \quad a^2)^2}$.

OR

10. Using Laplace transform, solve the equation $x \qquad {}^2 x \quad 0$ with $x(0) \quad C_1$ and $x(0) \quad C_2$.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

1. *(a)* State and prove Cauchy's integral formula. 1+3=4

(b) Using Cauchy's integral formula, evaluate the following integrals : 3+3=6

(i) $\oint_C \dfrac{4 \quad 3z}{z(z \quad 1)(z \quad 2)} dz$, where for circle C, $|z| \quad 3/2$

(ii) $\oint_C \dfrac{2 \quad z}{z(2 \quad z)} dz$, where for circle C, $|z| \quad 1$

OR

2. *(a)* Find the residue of $f(z) \quad \dfrac{z}{(z \quad 1)(z \quad 1)^2}$ at all the singularities. 3

(b) Use residue theorem to evaluate $\int_0^2 \dfrac{d}{a \quad b\cos}$; $a \quad b \quad 0$. Hence show that $\int_0^2 \dfrac{d}{2 \quad \cos} \quad \dfrac{2}{\sqrt{3}}$. 6+1=7

3. *(a)* Show that $x \quad 0$ is an irregular singular point and $x \quad 2$ is a regular singular point of the following differential equation : 2+2=4

$$x^3(x \quad 2)\frac{d^2y}{dx^2} \quad (x \quad 2)\frac{dy}{dx} \quad 3xy \quad 0$$

(b) Determine the series solution for the following differential equation about $x_0 \quad 0$: 6

$$y \quad y \quad 0$$

OR

4. *(a)* Show that the point of infinity is a regular singular point of the equation $x^2 y \quad (3x \quad 1) y \quad 3y \quad 0$. 4

(b) Solve the two-dimensional Laplace's equation in polar coordinates by the method of separation of variables. 6

5 *(a)* Show that when n is a positive integer, $J_n(x)$ is the coefficient of Z^n in the expansion of $e^{x(Z \quad \frac{1}{Z})/2}$ in ascending and descending powers of Z. 4

(b) For Hermite polynomials $H_n(x)$, prove the following : 3+3=6

(i) $2nH_{n \quad 1}(x) \quad H_n(x)$

(ii) $2nH_n(x) \quad 2nH_{n \quad 1}(x) \quad H_{n \quad 1}(x)$

OR

6. *(a)* Prove the following recurrence relations for Legendre polynomials : 4+4=8

(i) $nP_n \quad (2n \quad 1)xP_{n \quad 1} \quad (n \quad 1)P_{n \quad 1}$

(ii) $(2n \quad 1)P_n \quad P_{n \quad 1} \quad P_{n \quad 1}$

(b) Show that for the Bessel's function

$$J_{\frac{1}{2}}(x) \quad \sqrt{\frac{2}{x}} \cos x \qquad 2$$

7. *(a)* Find the Fourier series expansion of the periodic function of period 2 :

$$f(x) \quad x^2, \qquad x$$

Hence find the sum of the series $\frac{1}{1^2} \quad \frac{1}{2^2} \quad \frac{1}{3^2} \quad \frac{1}{4^2} \quad \cdots$. 5+1=6

(b) Find the infinite Fourier transform of $f(x)$ defined by $f(x) \quad \begin{matrix} 1, & |x| \quad 1 \\ 0, & |x| \quad 1 \end{matrix}$

and hence show that $\int_0 \frac{\sin x}{x} dx \quad \frac{}{2}$. 2+2=4

OR

8. *(a)* Discuss the application of Fourier series in full-wave rectifier and hence show that the lowest frequency term of the Fourier series has a frequency twice the frequency of the source. 6

(b) Show that the finite cosine transform of $f(x) \quad \frac{\quad}{3} \quad x \quad \frac{x^2}{2}$ in the interval $(0, \quad)$ is $\frac{1}{n^2}$. 4

9. *(a)* Find the Laplace transforms of the following functions : 2×3=6

(i) $f(t) \quad \sqrt{t}$

(ii) $f(t) \quad t^2 \cos 2t$

(iii) $f(t) \quad e^{\quad 4t} \frac{\sin 3t}{t}$

(b) Using Laplace transforms, find the solution of the differential equation : 4

$$y \quad 25y \quad 10\cos 5t \quad \text{with} \quad y(0) \quad 2, \quad y(0) \quad 0$$

OR

10. *(a)* Using Laplace transform, show that—

(i) $\int_0 t^2 e^{\quad t} \sin t\, dt \quad \frac{1}{2}$

(ii) $\int_0 \frac{\sin t}{t} dt \quad \frac{\quad}{2}$ 2+2=4

(b) Applying Cauchy's residue theorem, show that $L^{\quad 1} \frac{a}{s^2 \quad a^2} \quad \sinh at$. 3

(c) Use convolution theorem to find the functions whose Laplace transform is $\frac{s^2}{(s^2 \quad a^2)^2}$. 3

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