PHY/V/06 (R) **Student's Copy**

2018

(Pre-CBCS)

(5th Semester)

PHYSICS

SIXTH PAPER

(Quantum Mechanics—II)

(Revised)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. Conservation of probability in quantum mechanics is represented by the equation

(a) $\frac{\quad}{t} \quad \vec{} \ \vec{J} \quad 0$ ( ) (b) $\frac{\quad}{t} \quad \vec{} \ \vec{J} \quad 0$ ( )

(c) $\frac{\quad}{t} \quad \vec{} \ \vec{P} \quad 0$ ( ) (d) $\frac{\quad}{t} \quad \vec{} \ \vec{P} \quad 0$ ( )

2. Let be a wave function, the quantity represents

(a) probability density ( ) (b) charge density ( )

(c) energy density ( ) (d) wave intensity ( )

3. Let E_3 be energy of the third energy level of a free particle in one-dimensional infinite potential well. The relation between first energy level E_1 and third energy level E_3 is

(a) $E_3 \quad 3E_1$ ( ) (b) $E_3 \quad E_1$ ( )

(c) $E_1 \quad 9E_3$ ( ) (d) $E_3 \quad 9E_1$ ( )

4. For a free particle in step potential, let R and T be reflectance and transmittance, then

(a) $R \quad T \quad 1$ ( ) *(b)* $R \quad T$ ( )

(c) $R \quad T \quad 1$ ( ) *(d)* $R.T \quad 1$ ( )

5. The ground state energy of hydrogen atom is

(a) –16·3 eV ( ) *(b)* –13·6 eV ( )

(c) –1·36 eV ( ) *(d)* –64·1 eV ( )

6. Which of the following is not permissible set of quantum numbers for electrons in an atom?

(a) $n \quad 4, \; l \quad 0, \; m \quad 0, \; s \quad 1/2$ ( )

(b) $n \quad 5, \; l \quad 3, \; m \quad 0, \; s \quad 1/2$ ( )

(c) $n \quad 3, \; l \quad 2, \; m \quad 3, \; s \quad 1/2$ ( )

(d) $n \quad 3, \; l \quad 2, \; m \quad 2, \; s \quad 1/2$ ( )

7. Two kets $|a\rangle$ and $|b\rangle$ are orthonormal, if

(a) $\langle b|b\rangle \quad 0, \; \langle a|b\rangle \quad 0, \; \langle a|a\rangle \quad 1$ ( )

(b) $\langle b|b\rangle \quad 1, \; \langle a|b\rangle \quad 0, \; \langle a|a\rangle \quad 1$ ( )

(c) $\langle b|b\rangle \quad 1, \; \langle a|b\rangle \quad 1, \; \langle a|a\rangle \quad 1$ ( )

(d) $\langle b|b\rangle \quad 1, \; \langle a|b\rangle \quad 0, \; \langle a|a\rangle \quad 0$ ( )

8. The energy eigenvalue of a particle of mass m in three, dimensional box $(a \quad b \quad c)$ is

(a) $\dfrac{{}^2\hbar^2}{2m} \quad \dfrac{n_x}{a}{}^2 \quad \dfrac{n_y}{b}{}^2 \quad \dfrac{n_z}{c}{}^2$ ( )

(b) $\dfrac{\hbar^2}{2m} \quad \dfrac{n_x}{a}{}^2 \quad \dfrac{n_y}{b}{}^2 \quad \dfrac{n_z}{c}{}^2$ ( )

(c) $\dfrac{h^2}{2m} \quad \dfrac{n_x}{a}{}^2 \quad \dfrac{n_y}{b}{}^2 \quad \dfrac{n_z}{c}{}^2$ ( )

(d) $\dfrac{{}^2\hbar^2}{2m} \quad \dfrac{n_x}{2a}{}^2 \quad \dfrac{n_y}{2b}{}^2 \quad \dfrac{n_z}{2c}{}^2$ ( )

9. For electron, the number of possible spin states for Z component is

(a) 1 (   ) *(b)* 2 (   )
(c) 3 (   ) *(d)* 4 (   )

10. The commutation $[L_x, \ p_x]$ equals

(a) 0 (   ) *(b)* $i\hbar$ (   )
(c) $2i\hbar$ (   ) *(d)* $3i\hbar$ (   )

SECTION—B

(*Marks* : 15)

Write short answers to the following questions : 3×5=15

1. Explain complementary principles.

2. Show that $[x, \ p_x^n] \quad ni\hbar p_x^{n \quad 1}$, where x is position operator, p_x is x component of momentum operator.

3. What are the three quantum numbers associated with wave functions of a hydrogen atom? Give their significances.

4. Let $| \ \rangle \quad |u_1\rangle \quad 2|u_2\rangle \quad i|u_3\rangle$ and $| \ \rangle \quad 3|u_1\rangle \quad |u_2\rangle \quad 2|u_3\rangle$. Compute the inner product $\langle \ | \ \rangle$.

5. Show that electron spin magnetic moment is equal to Bohr magneton.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

1. *(a)* Show that de Broglie wavelength for a material particle of rest mass m_0 and charge q accelerated from rest through a potential difference of V volts relativistically is given by

$$\frac{h}{\sqrt{2m_0qV \quad 1 \quad \dfrac{qV}{2m_0qc^2}}}$$

5

(b) Show that material particle can only be represented by a group wave not by a single wave. 5

OR

2. *(a)* What is de Broglie hypothesis? Describe Davisson-Germer experiment for the study of electron diffraction. What are the results of the experiment? 1+6=7

(b) Write four basic postulates of Quantum mechanics. 3

3. A beam of particles of mass m and energy E is incident from the left on a rectangular potential barrier of the form

$$V(x) \quad \begin{matrix} 0 & , & x \quad 0 \\ V_0 & , & 0 \quad x \quad a \\ 0 & , & x \quad a \end{matrix}$$

where V_0 is the height and a is the thickness of the potential barrier. Discuss the solution for $E \quad V_0$ and explain how tunnelling can be understood. Give two examples of quantum tunnelling. 9+1=10

OR

4. *(a)* What do you mean by Hermitian operator? Show that eigenvalues of Hermitian operators are real. 1+4=5

(b) Show that if two Hermitian operators commute, their product is also Hermitian. 5

5. Solve the radial equation

$$\frac{1}{r^2}\frac{}{r} \quad r^2 \frac{R}{r} \qquad \frac{2mE}{\hbar^2} \quad \frac{2mV(r)}{\hbar^2} \quad \frac{l(l \quad 1)}{r^2} \quad R \quad 0$$

of the hydrogen atom, where the symbols have their usual meanings. Show that the energy values are exactly the same as those obtained by Bohr. 8+2=10

OR

6. Obtain the expression for energy eigenvalue of one-dimensional harmonic oscillator. What is zero point energy of harmonic oscillator? 8+2=10

7. *(a)* What are linear vector space and Hilbert space? 3

(b) Describe Gram-Schmidt orthonormalization process. Apply this process for a doubly degenerate system. 4+3=7

OR

8. *(a)* Consider the following two kets

$$| \quad \rangle \quad \begin{matrix} 5i \\ 2 \\ i \end{matrix} \quad \text{and} \quad | \quad \rangle \quad \begin{matrix} 3 \\ 8i \\ 9i \end{matrix}$$

(i) Find $| \quad \rangle$ and $\langle \quad |$.

(ii) Is $| \quad \rangle$ normalized? If not, normalize it.

(iii) Are the two kets orthogonal? 1+2+1=4

(b) Write down the addition and multiplication conditions to be satisfied by a vector space. 2

(c) Consider the state

$$| \quad \rangle \quad 3i|v_1\rangle \quad 7i|v_2\rangle$$
$$| \quad \rangle \quad |v_1\rangle \quad 2i|v_2\rangle$$

where $|v_1\rangle$ and $|v_2\rangle$ are orthonormal vectors :

(i) Calculate $| \qquad \rangle$ and $\langle \qquad |$.

(ii) Show that $\langle \quad | \quad \rangle \quad \langle \quad | \quad \rangle$. 2+2=4

9. *(a)* Show that square of angular momentum commutes with any one of the components of angular momentum, i.e., $[L^2, L_x] \quad 0$. What is the physical meaning of the commutation? 5+1=6

(b) State Uhlenbeck and Goudsmit's hypothesis of electron spin. What are Pauli spin operators? 1+3=4

OR

10. The expression for square of angular momentum is given by

$$L^2 \quad \hbar^2 \frac{1}{\sin} — \sin — \quad \frac{1}{\sin^2} \frac{{}^2}{{}^2}$$

Obtain the eigenvalue of L^2. 10

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