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( Pre-CBCS )

( 5th Semester )

**PHYSICS**

FIFTH PAPER

**( Mathematical Physics—I )**

( Revised )

Full Marks : 75

Time : 3 hours

**( PART : A—OBJECTIVE )**

( Marks : 25 )

*The figures in the margin indicate full marks for the questions*

SECTION—A

( Marks : 10 )

Tick (✓) the correct answer in the brackets provided :

1×10=10

1. The value of  $(1)$  is

(a) 0 ( )

(b) 1 ( )

(c) 1 ( )

(d) ( )

2. The value of  $(1, 3)$  is

(a)  $1/3$  ( )

(b)  $2/3$  ( )

(c)  $1/12$  ( )

(d) None of the above ( )

3. If  $z = e^i$ , then  $\cos z$  is given by

(a)  $\frac{1}{2i} \left( z - \frac{1}{z} \right)$  ( )                      (b)  $\frac{1}{2} \left( z + \frac{1}{z} \right)$  ( )

(c)  $\frac{1}{2i} \left( z + \frac{1}{z} \right)$  ( )                      (d)  $\frac{1}{2} \left( z - \frac{1}{z} \right)$  ( )

4. The function  $f(z) = \frac{e^z}{z^2 - 4}$  has

(a) two simple poles at  $z = 2i$  and at  $z = -2i$  ( )

(b) two simple poles at  $z = 2$  and at  $z = -2$  ( )

(c) a simple pole at  $z = 2$  and a pole of order 2 at  $z = -2$  ( )

(d) a simple pole at  $z = 2i$  and a pole of order 2 at  $z = -2i$  ( )

5. If  $H$  is a Hermitian matrix, then  $e^{iH}$  is a/an

(a) skew-Hermitian matrix ( )

(b) unitary matrix ( )

(c) orthogonal matrix ( )

(d) Hermitian matrix ( )

6. The eigenvalues of the matrix  $\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$  are

(a) 5, 4 ( )                      (b) 4, 1 ( )

(c) 1, 2 ( )                      (d) 4, -1 ( )

7. The scale factors for a cylindrical coordinate system are

(a)  $h_1 = 1, h_2 = r, h_3 = 1$  ( )

(b)  $h_1 = 1, h_2 = r, h_3 = 1$  ( )

(c)  $h_1 = 1, h_2 = r, h_3 = r \sin \theta$  ( )

(d)  $h_1 = 1, h_2 = r, h_3 = \sin \theta$  ( )

8. The number of components of the mixed tensor  $A_k^{ij}$  in a 4-dimensional space is

(a) 4 ( )                      (b) 16 ( )

(c) 64 ( )                      (d) 81 ( )

9. What are the values of  $x$  and  $y$  after executing the following C++ program segment?

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int x = 2, y = 3;
x = x + y;
y = x + y;
x = x + y;
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- (a)  $x = 2, y = 3$  ( )
- (b)  $x = 5, y = 3$  ( )
- (c)  $x = 3, y = 2$  ( )
- (d) None of the above ( )

10. Which of the following is true about C++?

It is a computer language based on

- (a) structured programming ( )
- (b) object-oriented programming ( )
- (c) procedural programming ( )
- (d) sequential programming ( )

SECTION—B

( Marks : 15 )

Answer the following questions :

3×5=15

1. Prove that  $(m, n) = (n, m)$ .
2. Show that  $f(z) = z^2$ , where  $z = x + iy$  is analytic function and satisfies the Cauchy-Riemann conditions.
3. Show that every tensor of rank 2 can be expressed as the sum of symmetric and skew-symmetric tensors of same rank.
4. Prove that the eigenvalues of a Hermitian matrix are real.
5. Mention different forms of data types in C++ and their memory sizes.

( PART : B—DESCRIPTIVE )

( Marks : 50 )

The figures in the margin indicate full marks for the questions

1. (a) Using the definition of  $\beta$ -function, show that  $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ . 4

(b) Show that  $\int_0^1 \frac{x^2 dx}{\sqrt{1-x^4}} = \int_0^1 \frac{dx}{\sqrt{1-x^4}} - \frac{1}{4\sqrt{2}}$ . 6

OR

2. (a) Prove that

$$\beta(m, n) = \int_0^1 \frac{x^{m-1} dx}{(1-x)^{m+n}}$$

Hence show that  $\int_0^1 \frac{x^8(1-x^6) dx}{(1-x)^{24}} = 0$ . 3+2=5

(b) Show that  $\int_0^{\pi/2} \frac{d}{\sqrt{\sin}} = \int_0^{\pi/2} \sqrt{\sin} d$ . 5

3. (a) State and prove Cauchy's integral theorem. 5

(b) Use Cauchy's integral theorem to evaluate  $\oint_C \frac{dz}{z}$ , where C is a simple closed curve. 5

OR

4. (a) State and prove Cauchy's residue theorem. 5

(b) Use Cauchy's residue theorem to show that

$$\int_0^{2\pi} \frac{\cos 3\theta d\theta}{5 - 4\cos\theta} = \frac{\pi}{12}$$
 5

5. (a) Show that any complex square matrix can be expressed as the sum of a Hermitian matrix and a skew-Hermitian matrix. 4

(b) Solve the following simultaneous equations by matrix method : 6

$$\begin{aligned} 2x - 3y + z &= 9 \\ x + y + z &= 6 \\ x + y + z &= 2 \end{aligned}$$

**OR**

6. (a) Show that the matrix  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$  is an orthogonal matrix. Diagonalize the matrix. 2+2=4

(b) Find the inverse of the matrix  $A = \begin{pmatrix} 1 & 3 & 2 \\ 3 & 0 & 5 \\ 2 & 5 & 0 \end{pmatrix}$ . 6

7. (a) Show that cylindrical coordinate system is orthogonal. 5

(b) Show that unit vectors in spherical polar coordinate system are related to unit vectors in Cartesian coordinate system as 5

$$\begin{matrix} \hat{r} & \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta & \hat{i} \\ \hat{\theta} & \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta & \hat{j} \\ \hat{\phi} & -\sin \phi & \cos \phi & 0 & \hat{k} \end{matrix}$$

**OR**

8. (a) Show that (i) velocity is a contravariant vector and (ii) gradient of a scalar function is a covariant vector. 3+3=6

(b) Show that, if  $A^i$  and  $B^j$  are two contravariant vectors, then the  $n^2$  quantities  $C^{ij} = A^i B^j$  are the components of a contravariant tensor of rank 2. 4

9. (a) Mention with symbols, the arithmetic operators, logical operators, relational operators and assignment operators in C++. 6

(b) Write a C++ program to enter two numbers and then print their sum, square of sum and square root of the sum. 4

**OR**

10. (a) With the help of appropriate flowchart diagram, describe how 'if', 'if else' and 'nested if' control statements are executed in C++ programs. 6

(b) Write a C++ program to read the length and breadth of a rectangle and print its area and perimeter. Take both length and breadth as float type data. 4

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