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(6th Semester)

MATHEMATICS

Paper : MATH–364(C)

(Astronomy)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—A

(*Marks* : 10)

Answer **all** questions

Each question carries 1 mark

Tick (✓) the correct answer in the brackets provided :

1. The number of great circles that can be drawn through two given points other than the extremities of a diameter is

(a) infinite ( )

(b) 0 ( )

(c) 1 ( )

(d) 2 ( )

2. The angles of a spherical triangle

(a) is necessarily equal to two right angles ( )

(b) cannot be greater than two right angles ( )

(c) cannot be smaller than two right angles ( )

(d) is necessarily equal to a right angle ( )

3. The duration of twilight at the equator during the equinox is

(a) 1 hour 12 minutes ( )

(b) 1 hour 24 minutes ( )

(c) 2 hours 12 minutes ( )

(d) 2 hours 24 minutes ( )

4. If a star rises tonight at 10 o'clock, at what time (approximately) will it rise 30 days hence?

(a) 6 o'clock ( )

(b) 8 o'clock ( )

(c) 10 o'clock ( )

(d) 12 o'clock ( )

5. 24 sidereal hours is equal to

(a) $24^h56^m4^s$ mean solar time ( )

(b) $24^h56^m45^s$ mean solar time ( )

(c) $23^h45^m4^s$ mean solar time ( )

(d) $23^h56^m4^s$ mean solar time ( )

6. If the apparent altitude of a star is 60° and the coefficient of refraction is 58 2 , then its true altitude is

(a) 59 59 26 4 ( )

(b) 58 59 26 5 ( )

(c) 58 58 26 4 ( )

(d) 59 58 26 5 ( )

7. The interval between two successive heliocentric conjunctions in longitudes is called

(a) periodic time of a planet ( )

(b) synodic period of a planet ( )

(c) orbital period of a planet ( )

(d) superior time of a planet ( )

8. The points where direct motion changes to retrograde motion and vice versa are called

(a) geocentric points ( )

(b) sidereal points ( )

(c) stationary points ( )

(d) elongation points ( )

9. The point of the apparent orbit of the sun at which the sun is nearest to the earth is called

(a) apogee ( )

(b) perigee ( )

(c) azimuth ( )

(d) eccentricity ( )

10. The angular distance between the visible horizon and the true astronomical horizon is called

(a) elongation of the horizon ( )

(b) latitude of the horizon ( )

(c) refraction of the horizon ( )

(d) dip of the horizon ( )

SECTION—B

(*Marks* : 15)

Answer **all** questions

Each question carries 3 marks

1. Show that in a spherical triangle *ABC*, where *AB* c, *BC* a, *CA* b

$$\sin c \cos B \quad \sin a \cos b \quad \cos a \sin b \cos C$$

2. A ship starts from a point on the equator and sails in great circle, cutting the equator at an angle of 45°. Find how much it has changed its longitude when it has reached a latitude of $\tan^{1} \frac{1}{2}$.

3. The horizontal parallax of the moon is 57 and her angular diameter is 31 5 . Find the diameter of the moon in kilometers assuming the radius of the earth is 6400 km.

4. If the line joining two planets to one another subtends an angle of 60° at the sun when the planets appear to each other to be stationary, show that $a^2 \quad b^2 \quad 7ab$, where a and b are the distances of the planets from the sun.

5. If T is the orbital period of a planet, using Kepler's third law, show that a small increase a in the semi-major axis a will produce an increase of $\frac{3T}{2a}$ a in the period.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

Answer *any* **one** question from each Unit

Each question carries 10 marks

UNIT—1

1. *(a)* Show that in a spherical triangle *ABC*, where *AB* *c*, *BC* *a*, *CA* *b*

$$\frac{\sin(a \quad b)}{\sin c} \quad \frac{\cos A \quad \cos B}{1 \quad \cos C}$$

5

(b) Prove that in a spherical triangle *ABC* right angled at *C*

$$\sin(A \quad B) \quad \frac{\cos b \quad \cos a}{1 \quad \cos b \cos a}$$

5

2. *(a)* Show that the sides and angles of a polar triangle are respectively supplements of the angles and sides of the primitive triangle. 5

(b) In a spherical triangle *ABC*, if angle *C* 120 and if the arc of a great circle drawn through *C* to meet *AB* at right angle is $\tan^{1} \frac{\sqrt{3}}{2}$, show that

$$\cot^2 a \quad \cot^2 b \quad \cot a \cot b \quad 1$$

5

UNIT—2

3. *(a)* Write short notes on the following : 2+2=4

(i) Twilight

(ii) Hour angle

(b) If the day is considered to begin and end when the sun is at angle below the horizon, show that the shortest day will not occur at the winter solstice if the latitude is less than , where sin sin sin and is the obliquity of the ecliptic. 6

4. *(a)* If at an equinox, the duration of twilight at a place is h hours, show that the latitude of the place is

$$\cos^{-1} \sin 18 \operatorname{cosec} \frac{h}{12}$$ 6

(b) If H is the hour angle of a star at rising, show that

$$\tan^2 \frac{H}{2} \quad \frac{\cos(\quad)}{\cos(\quad)}$$ 4

UNIT—3

5. *(a)* Two stars ($_1$, $_1$) and ($_2$, $_2$) have the same longitude. If is the obliquity of the ecliptic, prove that

$$\sin(\ _1 \quad \ _2) \quad \tan \ (\cos \ _1 \tan \ _2 \quad \cos \ _2 \tan \ _1)$$ 6

(b) If the declination of a star is unaffected by refraction at a given moment, prove that azimuth is then a minimum. 4

6. *(a)* Prove that the angular distance between two stars whose ecliptic coordinates are ($_1$, $_1$) and ($_2$, $_2$) is unaltered by aberration when the sun's longitude $\odot$ is given by

$$\cos \ _1 \sin(\odot \quad \ _1) \quad \cos \ _2 \sin(\odot \quad \ _2) \quad 0$$ 7

(b) Prove that the equation of time vanishes four times a year. 3

UNIT—4

7. *(a)* If a and b are the radii of the orbits assumed to be circular and coplanar of the earth E and a superior planet P and u and v are their respective linear velocities, prove that the square of the velocity of P relative to E at a stationary point is

$$\frac{(u^2 \quad v^2)(bu \quad av)}{bu \quad av}$$ 7

(b) Write short notes on direct and retrograde motion of a planet. 3

8. *(a)* Assuming that venus and earth describe circular orbits in the ecliptic, show that venus will appear brightest at an elongation given by

$$\cos \quad \frac{2}{3} \ (3 \quad a^2)^{\frac{1}{2}} \quad a$$

where a is the heliocentric distance of venus in astronomical unit. 5

(b) If x be the phase of the moon as seen from the earth and y be the phase of the earth as seen from the moon, prove that approximately

$$y \quad 2 \quad x \quad \frac{b}{a}(2x \quad x^2)$$

where the phase of full moon is 2 and a and b are respectively the radii of the orbits of the earth and the moon. 5

UNIT—5

9. State the three laws of Kepler. Deduce Kepler's third law from Newton's law of universal gravitation. 3+7=10

10. *(a)* The path of the rising sun makes an angle with the horizon. Prove that an observer whose height above the sea level is $\frac{1}{n}$ of the earth's radius will observe the time of sunrise $\frac{12\sqrt{2}\,\text{cosec} \quad \sec}{\sqrt{n}}$ hours earlier than an observer at sea level, where is the declination of the sun. 6

(b) Discuss the effect of the dip of the horizon in the rising and setting of a star. 4

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