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(6th Semester)

MATHEMATICS

Paper : Math–363

(Mechanics)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. The equation of the resultant of any number of coplanar forces acting on a rigid body is given by

(a) $xR_x \quad yR_y \quad G$ ( )

(b) $xR_y \quad yR_x \quad G$ ( )

(c) $G \quad yR_y \quad xR_x \quad 0$ ( )

(d) $G \quad yR_x \quad xR_y \quad 0$ ( )

2. If a body be on a point of sliding down on an inclined plane under its own weight, then

(a) $P \quad W \sin (\quad)$ ( )

(b) $P \quad W \sin (\quad)$ ( )

(c) ( )

(d) $P \quad W \sin (\quad)$ ( )

3. The centre of gravity of a circular arc of radius 4 cm subtending at an angle 90° lies on the axis of symmetry at a distance of

(a) $2\sqrt{2}$ from the centre ( )

(b) $4\sqrt{2}$ from the centre ( )

(c) $6\sqrt{2}$ from the centre ( )

(d) $8\sqrt{2}$ from the centre ( )

4. If the moments of inertia of a body about its three mutually perpendicular axes be A, B and C, then its moments of inertia about a line through O, having direction cosines , , is

(a) $A \ ^2 \quad B \ ^2 \quad C \ ^2$ ( )

(b) $A^2 \quad B^2 \quad C^2$ ( )

(c) $A^2 \ ^2 \quad B^2 \ ^2 \quad C^2 \ ^2$ ( )

(d) $A \quad B \quad C$ ( )

5. The position of a moving particle, starting at a distance x 3 m at t 0 from a fixed point with initial velocity 5 m/s, after 10 seconds, is (given that f 9 8 m/s^2)

(a) 513 m from the fixed point ( )

(b) 523 m from the fixed point ( )

(c) 533 m from the fixed point ( )

(d) 543 m from the fixed point ( )

6. The rate of change of velocity of a particle moving in cycloid is constant. Then

(a) the tangential acceleration is constant ( )

(b) the normal acceleration is constant ( )

(c) the resultant acceleration is constant ( )

(d) None of the above ( )

7. If a body is projected in a vertical plane with a velocity u and angle of projection , then the height of the directrix of the parabolic path above the plane of projection is

(a) $\frac{u}{2g}$ ( )

(b) $\frac{u^2}{2g}$ ( )

(c) $\frac{g}{2u}$ ( )

(d) $\frac{g^2}{2u}$ ( )

8. If the equation of motion of a body falling under gravity in a resisting medium is $v\frac{dv}{dx} \quad g \quad kv^2$, then the terminal velocity is

(a) the greatest velocity attained ( )

(b) the least velocity attained ( )

(c) the initial velocity ( )

(d) the velocity when the acceleration is greatest ( )

9. A smooth ball falling vertically from a height x impinges on a horizontal fixed plane. If e is the coefficient of restitution, then the ball rebounds to a height

(a) x ( )

(b) ex ( )

(c) ex^2 ( )

(d) e / x^2 ( )

10. A smooth sphere impinges directly with velocity u on another smooth sphere of equal mass at rest, if the spheres are perfectly elastic, the velocity of the 2nd sphere after collision will be

(a) u ( )

(b) $2u$ ( )

(c) eu ( )

(d) 0 ( )

SECTION—B

(*Marks* : 15)

Answer the following questions : 3×5=15

1. Find the least force required to pull up a body of weight 100 kg on an inclined plane with inclination of 60° with the horizontal. Given that the angle of friction is 30°.

2. Find the CG of the system of 3 particles of masses 1 kg, 3 kg and 4 kg placed at the points (0, 4, 4), (4, 0, 4) and (4, 4, 0) respectively.

3. The velocity of a point moving in a plane curve varies as the radius of curvature. Show that the direction of motion revolves with constant angular velocity.

4. If a particle is projected in all directions under gravity with a given velocity u, then prove that after time t all the particles lie on a circle of radius ut.

5. Prove that the rate of change of kinetic energy of a particle moving in a straight line is equal to its power.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Prove that a system of forces acting in one plane at different points of a rigid body can be reduced to a single force through a given point, and a couple. 5

(b) Forces proportional to 1, 2, 3 and 4 act along the sides *AB*, *BC*, *AD* and *DC* respectively of a square *ABCD*, the length of whose side is 2 ft. Find the magnitude and line of action of the resultant. 5

2. *(a)* A uniform rod rests on a fixed smooth sphere with its lower end pressing against a smooth vertical wall which touches the sphere. If be the angle which the rod makes with the vertical at equilibrium, then prove that

$$a \quad 2l \sin\frac{}{2} \cos^3\frac{}{2}$$

where l is the length of the rod and a is the radius of the sphere. 5

(b) A uniform ladder rests in limiting equilibrium with its upper end against a smooth wall. If be the inclination of the ladder to the vertical, prove that tan 2 , where is the coefficient of friction. 5

UNIT—II

3. *(a)* State and prove parallel axes theorem on moments of inertia. 5

(b) A thin uniform wire is bent into the form of a triangle *ABC*. Prove that its CG is the same as that of the three weights $\frac{b \quad c}{2}$, $\frac{c \quad a}{2}$ and $\frac{a \quad b}{2}$ placed at *A*, *B* and *C* respectively, where *a*, *b* and *c* are the lengths of the sides *BC*, *CA* and *AB* repectively. 5

4. *(a)* Find the centre of gravity of a uniform circular arc subtending an angle 2 at the centre. 5

(b) Let AB and AC are two uniform rods of length $2a$ and $2b$ respectively. If BAC , then prove that the distance of the CG from A of the two rods is

$$\frac{(a^4 \quad 2a^2b^2 \cos \quad b^4)^{\frac{1}{2}}}{a \quad b}$$ 5

UNIT—III

5. *(a)* Prove that the radial and transverse components of acceleration for a particle moving along a plane curve are $r \quad r^{2}$ and $\frac{1}{r}\frac{d}{dt}(r^2 \;)$. 5

(b) A point moves in a plane so that the tangential and normal accelerations are equal and angular velocity of the tangent is constant and equal to K. Show that the path is equiangular spiral $Ks \quad Ae \quad B$, where A and B are constant. 5

6. *(a)* The greatest possible acceleration of a train is 1 m/s^2 and the greatest possible retardation is $\frac{4}{3}$ m/s^2. Find the least time taken by the train to run two stations 12 km apart if the maximum speed of the train be 22 m/s. 5

(b) Show that a particle executing SHM requires $\frac{1}{6}$ of its period to move from the position of maximum displacement to one in which the displacement is half the amplitude. 5

UNIT—IV

7. *(a)* If be the angle between the tangents at extremities of any arc of the parabolic path, v and v are the velocities at those extremities and u is the constant horizontal velocity, then show that the time of describing the arc is $\frac{vv \ \sin}{ug}$. 5

(b) A particle is projected from a point on the ground level and its height is h when it is at horizontal distance a and $2a$ from its point of projection. Prove that the velocity of projection u is given by

$$u^2 \quad \frac{g}{4} \frac{4a^2}{h} \quad 9h$$

5

8. (a) For a given velocity of projection the maximum range down an inclined plane is three times the range up the inclined plane. Show that the inclination of the plane to the horizontal is 30°. 5

(b) A small stone of mass m, is thrown vertically upward with initial speed u. If the air resistance at speed v is mkv, show that the stone returns to its starting point with speed w given by the equation

$$g \quad kw \quad (g \quad kw) e^{\frac{k}{g}(u \quad w)}$$

5

UNIT—V

9. (a) Deduce work-energy equation. 5

(b) A uniform elastic string has the length a_1 where the tension is T_1 and the length a_2 when the tension is T_2. Show that its natural length is $\frac{a_2T_1 \quad a_1T_2}{T_1 \quad T_2}$, and the amount of work done in stretching it from its natural length to a length $a_1 \quad a_2$ is

$$\frac{1}{2} \frac{(a_1T_1 \quad a_2T_2)^2}{(T_1 \quad T_2)(a_1 \quad a_2)}$$

5

10. (a) Two spheres of masses M and m impinge directly when moving in opposite directions with velocities u and v respectively. If the sphere of mass m is brought to rest by collision, show that $v(m \quad eM) \quad M(1 \quad e)u$. 5

(b) A sphere impinges obliquely on another sphere at rest. If the two spheres are smooth, elastic and equal in mass, then prove that they move at right angles to each other after impact. 5

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