V/MAT (v)

2016

(5th Semester)

MATHEMATICS

FIFTH PAPER (MATH–351)

(Computer-oriented Numerical Analysis)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Find a real root of the equation $x^3 \quad 3x \quad 1 \quad 0$ lying between 1 and 2 to two places of decimal by bisection method. 5

(b) Find the second difference of the polynomial

$$f(x) \quad x^4 \quad 12x^3 \quad 42x^2 \quad 30x \quad 9$$

taking h 2. Express the result in normal polynomial form. 4+1=5

2. *(a)* Obtain the regula falsi formula to find the root of an algebraic or transcendental equation with an illustrative diagram/curve. 4+1=5

(b) Let $f(x)$ be a polynomial of degree n. Then prove that the nth difference of $f(x)$ is a constant and all higher order differences are zero, i.e.

$$^{r}[f(x)] \quad \begin{matrix} \text{constant} & \text{if } r \quad n \\ 0 & \text{if } r \quad n \end{matrix}$$ 5

UNIT—II

3. *(a)* Obtain Newton's forward interpolation formula for interpolation with equal intervals of the argument. 6

(b) Find the form of the function $f(x)$ using Lagrange's interpolation formula from the following table : 4

x	3	2	1	–1
$f(x)$	3	12	15	–21

4. *(a)* Obtain Newton's divided difference interpolation formula for non-equal intervals of the argument. 6

(b) The population of a country in the decimal census were as under :

Year	1941	1951	1961	1971	1981
Population	46	67	83	95	102

Estimate the population for the year 1975. 4

UNIT—III

5. *(a)* Solve the following system of equations by Gaussian elimination method : 5

$$2x \quad y \quad 4z \quad 12$$
$$8x \quad 3y \quad 2z \quad 20$$
$$4x \quad 11y \quad z \quad 33$$

(b) By Crout's method, solve the following system of simultaneous equations : 5

$$x \quad y \quad z \quad 1$$
$$3x \quad y \quad 3z \quad 5$$
$$x \quad 2y \quad 5z \quad 10$$

6. *(a)* Solve the following by Gauss-Jordan method : 4

$$x \quad y \quad z \quad 9$$
$$2x \quad 3y \quad 4z \quad 13$$
$$3x \quad 4y \quad 5z \quad 40$$

(b) Explain Gauss-Seidel method with the proper algorithm. 6

UNIT—IV

7. *(a)* Obtain the formula for Simpson's one-third rule for numerical integration. 5

(b) Find the second derivative of $f(x)$ at $x \quad 3 \quad 0$ from the following table : 5

x	3·0	3·2	3·4	3·6	3·8	4·0
y	–14·000	–10·032	–5·296	–0·256	6·672	14·000

8. *(a)* Evaluate

$$I \quad {}_0^{\ /3} \frac{x}{\cos x}\, dx$$

by using trapezoidal rule up to four decimal places. 5

(b) Find the first derivative of $f(x)$ at $x \quad 0 \quad 4$ from the following table : 5

x	0·1	0·2	0·3	0·4
y	1·10517	1·22140	1·34986	1·49182

UNIT—V

9. *(a)* Compute $y(0\ \ 1)$ by Runge-Kutta method of fourth-order for the differential equation $\frac{dy}{dx}\ \ xy\ \ y^2$ with $y(0)\ \ 1$. 5

(b) Apply Euler's method with $h\ \ 0\ 025$ to find the solution of the equation $\frac{dy}{dx}\ \ \frac{y\ \ x}{y\ \ x}$ with initial condition $y(0)\ \ 1$ in the range $0\ \ x\ \ 0\ 1$. 5

10. *(a)* Solve $y\ \ \ y$ with $y(0)\ \ 1$ by using Milne's method $x\ \ 0\ 1$ to $x\ \ 2\ 7$ with $h\ \ 0\ 3$. 10

Or

(b) Given

$$\frac{dy}{dx}\ \ \frac{y}{x}\ \ \frac{1}{x^2},\ y(1)\ \ 1$$

evaluate $y(1\ 3)$ by modified Euler's method.

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Subject Code : V/MAT (v)

Booklet No. A

To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce / ....) Exam., **2016**

Subject ....

Paper ....

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2 0 1 6

(5th Semester)

MATHEMATICS

FIFTH PAPER (MATH–351)

(Computer-oriented Numerical Analysis)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. By definition of forward difference operator, $^2 f(x)$ equals to

(a) $f(x \quad h) \quad f(x)$ ☐

(b) $f(x \quad 2h) \quad f(x \quad h) \quad f(x)$ ☐

(c) $f(x \quad 2h) \quad 2f(x \quad h) \quad f(x)$ ☐

(d) $f(x \quad 2h) \quad 2f(x \quad h) \quad f(x)$ ☐

2. A factorial function, denoted by $x^{(n)}$, where n is a positive integer, is a product of the form given by

(a) $(x \quad h)(x \quad 2h)\cdots(x \quad nh)$ ☐

(b) $x(x \quad h)(x \quad 2h)\cdots[x \quad (n \quad 1)h]$ ☐

(c) $(x \quad h)(x \quad 2h)\cdots[x \quad (n \quad 1)h]$ ☐

(d) $x(x \quad h)(x \quad 2h)\cdots(x \quad nh)$ ☐

3. Second divided difference with arguments 2, 4, 9 of the function $f(x) \quad x^2$ is

(a) 2 ☐

(b) 1 ☐

(c) 0 ☐

(d) –1 ☐

4. Let observations for the function $y \quad f(x)$ at the points $x \quad a, a \quad h, a \quad 2h, \cdots, a \quad nh$ be $f(a), f(a \quad h), f(a \quad 2h), \cdots, f(a \quad nh)$. Then the method of finding $f(m)$ at $x \quad m$, where m lies in the range of a and $a \quad nh$ is known as

(a) intrapolation ☐

(b) extrapolation ☐

(c) interpolated value ☐

(d) interpolation ☐

5. Back-substitution procedure of solving a simultaneous linear equation is given by

(a) Gauss elimination method ☐

(b) Crout's method ☐

(c) Gauss-Seidel method ☐

(d) None of the above ☐

6. Indirect method of solving a simultaneous linear equation be represented by

(a) Gauss elimination method ☐

(b) Gauss-Jordan method ☐

(c) Gauss-Seidel method ☐

(d) None of the above ☐

7. Simpson's rule is based on approximating the function $f(x)$ by fitting quadratics through sets of

(a) two points ☐

(b) three points ☐

(c) four points ☐

(d) two or four points ☐

8. The value of f (4) from the table

x	1	2	3	4
y	6	12	20	30

is

(a) 1 ☐

(b) 10 ☐

(c) 11 ☐

(d) 21 ☐

9. Which of the following statements is correct?

(a) Runge-Kutta method is not self-starting ☐

(b) Predictor-corrector method is not self-starting ☐

(c) Both Runge-Kutta method and predictor-corrector method are self-starting ☐

(d) Neither Runge-Kutta method nor predictor-corrector method is self-starting ☐

10. The ordinary differential equation

$$\frac{dy}{dx}^{2} \quad y \quad x$$

is of

(a) first order and first degree ☐

(b) second order and first degree ☐

(c) second order and second degree ☐

(d) first order and second degree ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

1. If $x^{(r)}$ is the factorial notation of x raised to power r factorial, then show that $x^{(r)} \quad rhx^{(r \quad 1)}$, where the interval of differencing is h.

2. If $f(x) \quad \frac{1}{x^2}$, then find the divided difference of $[a, b, c]$.

3. Solve the given equations by Gauss-Jordan method :

$$x \quad 2y \quad 4$$
$$x \quad y \quad 13$$

4. Evaluate

$$_{0}^{1} \frac{dx}{1 \quad x^2}$$

using trapezoidal rule with $h \quad 0 \;\; 2$. Hence, determine the value of .

5. Using Taylor's method, find $y(0\ \ 1)$ correct to three decimal places from the equation

$$\frac{dy}{dx} \quad 2xy \quad 1 \text{ with } y_0 \quad 0$$

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V/MAT (vi)

2 0 1 6

(5th Semester)

MATHEMATICS

SIXTH PAPER (MATH–352)

(Real Analysis)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. State and prove Lindelof covering theorem. 2+8=10

2. *(a)* Let $\{G_n\}$ be a sequence of non-empty closed sets such that—

(i) $G_{n \quad 1} \quad G_n \quad n$

(ii) G_1 is bounded

Then prove that the intersection $G_n, \ n \quad N$, is nonempty. 5

(b) Prove that a set is compact if and only if every infinite subset thereof has a limit point in the set. 5

UNIT—II

3. *(a)* Prove that the range of a function continuous on a compact set is compact. 7

(b) If

$$f(x, y) \quad \begin{cases} \dfrac{x^2}{x^2 \quad y^2 \quad x}, & (x, y) \quad (0, 0) \\ 0, & (x, y) \quad (0, 0) \end{cases}$$

then show that f is not continuous at (0, 0). 3

4. *(a)* Prove that a function continuous on a compact domain is uniformly continuous. 6

(b) If

$$f(x, y) \quad \begin{cases} 3xy, & (x, y) \quad (2, 3) \\ 6, & (x, y) \quad (2, 3) \end{cases}$$

then show that f has a removable discontinuity at (2, 3) and redefine the function to make it continuous. 4

UNIT—III

5. *(a)* If $u_1, u_2, u_3, \ldots, u_n$ are functions of $y_1, y_2, y_3, \ldots, y_n$ and $y_1, y_2, y_3, \ldots, y_n$ are functions of $x_1, x_2, x_3, \ldots, x_n$, then prove that

$$\frac{(u_1, u_2, u_3, \ldots, u_n)}{(x_1, x_2, x_3, \ldots, x_n)}$$

$$\frac{(u_1, u_2, u_3, \ldots, u_n)}{(y_1, y_2, y_3, \ldots, y_n)} \quad \frac{(y_1, y_2, y_3, \ldots, y_n)}{(x_1, x_2, x_3, \ldots, x_n)}$$ 6

(b) Prove that a function which is differentiable at a point is also continuous at the point. 4

6. *(a)* Consider the function $f : R^2 \quad R$ defined by

$$f(x, y) \quad \begin{matrix} \dfrac{xy^2}{x^2 \quad y^4}, & \text{if } x \quad 0 \\ 0, & \text{if } x \quad 0 \end{matrix}$$

Show that the directional derivatives of f at $(0, 0)$ in all directions exist but the function is not continuous at $(0, 0)$. 5

(b) Prove that a function which is differentiable at a point admits of partial derivatives at the point. 5

UNIT—IV

7. State and prove Schwarz's theorem. 2+8=10

8. *(a)* Show that for the function

$$f(x, y) \quad \begin{matrix} \dfrac{x^2 y^2}{x^2 \quad y^2}, & (x, y) \quad (0, 0) \\ 0, & (x, y) \quad (0, 0) \end{matrix}$$

$f_{xy}(0, 0) \quad f_{yx}(0, 0)$, even though the conditions of Schwarz's theorem are not satisfied. 6

(b) Examine the function

$$f(x, y) \quad y^2 \quad x^2 y \quad ax^4$$

for extreme values. 4

UNIT—V

9. *(a)* Prove that the space R^n of all ordered n-tuples with the metric d, where

$$d(x, y) \quad {}_{i\ 1}^{\ n} (x_i \quad y_i)^2 \ ^{1/2}$$

is a complete metric space. 8

(b) Give an example to show that the intersection of an infinite number of open sets is not open. 2

10. *(a)* Show that every compact subset of a metric space (X, d) is closed. 7

(b) Prove that every closed subset of a compact metric space is compact. 3

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Subject Code : V/MAT (vi)

Booklet No. A

Date Stamp ....

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(Arts / Science / Commerce / ....) Exam., **2016**

Subject ....
Paper ....

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V/MAT (vi)

2 0 1 6

(5th Semester)

MATHEMATICS

SIXTH PAPER (MATH–352)

(Real Analysis)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. Every infinite and bounded set has at least one limit point. This is the statement of

(a) Bolzano-Weierstrass theorem ☐

(b) Heine-Borel theorem ☐

(c) Lindelof covering theorem ☐

(d) Cantor intersection theorem ☐

2. If $E \quad R^n$, then a point $a \quad R^n$ is called a limit point of E, if every neighbourhood of the point a contains

(a) an infinite number of points of the set E ☐

(b) a finite number of points of the set E ☐

(c) no point of the set E ☐

(d) None of the above ☐

3. If $\lim\limits_{x \quad a} f(x) \quad b$, where $x \quad (x_1, x_2)$, $a \quad (a_1, a_2)$, $b \quad (b_1, b_2)$ and $f \quad (f_1, f_2)$, then

(a) $\lim\limits_{x \quad a} f_1(x) \quad b_2$ ☐

(b) $\lim\limits_{x \quad a} f_2(x) \quad a_2$ ☐

(c) $\lim\limits_{x \quad a} f_1(x) \quad b_1$ and $\lim\limits_{x \quad a} f_2(x) \quad b_2$ ☐

(d) None of the above ☐

4. A set is said to be compact if it is

(a) bounded ☐

(b) both bounded and closed ☐

(c) open ☐

(d) None of the above ☐

5. If $u \quad \cos x,\ v \quad \sin x \cos y,\ w \quad \sin x \sin y \sin z,$ then

$$\frac{(u, v, w)}{(x, y, z)}$$

is equal to

(a) $\sin^3 x \sin^2 y \sin z$ ☐

(b) $\sin x \sin^2 y \sin^3 z$ ☐

(c) $\quad \sin^3 x \sin^2 y \sin z$ ☐

(d) None of the above ☐

6. If

$$f(x, y) \qquad \begin{array}{ll} xy \dfrac{x^2 \quad y^2}{x^2 \quad y^2}, & (x, y) \quad (0, 0) \\ 0, & (x, y) \quad (0, 0) \end{array}$$

then $f_y(x, 0)$ is equal to

(a) x ☐

(b) y ☐

(c) 0 ☐

(d) None of the above ☐

7. If (a, b) be a point of the domain contained in R^2 of a function f such that f_x and f_y are both differentiable at (a, b), then

(a) $f_{xy}(a, b) \quad f_{yx}(a, b)$ ☐

(b) $f_{xy}(a, b) \quad f_{yx}(a, b)$ ☐

(c) $f_{xy}(a, b) \quad f_{yx}(a, b)$ ☐

(d) None of the above ☐

8. The function $f(x, y) \quad y^2 \quad 4xy \quad 3x^2 \quad x^3$ has

(a) a minimum at (0, 0) ☐

(b) a maximum at (0, 0) ☐

(c) neither a minimum nor a maximum at (0, 0) ☐

(d) None of the above ☐

9. Let (X, d) be a complete metric space and Y be a subspace of X. Then Y is complete if and only if it is

(a) closed in (X, d) ☐

(b) open in (X, d) ☐

(c) both closed and open in (X, d) ☐

(d) None of the above ☐

10. Let A and B be two subsets of a metric space (X, d). Then

(a) $A \quad B \quad \overline{B} \quad \overline{A}$ ☐

(b) $\overline{A \quad B} \quad \overline{A} \quad \overline{B}$ ☐

(c) $\overline{A \quad B} \quad \overline{A} \quad \overline{B}$ ☐

(d) None of the above ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

Answer the following :

1. Show that a set is closed if and only if its complement is open.

2. Define convex set and state the intermediate value theorem.

3. Show that the function f, where

$$f\,(x, y) \qquad \begin{matrix} \dfrac{x^3 \quad y^3}{x^2 \quad y^2}, & (x, y) \quad (0, 0) \\ 0\,, & (x, y) \quad (0, 0) \end{matrix}$$

is continuous at (0, 0).

4. Give an example of a function $f(x, y)$ such that the conditions of Young's theorem are not satisfied but $f_{xy}(0, 0) \quad f_{yx}(0, 0)$. Justify it.

5. In any metric space (X, d), show that the union of an arbitrary family of open sets is open.

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V/MAT (vii)

2016

(5th Semester)

MATHEMATICS

SEVENTH PAPER (MATH–353)

(Complex Analysis)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Prove that

$$\left|\frac{z_1 \quad z_2}{1 \quad \bar{z}_1 z_2}\right| \quad 1$$

if $|z_1| \quad 1$ and $|z_2| \quad 1$. 5

(b) Prove that the area of the triangle whose vertices are the points represented by the complex numbers z_1, z_2, z_3 on the Argand diagram is

$$[(z_2 \quad z_3)|z_1|^2 / 4iz_1]$$ 5

2. *(a)* Find the equations in complex variables of all the circles which are orthogonal to $|z| \quad 1$ and $|z \quad 1| \quad 4$. 5

(b) Find the regions of Argand diagram defined by

$$|z \quad 1| \quad |z \quad 1| \quad 4$$ 5

UNIT—II

3. *(a)* If n is real, then show that

$$r^n(\cos n \quad i\sin n \,)$$

is analytic except when $r \quad 0$ and find its derivatives. 5

(b) If $u \quad e^x(x\cos y \quad y\sin y)$, then find the analytic function $u \quad iv$. 5

4. *(a)* If $f(z) \quad u \quad iv$ is analytic function and $u \quad v \quad e^x(\cos y \quad \sin y)$, then find $f(z)$ in terms of z. 4

(b) If $f(z) \quad u \quad iv$ is analytic function of z in any domain, then prove that

$$\frac{2}{x^2} \quad \frac{2}{y^2} \; |f(z)|^p \quad p^2 |f(z)|^{p \quad 2} |f(z)|^2$$ 6

UNIT—III

5. (a) Show that the power series $\quad a_n z^n$ and its derivative $\quad n a_n z^{n \quad 1}$ have same radius of convergence. 5

(b) Find the radius of convergence of the power series

$$f(z) \quad {}_{0} \frac{z^n}{2^n \quad 1}$$

and prove that $(2 \quad z) f(z) \quad 2 \quad 0$ as $z \quad 0.$ 5

6. (a) Find the region of convergence of the series

$$_{n \quad 1} \frac{(z \quad 2)^{n \quad 1}}{(n \quad 1)^3 4^n}$$ 5

(b) Find the domain of convergence of the series

$$_{1} \frac{1 \; 3 \; 5 \; \ldots \; (2n \quad 1)}{\lfloor n} \; \frac{1 \quad z}{z}^{\,n}$$ 5

UNIT—IV

7. (a) Evaluate

$$_{(0,\,3)}^{(2,\,4)} [(2y \quad x^2)\, dx \quad (3x \quad y)\, dy]$$

using the substitution $x \quad 2t,\ y \quad t^2 \quad 3.$ 4

(b) State and prove Cauchy's fundamental theorem. 6

8. (a) Verify Cauchy's theorem for the function $5 \sin 2z$ if C is the square with vertices at $1 \quad i, \quad 1 \quad i.$ 5

(b) If $f(z)$ is analytic within and on a closed contour C and a is any point within C, then show that

$$f(a) \quad \frac{1}{2 \quad i} {}_{C} \frac{f(z)\, dz}{(z \quad a)}$$ 5

UNIT—V

9. (a) State and prove Liouville's theorem. 4

(b) State and prove Taylor's theorem. 6

10. *(a)* Obtain the Laurent's series which represents the function

$$f(z) \quad \frac{z^2 \quad 1}{(z \quad 2)(z \quad 3)}$$

in the regions—

(i) $|z| \quad 2$

(ii) $|z| \quad 3$ 5

(b) Find the singularities of the following functions : 5

(i) $\frac{\cot \ z}{(z \quad a)^2}$ at $z \quad 0, \ z$

(ii) $\tan \frac{1}{z}$ at $z \quad 0$

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Subject Code : V/MAT (vii)

Booklet No. A

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V/MAT (vii)

2 0 1 6

(5th Semester)

MATHEMATICS

SEVENTH PAPER (MATH–353)

(Complex Analysis)

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. In an Argand plane, the radius of the circle $|5z \quad 15 \quad 16i| \quad 20$ is

(a) 2 ☐

(b) 20 ☐

(c) 4 ☐

(d) 10 ☐

2. The value of $\arg z \quad \arg \bar{z}$ is

(a) $2n$ ☐

(b) $\quad 2n$ ☐

(c) n ☐

(d) $\quad n$ ☐

3. If $f(z) \quad u \quad iv$ is analytic function in a finite region and $u \quad x^3 \quad 3xy^2$, then v is

(a) $3x^2y^2 \quad y^3 \quad c$ ☐

(b) $3x^2y \quad y^3 \quad c$ ☐

(c) $3x^2y \quad y^2 \quad c$ ☐

(d) None of the above ☐

4. The analytic function whose real part is $e^x \cos y$ is

(a) xe^z ☐

(b) $3e^z$ ☐

(c) e^{2z} ☐

(d) $e^z \quad c\,i$ ☐

5. If the power series $a_n z^n$ is convergent but the series $|a_n z^n|$ is not convergent, then the series $\underset{n\ 0}{} a_n z^n$ is said to be

(a) conditionally convergent ☐

(b) divergent ☐

(c) oscillatory ☐

(d) None of the above ☐

6. For the series $\frac{n!}{n^2} z^n$, the radius of convergence R is

(a) e ☐

(b) ☐

(c) 1 ☐

(d) 0 ☐

7. If $f(z)$ is analytic in a simply connected domain D enclosed by a rectifiable Jordan curve C and $f(z)$ is continuous on C, then for any point z_0 in D, we have $f(z_0)$

(a) $\frac{1}{2} {}_C \frac{f(z)\,dz}{z \quad z_0}$ ☐

(b) $\frac{1}{2\ i} {}_C \frac{f(z)\,dz}{z \quad z_0}$ ☐

(c) $2\ i \frac{f(z)\,dz}{z \quad z_0}$ ☐

(d) $2 \frac{f(z)\,dz}{z \quad z_0}$ ☐

8. If C is a circle $|z| \;\; 1$, then $\underset{C}{\;\;\bar{z}\, dz}$ is

(a) $\;\; i$ ☐

(b) $2 \;\; i$ ☐

(c) 0 ☐

(d) ☐

9. For the function $f(z) \;\; e^{z}$, $z \;\;\; i$ is

(a) isolated essential singularity ☐

(b) pole ☐

(c) ordinary point ☐

(d) None of the above ☐

10. The number of isolated singular points of

$$f(z) \;\; \frac{z \;\; 3}{z^2(z^2 \;\; 2)}$$

is

(a) 3 ☐

(b) 4 ☐

(c) infinite ☐

(d) 6 ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

Answer **all** questions

Answer the following :

1. Show that for two complex numbers z_1 and z_2

$$|z_1 \quad z_2| \quad |z_1| \quad |z_2|$$

2. Show that the function $f(z) \quad xy \quad iy$ is everywhere continuous but is not analytic.

3. Find the centre and radius of convergence of the power series

$$\frac{(\quad 1)^n}{n}(z \quad 2i)^n$$

4. Evaluate $(\bar{z})^2 dz$ around the circle $|z \quad 1| \quad 1$.

5. Find the zeros and poles of

$${\frac{z \quad 1}{z^{2} \quad 1}}^{2}$$

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V/MAT (viii) (A)

2016

(5th Semester)

MATHEMATICS

Paper : MATH–354(A)

(Operations Research)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Two products A and B are to be manufactured. One unit of product A requires 2·4 minutes of punch press time and 5 minutes of assembly time. The profit for product A is ₹ 0·60 per unit. One unit of product B requires 3 minutes of punch press time and 2·5 minutes of

welding time. The profit for product B is ₹ 0·70 per unit. The capacity of the punch press department available for these products is 1200 minutes per week. The welding department has a capacity of 600 minutes per week and assembly department has 1500 minutes per week.

(i) Formulate the problem as linear programming problem.

(ii) Determine the quantities of products A and B so that the total profit is maximized. 5

(b) An agriculturist has a firm of 125 acres. He produces radish, muttar and potato. Whatever he raises is fully sold in the market. He gets ₹ 5 for radish per kg, ₹ 4 for muttar per kg and ₹ 5 for potato per kg. The average yield is 1500 kg of radish per acre, 1800 kg of muttar per acre and 1200 kg of potato per acre. To produce each 100 kg of radish, muttar and to produce each 80 kg of potato, a sum of ₹ 12·50 has to be used for manure. Labour required for each acre to raise the crop is 6 man-days for radish and potato each and 5 man-days for muttar. A total of 500 man-days of labour at the rate of ₹ 40 per man-day is available.

Formulate this as an LPP to maximize the agriculturist's total profit. 5

2. *(a)* A firm is engaged in breeding pigs. The pigs are fed on various products grown on the firm. In view of the need to ensure certain nutrient constituents (call them *X*, *Y* and *Z*), it is necessary to buy two additional products *A* and *B*. One unit of product *A* contains 36 units of *X*, 3 units of *Y* and 20 units of *Z*. One unit of *B* contains 6 units of *X*, 12 units of *Y* and 10 units of *Z*. The minimum requirements of *X*, *Y* and *Z* are 108 units, 36 units and 100 units respectively. Product *A* costs ₹20 per unit and product *B* costs ₹40 per unit.

Formulate the above as a linear programming problem to minimize the total cost, and solve the problem by using graphic method. 5

(b) A diet is to contain at least 400 units of carbohydrate, 500 units of fat and 300 units of protein. Two foods *A* and *B* are available. *A* costs ₹2 per unit and *B* costs ₹4 per unit. A unit of food *A* contains 10 units of carbohydrate, 20 units of fat and 15 units of protein; a unit of food *B* contains 25 units of carbohydrate, 10 units of fat and 20 units of protein. Find the minimum

cost for diet that consists of a mixture of these two foods and also meets the minimum requirements. Formulate the problem as a linear programming problem and solve it. 5

UNIT—II

3. *(a)* Use simplex method to solve the following problem : 5

Maximize $Z \quad 2x_1 \quad 5x_2$

subject to

$$x_1 \quad 4x_2 \quad 24$$
$$3x_1 \quad x_2 \quad 21$$
$$x_1 \quad x_2 \quad 9$$

and $x_1, x_2 \quad 0$.

(b) A firm produces three types of biscuits *A*, *B* and *C*. It packs them in assortments of two sizes *I* and *II*. The size *I* contains 20 biscuits of type *A*, 50 of type *B* and 10 of type *C*. The size *II* contains 10 biscuits of type *A*, 80 of type *B* and 60 of type *C*. A buyer intends to buy at least 120 biscuits of type *A*, 740 of type *B* and 240 of type *C*. Determine the least number of packets he should buy. Use simplex method and concept of dual. 5

4. The following data are available for the firms which manufacture three items *A*, *B* and *C* products :

Product	*Time required* (in hour)		*Profit*
	Assembly	*Finishing*	
A	10	2	80
B	4	5	60
C	5	4	30
Firm capacity	2000	1009	

Find the optimum solution of the above data using simplex method. 10

UNIT—III

5. A company has four jobs *A*, *B*, *C* and *D* to be done on four machines *W*, *X*, *Y* and *Z*. Each job must be done on one and only one machine. The cost (in ₹) of each job on each machine is given in the following cost table :

Cost Table

		W	*X*	*Y*	*Z*
Job	*A*	7	9	8	13
	B	16	16	15	11
	C	16	19	10	15
	D	16	17	14	16

Using Hungarian method of assignment, determine the job assignments to the machine so as to minimize the total cost. 10

6. The following data describe a transportation problem :

From \ To	D_1	D_2	D_3	D_4	*Supply*
S_1	21	16	15	3	11
S_2	17	18	14	13	13
S_3	32	27	18	41	19
Demand	6	10	12	15	43 / 43

Find the initial solution by using—

(a) least cost method;

(b) Vogel's approximation method. 10

UNIT—IV

7. Use branch and bound technique algorithm to solve the following mixed integer problem : 10

Maximize $Z \quad 5x_1 \quad 4x_2$

subject to

$x_1 \quad x_2 \quad 5$

$10x_1 \quad 6x_2 \quad 45$

$x_1, x_2 \quad 0$ and integers

8. Use Gomory's cutting algorithm to solve the following problem : 10

Maximize $Z \quad 5x_1 \quad 7x_2$

subject to

$2x_1 \quad 3x_2 \quad 6$

$6x_1 \quad x_2 \quad 30$

$x_1, x_2 \quad 0$ and integers

UNIT—V

9. Solve the following game by the rule of dominance : 10

Player B

		I	*II*	*III*
	I	4	6	3
Player A	*II*	3	3	4
	III	2	3	4

10. Solve the game by graphical method, whose profit matrix is given below : 10

Player B

		B_1	B_2	B_3	B_4	B_5
Player A	A_1	5	5	0	1	8
	A_2	8	4	1	6	5

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Subject Code : V/MAT (viii) (A)

Booklet No. A

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DEGREE 5th Semester
(Arts / Science / Commerce / ....) Exam., **2016**

Subject ....
Paper ....

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DEGREE 5th Semester
(Arts / Science / Commerce / ....) Exam., **2016**

Roll No. ....
Regn. No. ....

Subject ....
Paper ....

Descriptive Type
Booklet No. B ....

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/144

2016

(5th Semester)

MATHEMATICS

Paper : MATH–354(A)

(Operations Research)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—I

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. Objective function of an LPP is

(a) a constant ☐

(b) a function to be optimized ☐

(c) a relation between the variables ☐

(d) a variable ☐

2. An LPP must have

(a) objective that we aim to maximize or minimize ☐

(b) constraints that we need to be specify ☐

(c) decision variables that we need to be determine ☐

(d) All of the above ☐

3. In a simplex method, if there is tie between a decision variable and a slack (or surplus) variable, then

(a) decision variable should be selected ☐

(b) slack variable should be selected ☐

(c) surplus variable should be selected ☐

(d) All of the above ☐

4. At any iteration of the usual simplex method, if there is at least one basic variable in the basis at zero level and all $z_j \quad c_j \quad 0$, the current solution is

(a) infeasible ☐

(b) unbounded ☐

(c) non-degenerate ☐

(d) degenerate ☐

5. The dual of the primal maximization LPP having *m* constraints and *n* non-negative variables should

(a) be a minimization LPP ☐

(b) have *n* constraints and *m* non-negative variables ☐

(c) Both *(a)* and *(b)* ☐

(d) be a maximization LPP ☐

6. The transportation problem deals with the transportation of

(a) a single product from several sources to a destination ☐

(b) a multiproduct from several sources to several destinations ☐

(c) a single product from a source to several destinations ☐

(d) a single product from several sources to several destinations ☐

7. In a mixed integer programming problem

(a) only few of the decision variables require integer solutions ☐

(b) different objective functions are mixed together ☐

(c) all of the decision variables require integer solution ☐

(d) None of the above ☐

8. While solving an IPP, any non-integer variables in the solution is picked up to

(a) enter the solution ☐

(b) leave the solution ☐

(c) obtain the cut constraints ☐

(d) All of the above ☐

9. A game is said to be fair, if

(a) upper and lower values of the game are same and zero ☐

(b) upper and lower values of the game are not equal ☐

(c) upper value is more than lower value of the game ☐

(d) None of the above ☐

10. Consider the game G of the following payoff matrix :

Player B

Player A $\begin{matrix} 3 & 8 \\ 3 & \end{matrix}$

Then the value of the game G is

(a) 3 ☐

(b) 3 ☐

(c) 8 ☐

(d) 3 24 ☐

SECTION—II

(*Marks* : 15)

Each question carries 3 marks

1. Express the following LPP in the standard form :

Maximize $Z \quad 3x_1 \quad 5x_2 \quad 2x_3$

subject to

$$x_1 \quad 2x_2 \quad x_3 \quad 4$$
$$5x_1 \quad 6x_2 \quad 7x_3 \quad 5$$
$$2x_1 \quad x_2 \quad 3x_3 \quad 10$$

$x_1,\ x_2 \quad 0,\ x_3$ unrestricted in sign

2. Define pivot column and pivot row. How do you derive a new tableau?

3. Obtain the dual problem from the following LPP :

Minimize $Z \quad 2x_1 \quad 5x_2$

subject to the constraints

$$x_1 \quad x_2 \quad 2$$

$$2x_1 \quad x_2 \quad 6x_3 \quad 6$$

$$x_1 \quad x_2 \quad 3x_3 \quad 4$$

and $x_1,\ x_2,\ x_3 \quad 0$

4. Distinguish between pure and mixed integer programming problems.

5. Solve the game whose payoff matrix is

		Player B	
	15	2	3
Player A	6	5	7
	7	4	0

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V/MAT (viii) (B)

2016

(5th Semester)

MATHEMATICS

Paper : MATH–354(B)

(Probability Theory)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* State the axiomatic definition of probability. For any two events A and B, prove that

$$P(\bar{A} \quad B) \quad P(B) \quad P(A \quad B)$$ 5

(b) From a city population, the probability of selecting

(i) a male or a smoker is 7/10

(ii) a male smoker is 2/5

(iii) A male, if a smoker is already selected is 2/3

Find the probability of selecting

(1) a non-smoker;

(2) a male;

(3) a smoker, if a male is first selected. 5

2. *(a)* State and prove Bayes' theorem. 7

(b) A factory produces a certain type of outputs by three types of machine. The respective daily production figures are :

Machine I : 3000 units

Machine II : 2500 units

Machine III : 4500 units

Past experience shows that 1 percent of the output produced by Machine I is defective. The corresponding fraction of defectives for the other two machines are 1·2 percent and 2 percent respectively. An item is drawn at random from the day's production run and is found to be defective. What is the probability that it comes from the output of Machine I ? 3

UNIT—II

3. *(a)* A continuous random variable X has probability distribution function $f(x) \quad 3x^2, 0 \quad x \quad 1$. Find a and b such that

(i) $P(X \quad a) \quad P(X \quad a)$

(ii) $P(X \quad b) \quad 0 \; 05$ 5

(b) Ten coins are thrown simultaneously. Find the probability of getting at least 7 heads. 5

4. A random variable X is distributed at random between the values 0 and 4 and its probability density function is given by

$$f(x) \quad kx^3 (4 \quad x)^2$$

Find the value of k, the mean, the variance and the standard deviation. 5+3+2=10

UNIT—III

5. For the joint probability distribution of two random variables X and Y given below : 5+5=10

X \ Y	1	2	3	4	Total
1	4/36	3/36	2/36	1/36	10/36
2	1/36	3/36	3/36	2/36	9/36
3	5/36	1/36	1/36	1/36	8/36
4	1/36	2/36	1/36	5/36	9/36
Total	11/36	9/36	7/36	9/36	1

(a) Find the marginal distribution of X and Y.

(b) Find the conditional distribution of X, given the value of $Y \quad 1$ and that of Y given the value of $X \quad 2$.

6. Suppose that a two-dimensional continuous random variable (X, Y) has joint probability density function given by : 5+5=10

$$f(x, y) \quad \begin{matrix} 6x^2y, & 0 \quad x \quad 1, \; 0 \quad y \quad 1 \\ 0 \;\;, & \text{elsewhere} \end{matrix}$$

(a) Verify that $\quad {}_0^1 \; {}_0^1 \; f(x, y)\, dxdy \quad 1$.

(b) Find—

(i) $P\ 0\quad X\quad \frac{3}{4}, \frac{1}{3}\quad Y\quad 2$

(ii) $P(X\quad Y\quad 1)$

(iii) $P(X\quad Y)$

(iv) $P(X\quad 1|Y\quad 2)$

UNIT—IV

7. State and prove Chebyshev's inequality. 2+8=10

8. Calculate the correlation coefficients for the following heights (in inches) of fathers (X) and their sons (Y) : 10

X	:	65	66	67	67	68	69	70	72
Y	:	67	68	65	68	72	72	69	71

UNIT—V

9. *(a)* For a Poisson distribution, prove that

$$_{r\ 1}\quad r\quad _{r\ 1}\quad \frac{d\ _{r}}{d}$$ 7

(b) X is a normal variate with mean 30 and standard deviation 5. Find the probability that $X\quad 45$. 3

10. *(a)* If X and Y are independent Poisson variates, show that the conditional distribution of X given $X\quad Y$, is binomial. 7

(b) Find the moment-generating function of the gamma distribution about origin. 3

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Subject Code : V/MAT (viii) (B)

Booklet No. A

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V/ **MAT (viii) (B)**

2016

(5th Semester)

MATHEMATICS

Paper : MATH–354(B)

(Probability Theory)

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—A (Multiple choice)

(*Marks* : 10)

Each question carries 1 mark

Answer **all** questions

Put a Tick ☑ mark against the correct answer in the box provided : 1×10=10

1. A letter of the English alphabet is chosen at random. The probability that the letter so chosen precedes *m* and is a vowel is

(a) $\frac{5}{26}$ ☐

(b) $\frac{3}{26}$ ☐

(c) $\frac{12}{26}$ ☐

(d) None of the above ☐

2. If $P\,(A \quad B) \quad P\,(A) \quad P\,(B)$, then the two events A and B are

(a) mutually exclusive ☐

(b) independent ☐

(c) dependent ☐

(d) None of the above ☐

3. For the binomial distribution

(a) mean < variance ☐

(b) variance < mean ☐

(c) mean = variance ☐

(d) None of the above ☐

4. The parameters of a binomial distribution with mean 4 and variance 3 are

(a) $n \quad 16,\ p \quad \frac{1}{2}$ ☐

(b) $n \quad 16,\ p \quad \frac{1}{4}$ ☐

(c) $n \quad 32,\ p \quad \frac{1}{2}$ ☐

(d) $n \quad 32,\ p \quad \frac{1}{4}$ ☐

5. Two random variables X and Y with joint p.d.f. (p.m.f.) $f_{XY}(x, y)$ and marginal p.d.f.'s (p.m.f.'s) $f_X(x)$ and $g_Y(y)$ respectively, are said to be stochastically independent if and only if

(a) $f_{XY}(x, y) \quad f_X(x)\, g_Y(y)$ ☐

(b) $f_{XY}(x, y) \quad f_X(x) / g_Y(y)$ ☐

(c) $f_{XY}(x, y) \quad f_X(x) \quad g_Y(y)$ ☐

(d) $f_{XY}(x, y) \quad f_X(x) \quad g_Y(y)$ ☐

6. The conditional probability density function of Y given X for two random variables X and Y which are jointly continuously distributed is given by

(a) $f_{X|Y}(x|y) \quad \frac{\quad}{x} F_{Y|X}(y|x)$ ☐

(b) $f_{X|Y}(x|y) \quad \frac{\quad}{y} F_{X|Y}(y|x)$ ☐

(c) $f_{Y|X}(y|x) \quad \frac{\quad}{x} F_{Y|X}(y|x)$ ☐

(d) $f_{Y|X}(y|x) \quad \frac{\quad}{y} F_{Y|X}(y|x)$ ☐

7. For two random variables X and Y, var $(X \quad Y)$ is equal to

(a) var (X) var (Y) ☐

(b) var (X) var (Y) ☐

(c) var (X) var (Y) $2\,\mathrm{cov}(X, Y)$ ☐

(d) var (X) var (Y) $2\,\mathrm{cov}(X, Y)$ ☐

8. For two random variables X and Y, the relation $E(XY) \quad E(X) \; E(Y)$ holds good

(a) if X and Y are independent ☐

(b) if X and Y are identical ☐

(c) for all X and Y ☐

(d) None of the above ☐

9. The mean of a geometric distribution is

(a) pq ☐

(b) p / q ☐

(c) q / p ☐

(d) None of the above ☐

10. For exponential distribution, when   1

(a) variance < mean ☐

(b) variance = mean ☐

(c) variance > mean ☐

(d) None of the above ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

Answer **all** questions

1. If A and B are mutually independent events, then A^c and B are also mutually independent. State true or false and justify your answer.

2. If X is uniformly distributed over the interval $[a, b]$, prove that

$$E(X) \quad \frac{a \quad b}{2}$$

3. If X and Y are random variables having joint density function

$$f(x, y) \quad \begin{matrix} \frac{1}{8}(6 \quad x \quad y), & 0 \quad x \quad 2, \quad 2 \quad y \quad 4 \\ 0 \qquad , & \text{otherwise} \end{matrix}$$

find $P(X \quad 1 \quad Y \quad 3)$.

4. For random variables X and Y, prove that $E(X \quad Y) \quad E(X) \quad E(Y)$, provided all the expectations exist.

5. If X is a Poisson variate such that

$$P(X \quad 2) \quad 9P(X \quad 4) \quad 90P(X \quad 6)$$

find the mean.

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