2015

(5th Semester)

MATHEMATICS

Paper : MATH–351

(Computer-oriented Numerical Analysis)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer *any* **one** question from each Unit

UNIT—I

1. *(a)* Can we find the real root of the equation $x \quad 2^x$? In the light of this function, can the real root of $x^3 \quad x^2 \quad 1 \quad 0$ in the interval [0, 1] be obtained by the method of successive iteration? 2+3=5

(b) Express

$$3x^4 \quad 4x^3 \quad 6x^2 \quad 2x \quad 1$$

in terms of a factorial polynomial and find its fourth-order difference. 2+3=5

2. *(a)* Write an algorithm for Newton-Raphson formula to find the root of algebraic equation. Further find the cube root of 10 using Newton-Raphson formula. 3+2=5

(b) If

$$y \quad \frac{1}{x(x \quad 3)(x \quad 6)}$$

then show that

$$^2y \quad \frac{108}{x(x \quad 3)(x \quad 6)(x \quad 9)(x \quad 12)}$$

where is the forward difference operator. 5

UNIT—II

3. *(a)* From the data given below, find the number of students whose weight is between 60 and 70 : 4

Weight	:	0–40	40–60	60–80	80–100	100–120
No. of Students	:	250	120	100	70	50

(b) Obtain Newton's divided difference interpolation formula for interpolation with non-equal intervals of the argument. 6

4. *(a)* Use Lagrange's formula to find a polynomial of degree three which takes the values prescribed below and hence find the value of $f(x)$ when $x \quad 2$: 4+1=5

x	:	0	1	3	4
$f(x)$	:	12	0	6	12

(b) The area A of a circle of diamater d is given for the following values :

d	:	80	85	90	95	100
A	:	5026	5674	6362	7088	7854

Find the approximate value for the area of a circle of diameter 97. 5

UNIT—III

5. *(a)* Write an algorithm for Gauss elimination method. Further show the back substitution procedure in the Gaussian elimination method of solving a simultaneous linear equation. 2+3=5

(b) By Crout's method, solve the following system of simultaneous equations : 5

$$x \quad y \quad z \quad 9$$
$$2x \quad 3y \quad 4z \quad 13$$
$$3x \quad 4y \quad 5z \quad 40$$

6. *(a)* What is diagonally dominant matrix? Make the following system of equations diagonally dominant and write the corresponding system of simultaneous equations : 2+3=5

$$3x \quad 9y \quad 2z \quad 10$$
$$4x \quad 2y \quad 13z \quad 19$$
$$4x \quad 2y \quad z \quad 3$$

(b) Solve the system of following linear equations by Gauss-Jordan method : 5

$$10x \quad y \quad z \quad 12$$
$$2x \quad 10y \quad z \quad 13$$
$$x \quad y \quad 5z \quad 7$$

UNIT—IV

7. *(a)* Compute by Simpson's rule the value of the integral

$$I \quad {}_0^1 \frac{x^2}{1 \quad x^2} dx$$

by dividing into four equal parts. 5

(b) Obtain the general quadrature formula for equidistant points to find the approximate integration of any function for which numerical values are known. 5

8. *(a)* Obtain the first and second derivatives of the function tabulated below, at the point $x \quad 0 \; 51$: 5

x :	0·4	0·5	0·6	0·7	0·8
y :	1·5836494	1·7974426	2·0442376	2·3275054	2·6510818

(b) Compute the value of the integral

$$I \quad {}_0^{\;/3} \sqrt{\cos \quad} \; d$$

by trapezoidal rule. 5

UNIT—V

9. *(a)* Solve

$$\frac{dy}{dx} \quad 1 \quad y, \; y(0) \quad 0$$

using Euler's method. Find y at $x \quad 0 \; 1$ and $x \quad 0 \; 2$. Compare the result with results of the exact solution. 3+2=5

(b) Using Taylor's method, find $y(0 \; 1)$ correct to 3-decimal places from

$$\frac{dy}{dx} \quad 2xy \quad 1, \; y_0 \quad 0$$

5

10. Using any Predictor-Corrector method, find $y(0 \; 4)$ for the differential equation

$$\frac{dy}{dx} \quad 1 \quad xy, \; y(0) \quad 2$$

10

★★★

Subject Code : V/MAT (v)

Booklet No. A

Date Stamp ....

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DEGREE 5th Semester
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Subject ....

Paper ....

INSTRUCTIONS TO CANDIDATES

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Roll No. ....

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Descriptive Type
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2 0 1 5

(5th Semester)

MATHEMATICS

Paper : MATH–351

(Computer-oriented Numerical Analysis)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. By definition of backward difference operator, $f(x)$ equals to

(a) $f(x \quad h) \quad f(x)$ ☐

(b) $f(x \quad h) \quad f(x)$ ☐

(c) $f(x) \quad f(x \quad h)$ ☐

(d) $f(x) \quad f(x \quad h)$ ☐

2. If

$$f(x) \quad \frac{1}{(x \quad 1)(x \quad 2)(x \quad 3)}$$

then the value of $^{3}f(x)$ is

(a) $60x^{(\quad 6)}$ ☐

(b) $12x^{(\quad 5)}$ ☐

(c) $360x^{(\quad 7)}$ ☐

(d) $12x^{(\quad 5)}$ ☐

3. If $f(0) \quad 5$, $f(1) \quad 1$, $f(2) \quad 9$, $f(3) \quad 25$ and $f(4) \quad 55$, then the value of $f(5)$ is

(a) 105 ☐

(b) 115 ☐

(c) 125 ☐

(d) None of the above ☐

4. If $u_1 \quad 1$, $u_3 \quad 17$, $u_4 \quad 43$ and $u_5 \quad 89$, then the value of u_2 is

(a) 10 ☐

(b) 15 ☐

(c) 6 ☐

(d) 5 ☐

5. The method for obtaining the solution of the system of simultaneous equations by Gauss-Jordan elimination method depends on reducing the coefficient matrix to a/an

(a) lower triangular matrix ☐

(b) upper triangular matrix ☐

(c) diagonal matrix ☐

(d) diagonally dominant matrix ☐

6. The coefficient matrix obtained from the simultaneous equations

$$\begin{array}{cccc} a_{11}x & a_{12}y & a_{13}z & d_1 \\ b_{21}x & b_{22}y & b_{23}z & d_2 \\ c_{31}x & c_{32}y & c_{33}z & d_3 \end{array}$$

will be a diagonally dominant matrix, if

(a) $\begin{array}{ccc} |a_{11}| & |a_{12}| & |a_{13}| \\ |b_{21}| & |b_{22}| & |b_{23}| \\ |c_{31}| & |c_{32}| & |c_{33}| \end{array}$ ☐

(b) $\begin{array}{ccc} |a_{11}| & |a_{12}| & |a_{13}| \\ |b_{22}| & |b_{21}| & |b_{23}| \\ |c_{33}| & |c_{31}| & |c_{32}| \end{array}$ ☐

(c) $\begin{array}{ccc} |a_{11}| & |a_{12}| & |a_{13}| \\ |b_{22}| & |b_{21}| & |b_{23}| \\ |c_{33}| & |c_{31}| & |c_{32}| \end{array}$ ☐

(d) $\begin{array}{cccc} |a_{11}| & |a_{12}| & |a_{13}| & |d_1| \\ |b_{21}| & |b_{22}| & |b_{23}| & |d_2| \\ |c_{31}| & |c_{32}| & |c_{33}| & |d_3| \end{array}$ ☐

7. In the general quadrature formula, Simpson's 1/3rd rule is obtained by putting

(a) n 1 ☐

(b) n 4 ☐

(c) n 2 ☐

(d) n 2 and 4 both ☐

8. When numerical integration is applied for the integration of a function of single variable, the method is called

(a) mechanical quadrature ☐

(b) general quadrature ☐

(c) Simpson's $\frac{1}{3}$rd rule ☐

(d) trapezoidal rule ☐

9. Taylor series method is a powerful single-step method if we are able to find easily the successive

(a) integration ☐

(b) derivatives ☐

(c) continuity ☐

(d) partial derivatives ☐

10. Which of the following statements is wrong?

(a) Modified Euler's method is Runge-Kutta method of second order. ☐

(b) Euler's method is the Rung-Kutta method of first order. ☐

(c) For solving ordinary differential equation numerically, the most reliable and most accurate method is Runge-Kutta method. ☐

(d) For solving ordinary differential equation numerically, Euler's method needs h to be very large to get a reasonable accuracy. ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

1. Obtain the relation $^{n}f(x \quad nh) \quad ^{n}f(x)$, where is the forward difference operator, and is the backward difference operator.

2. Show that the divided differences are independent of the order of arguments, i.e., $(x_0, x_1) \quad (x_1, x_0)$. Is it true for more than two arguments also?

3. Solve the given equations by Gauss elimination method :

$$x \quad y \quad 2;\ 2x \quad 3y \quad 5$$

4. Find the value of $\log 2^{1/3}$ from

$$\int_0^1 \frac{x^2}{1+x^3}\,dx$$

using Simpson's $\frac{1}{3}$rd rule with $h = 0.25$.

5. Using Picard's method, solve the differential equation

$$\frac{dy}{dx} \quad 1 \quad xy \text{ with } y(0) \quad 2$$

and find $y(0 \;\; 2)$.

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V/MAT (vi)

2 0 1 5

(5th Semester)

MATHEMATICS

Paper : MATH–352

(Real Analysis)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. Define a limit point of a set. Prove that every infinite and bounded set has at least one limit point. 2+8=10

2. *(a)* Prove that every open cover of a compact set admits of a finite subcover. 6

(b) Show that a set is closed if and only if its complement is open. 4

UNIT—II

3. *(a)* Prove that a function continuous on a compact domain is uniformly continuous. 6

(b) Let $f : R^2 \quad R$ be a function defined by

$$f(x, y) \quad \frac{x^2 y}{x^4 \quad y^2}, \quad (x, y) \quad (0, 0)$$
$$0 \quad , \quad (x, y) \quad (0, 0)$$

Then test the continuity of f at (0, 0). 4

4. *(a)* State and prove intermediate value theorem. 1+5=6

(b) Let

$$\lim_{x \quad a} f(x) \quad b \text{ and } b \quad (b_1, b_2, \cdots, b_m)$$
$$f \quad (f_1, f_2, \cdots, f_m)$$

Then show that

$$\lim_{x \quad a} f_i(x) \quad b_i, \ 1 \quad i \quad m$$ 4

UNIT—III

5. *(a)* If , , are the roots of the equation in t, such that

$$\frac{u}{a \quad t} \quad \frac{v}{b \quad t} \quad \frac{w}{c \quad t} \quad 1$$

then prove that

$$\frac{(u, v, w)}{(\ , \ , \ )} \quad \frac{(\quad)(\quad)(\quad)}{(b \quad c)(c \quad a)(a \quad b)}$$ 6

(b) Prove that a function which is differentiable at a point admits of partial derivatives at the point. 4

6. If

$$f(x, y) = \begin{cases} \dfrac{x^3}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

show that—

(i) f is continuous at (0, 0);

(ii) directional derivative of f exists at (0, 0) in every direction;

(iii) f is not differentiable at (0, 0).

3+2+5=10

UNIT—IV

7. State and prove Young's theorem. 2+8=10

8. *(a)* Show that the function

$$f(x, y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

does not satisfy the conditions of Schwarz's theorem and

$$f_{xy}(0, 0) \neq f_{yx}(0, 0)$$ 7

(b) Show that

$$f(x, y) = (y - x)^4 + (x - 2)^4$$

has a minimum at (2, 2). 3

UNIT—V

9. *(a)* Let (X, d) be any metric space. Then show that the function d_1 defined by

$$d_1(x, y) = \frac{d(x, y)}{1 + d(x, y)}, \quad x, y \in X$$

is a metric on X. 6

(b) Let (X, d) be a metric space and let x, y, z be any three points of X, then show that

$$d(x, y) \geq |d(x, z) - d(z, y)|$$ 4

10. *(a)* Prove that every compact subset F of a metric space (X, d) is closed. 6

(b) In a metric space (X, d), prove that the intersection of an arbitrary family of closed sets is closed. 4

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Subject Code : V/MAT (vi)

Booklet No. A

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DEGREE 5th Semester
(Arts / Science / Commerce / ....) Exam., **2015**

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V/ MAT (vi)

2 0 1 5

(5th Semester)

MATHEMATICS

Paper : MATH–352

(Real Analysis)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. In R^2, the limit point of the set

$$\frac{1}{m}, \frac{1}{n} \; ; m \quad N, n \quad N$$

is

(a) (0, 0) ☐

(b) (1, 1) ☐

(c) (0, 1) ☐

(d) None of the above ☐

2. A set is said to be compact if and only if it is

(a) bounded ☐

(b) both bounded and closed ☐

(c) open ☐

(d) None of the above ☐

3. If every open cover of the set admits a finite subcover, it is said to have

(a) the Cantor intersection property ☐

(b) the Lindeloff covering property ☐

(c) the Heine-Borel property ☐

(d) None of the above ☐

4. If $x \quad y \quad z \quad u$, $y \quad z \quad uv$, $z \quad uvw$, then

$$\frac{(x,\ y,\ z)}{(u,\ v,\ w)}$$

is equal to

(a) u^2v ☐

(b) uv^2 ☐

(c) uv ☐

(d) u^2v^2 ☐

5. Let f be a real valued function with an open set $D \quad R^n$ as its domain. Then the function admits of directional derivative at every point where it admits of

(a) continuous first-order partial derivatives ☐

(b) first-order partial derivatives ☐

(c) second-order partial derivatives ☐

(d) None of the above ☐

6. Let X be a non-empty set and d is a function from $X \quad X$ into R such that $d(x, y) \quad 0$ if and only if $x \quad y$. Then (X, d) is a metric space if $\quad x, y, z \quad X$

(a) $d(x, y) \quad d(y, x)$ ☐

(b) $d(x, y) \quad d(x, z) \quad d(y, z)$ ☐

(c) $d(x, z) \quad d(z, y) \quad d(x, y)$ ☐

(d) None of the above ☐

7. If

$$f(x, y) \quad \begin{matrix} \dfrac{xy^2}{x^2 \quad y^4}, & (x, y) \quad (0, 0) \\ 0 \quad , & (x, y) \quad (0, 0) \end{matrix}$$

then the directional derivative of f at (0, 0)

(a) does not exist ☐

(b) exists in all directions ☐

(c) exists in a particular direction ☐

(d) None of the above ☐

8. If (a, b) be a point of the domain $D \quad R^2$ of a real valued function f such that f_x exists in a certain neighbourhood of (a, b) and f_{xy} is continuous at (a, b), then

(a) $f_{yx}(a, b) \quad f_{xy}(a, b)$ ☐

(b) $f_{yx}(a, b) \quad f_{xy}(a, b)$ ☐

(c) $f_{yx}(a, b) \quad f_{xy}(a, b)$ ☐

(d) None of the above ☐

9. A metric space (X, d) is said to be complete if

(a) every Cauchy sequence in X diverges to a point of X ☐

(b) every Cauchy sequence in X converges to a point of X ☐

(c) there exists no Cauchy sequence in X ☐

(d) None of the above ☐

10. If $f(x, y) \quad x^3 \quad y^3 \quad 3x \quad 12y \quad 20$, then f is

(a) maximum at (1, 2) ☐

(b) minimum at (1, 2) ☐

(c) minimum at (1, 2) ☐

(d) None of the above ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

Answer the following :

1. Show that $f(x, y) \quad y^2 \quad x^2y \quad x^4$ has a minimum at $(0, 0)$.

2. Show that the intersection of any finite family of open sets is open.

3. Define closure of a set. Show that a set A is closed if and only if $\overline{A} \quad A$.

4. Show that the function

$$f(x,\ y) \quad \begin{matrix} \dfrac{xy}{\sqrt{x^2 \quad y^2}}, & (x,\ y) \quad (0,\ 0) \\ 0 \qquad , & (x,\ y) \quad (0,\ 0) \end{matrix}$$

is not differentiable at (0, 0).

5. Prove that every closed subset of a compact metric space is compact.

★★★

2015

(5th Semester)

MATHEMATICS

Paper : MATH–353

(Complex Analysis)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Show that the modulus of sum of two complex numbers is always less than or equal to the sum of their moduli. 5

(b) Find the centre and radius of the circle passing through the points $1,\ i,\ 1 \quad i$. 5

2. *(a)* If z_1 and z_2 are two complex numbers, then prove that

$$|z_1 \quad z_2|^2 \quad |z_1|^2 \quad |z_2|^2$$

if and only if $z_1\bar{z}_2$ is purely imaginary. 5

(b) If z_1, z_2, z_3 are the vertices of an isoceles triangle, right angled at the vertex z_2, then prove that

$$z_1^2 \quad z_2^2 \quad z_3^2 \quad 2(z_1 \quad z_3)z_2$$ 5

UNIT—II

3. *(a)* For what value of z the function defined by the equation

$$z \quad \sin u \cosh v \quad i \cos u \sinh v, \qquad u \quad iv$$

ceases to be analytic? 5

(b) Show that the function $f(z) \quad xy \quad iy$ is everywhere continuous but not analytic. 5

4. *(a)* If $u \quad x^3 \quad 3xy^2$, show that there exists a function $v(x,\ y)$ such that $\quad u \quad iv$ is an analytic function in a finite region. 5

(b) Show that the function $f(z) \quad \sqrt{|xy|}$ is not analytic at the origin although Cauchy-Riemann equations are satisfied at that point. 5

UNIT—III

5. *(a)* State and prove Cauchy-Hadamard formula for the radius of convergence. 4

(b) Find the radius of convergence of the following power series : 6

(i) $\frac{z^n}{2^n \quad 1}$

(ii) $1 \quad \frac{a \; b}{1 \; c} z \quad \frac{a \; (a \quad 1) b \; (b \quad 1)}{1 \; 2 \; c \; (c \quad 1)} z^2 \quad \cdots$

6. *(a)* Find the domain of convergence of the power series

$$\frac{2i}{z \quad 1 \quad i}^{\;n}$$ 5

(b) Find the region of convergence of the power series

$$_{n \quad 1} \frac{(z \quad 2)^{n \quad 1}}{(n \quad 1)^3 4^n}$$ 5

UNIT—IV

7. *(a)* Using the definition of the integral of $f(z)$ on a given path, evaluate

$$_{2 \quad i}^{5 \quad 3i} z^3 dz$$ 5

(b) If $f(z)$ is analytic within and on a closed contour C and a is any point within C, then show that

$$f(a) \quad \frac{1}{2 \quad i} {}_C \frac{f(z) dz}{(z \quad a)}$$ 5

8. *(a)* Write down the Cauchy's integral formula for the derivative of an analytic function; hence show that for a function $f(z)$ which is analytic in a region D and if $f(z)$ has, at any point $z \quad a$ of D, derivatives of all orders, all of which are again analytic functions in D, their values are given by

$$f^n(a) \quad \frac{\underline{|n}}{2 \quad i} {}_C \frac{f(z) dz}{(z \quad a)^{n \quad 1}}$$

where C is any closed contour in D surrounding the point $z \quad a$. 6

(b) Evaluate by Cauchy integral formula

$$_C \frac{z dz}{(9 \quad z^2)(z \quad i)}$$ 4

UNIT—V

9. *(a)* Expand $\frac{z \quad 3}{z(z^2 \quad z \quad 2)}$ for the region $|z| \quad 2$. 4

(b) Examine the nature of the following functions : 6

(i) $\frac{1}{1 \quad e^z}$ at $z \quad 2 \quad i$

(ii) $\frac{1}{\sin z \quad \cos z}$ at $z \quad \frac{}{4}$

10. State and prove Liouville's theorem. Use this result to prove the fundamental theorem of algebra. 1+4+5=10

★★★

Subject Code : V/MAT (vii)

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

DEGREE 5th Semester
(Arts / Science / Commerce / ....) Exam., **2015**

Subject ....
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2 0 1 5

(5th Semester)

MATHEMATICS

Paper : MATH–353

(Complex Analysis)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. If $\bar{z}$ is the conjugate of z, then

(a) $|z| \quad |\bar{z}|$ ☐

(b) $|z| \quad |\bar{z}|$ ☐

(c) $|z| \quad |\bar{z}|$ ☐

(d) $|z| \quad |\bar{z}|$ ☐

2. The real part of $\dfrac{5}{3 \quad 4i}$ is

(a) $\dfrac{3}{5}$ ☐

(b) $\quad \dfrac{3}{5}$ ☐

(c) $\quad \dfrac{4}{5}$ ☐

(d) $\dfrac{4}{5}$ ☐

3. The analytic function whose imaginary part in $e^x \cos y$ is

(a) e^z ☐

(b) ie^z ☐

(c) $ie^{\quad z}$ ☐

(d) $e^{\quad z}$ ☐

4. The function $\sin x \cosh y \quad i \cos x \sinh y$ is

(a) neither continuous nor analytic ☐

(b) continuous but not analytic ☐

(c) continuous as well as analytic everywhere ☐

(d) not analytic everywhere ☐

5. If $\lim\limits_{n} |u_r|^{1/n}\quad l$, then the series $\quad u_n$ is absolutely convergent for

(a) $l \quad 1$ ☐

(b) $l \quad 1$ ☐

(c) $l \quad 1$ ☐

(d) $l \quad 1$ ☐

6. The power series $\quad \lfloor\!\underline{n}\; z^n$ will converge

(a) if $z \quad 0$ ☐

(b) if $|z| \quad 1$ ☐

(c) if $|z| \quad 1$ ☐

(d) for all real values of z ☐

7. A Jordan curve consisting of continuous chain of a finite number of regular arcs is called a

(a) continuous arc ☐

(b) contour ☐

(c) rectifiable arc ☐

(d) multiple arc ☐

8. The value of the integral $_{C}\frac{dz}{z \quad a}$, while C is the circle $|z \quad a|$ is

(a) 2 ☐

(b) 2 ☐

(c) 2 i ☐

(d) 2 i ☐

9. The function $\frac{z \quad 1}{z(z \quad 2)}$ has/have singularity/singularities at

(a) z 0 only ☐

(b) z 2 only ☐

(c) z 0 and z 2 only ☐

(d) z 1 only ☐

10. The nature of the function $\frac{\sin(z \quad a)}{(z \quad a)}$ at $z \quad a$ is

(a) removable singularity ☐

(b) non-isolated singularity ☐

(c) isolated singularity ☐

(d) pole ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

Answer **all** questions

Answer the following :

1. Prove that for any complex number z, $|z|^2 \quad z\bar{z}$, where $\bar{z}$ is the conjugate of z.

2. With suitable example, show that continuity is not a sufficient condition for the existence of a finite derivative.

3. Examine the convergence of the series z^n.

4. Show that $_{C}\frac{dz}{z} \quad 2\ i$, where C is a complete circle.

5. Define non-isolated singularity with a suitable example.

★★★

2015

(5th Semester)

MATHEMATICS

Paper : MATH–354(A)

(Operations Research)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The questions are of equal value

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. A firm manufactures two types of product A and B, and sells them at a profit of ₹ 2 on type A and ₹ 3 on type B. Each product is processed on two machines M_1 and M_2. Type A requires one minute of processing time on M_1 and two minutes on M_2; type B requires one minute on M_1 and one minute on M_2. The machine M_1 is available for not more than 6 hours 40 minutes while machine M_2 is available for 10 hours during any working day.

Formulate the problem as linear programming problem and find how many products of each type should the firm produce each day in order to get maximum profit by graphically method.

2. A company makes two kinds of leather belts. Belt A is high quality belt and belt B is of lower quality. The respective profits are ₹ 4 and ₹ 3 per belt. Each belt of type A requires twice as much as that type of belt B. If all were of type B, the company could make 1000 belts per day. The supply of leather is sufficient for only 800 belts per day. Belt A requires a fancy buckle and only 400 buckles are available. There are only 700 buckles a day available for belt B. How should the company manufacture the two types of belts in order to have a maximum profit? (Use graphic method)

UNIT—II

3. Solve the given LPP by using simplex method :

Maximize $Z \quad 3x_1 \quad 5x_2 \quad 4x_3$

subject to

$$2x_1 \quad 3x_2 \quad 8$$
$$2x_2 \quad 5x_3 \quad 10$$
$$3x_1 \quad 2x_2 \quad 4x_3 \quad 15$$

and $x_1, x_2, x_3 \quad 0$.

4. Solve the following LPP by Big-M method :

$$\text{Maximize } Z \quad x_1 \quad 2x_2 \quad 3x_3 \quad x_4$$

subject to

$$x_1 \quad 2x_2 \quad 3x_3 \quad 15$$
$$2x_1 \quad x_2 \quad 5x_3 \quad 20$$
$$x_1 \quad 2x_2 \quad x_1 \quad x_4 \quad 10$$

and $x_1, x_2, x_3, x_4 \quad 0$.

UNIT—III

5. Four different jobs can be done on four different machines. The set up and take-down time costs are assumed to be prohibitively high for changeovers. The matrix below gives the cost in rupees of producing job i on machine j :

	M_1	M_2	M_3	M_4
J_1	5	7	11	6
J_2	8	5	9	6
J_3	4	7	10	7
J_4	10	4	8	3

How should the jobs be assigned to the various machines so that the total cost is minimized?

6. The following data describe a transportation problem :

Source	D_1	D_2	D_3	*Supply*
S_1	9	8	5	25
S_2	6	8	4	35
S_3	7	6	9	40
Demand	30	25	45	100

Find the initial solution by using *(a)* North-West corner method, *(b)* least cost method and *(c)* Vogel's approximation method.

UNIT—IV

7. Solve the following LPP by Gomory technique :

$$\text{Maximize } Z \quad 3x_2$$

subject to the constraints

$$3x_1 \quad 2x_2 \quad 7$$
$$x_1 \quad x_2 \quad 2$$

$x_1, x_2 \quad 0$ and are integers.

8. Use branch and bound technique to solve the following mixed integer problem :

$$\text{Maximize } Z \quad x_1 \quad x_2$$

subject to

$$2x_1 \quad 5x_2 \quad 16$$
$$6x_1 \quad 5x_2 \quad 30$$

$x_1 \quad 0, x_2 \quad 0$ and are integers.

UNIT—V

9. Solve the following game by using the principle of dominance :

Player B

Player A

4	2	0	2	1	1
4	3	1	3	2	2
4	3	7	5	1	2
4	3	4	1	2	2
4	3	3	2	2	2

10. Solve the game by graphical method, whose payoff matrix is given below :

Player B

Player A

4	2	3	1
1	2	0	1
2	1	2	0

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Subject Code : V/MAT (viii) (A)

Booklet No. A

Date Stamp ....

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DEGREE 5th Semester
(Arts / Science / Commerce / ....) Exam., **2015**

Subject ....

Paper ....

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Paper ....

Descriptive Type
Booklet No. B ....

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/141

2015

(5th Semester)

MATHEMATICS

Paper : MATH–354(A)

(Operations Research)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—I

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. Operation Research (OR), which is a very powerful tool for

(a) research ☐

(b) decision-making ☐

(c) operations ☐

(d) None of the above ☐

2. Which of the following statements is correct?

(a) If an LPP has two optional solutions, then it has infinitely many solutions ☐

(b) Every LPP has at least one optimal solution ☐

(c) Every LPP has a unique optimal solution ☐

(d) An LPP can have only to decision variables ☐

3. A minimization problem can be converted in a maximization problem by changing the sign of coefficients in the

(a) constraints ☐

(b) objective function ☐

(c) Both *(a)* and *(b)* ☐

(d) None of the above ☐

4. In a simplex method, if there is a tie between a decision variable and slack (or surplus) variable

(a) slack variable should be selected ☐

(b) decision variable should be selected ☐

(c) surplus variable should be selected ☐

(d) None of the above ☐

5. The solution of transportation problem with m sources and n destinations is feasible, if the number of allocations are

(a) $m \quad n \quad 1$ ☐

(b) $m \quad n \quad 1$ ☐

(c) $m \quad n$ ☐

(d) $m \quad n \quad 1$ ☐

6. Which of the following is not correct?

(a) Associated with every LPP, there is always another LPP which is based on the same data and having the same solution ☐

(b) It is always necessary to convert the inequality constraints into equality constraints for writing the dual to an LPP ☐

(c) The dual of dual is primal ☐

(d) Given LPP is called the primal problem while the associated LPP is called its dual ☐

7. In a mixed integer programming problem

(a) all decision variables have integer solution values ☐

(b) some decision variables have integer solution values ☐

(c) decision variables have no integer solution value ☐

(d) None of the above ☐

8. Branch and bound method divides the feasible solution space into smaller parts by

(a) bounding ☐

(b) enumerating ☐

(c) branching ☐

(d) All of the above ☐

9. Let (a_{ij}) be the $m \quad n$ payoff matrix for a two-person zero-sum game. Then which of the following inequalities holds?

(a) $\min_j (\max_i (a_{ij})) \quad \max_i (\min_j (a_{ij}))$ ☐

(b) $\max_i (\min_j (a_{ij})) \quad \min_j (\max_i (a_{ij}))$ ☐

(c) $\min_j (\max_i (a_{ij})) \quad \max_i (\min_j (a_{ij}))$ ☐

(d) $\min_j (\max_i (a_{ij})) \quad \max_i (\min_j (a_{ij}))$ ☐

10. In a two-person zero-sum game, which of the following characteristics is not correct?

(a) Only two players are involved ☐

(b) Each player has an infinite number of strategies to use ☐

(c) Total payoff to the players at the end of each play is zero ☐

(d) None of the above ☐

SECTION—II

(*Marks* : 15)

Each question carries 3 marks

Answer the following :

1. Express the following LPP into standard form :

Maximize $Z \quad 3x_1 \quad 2x_2 \quad 5x_3$

subject to

$$2x_1 \quad 3x_2 \quad 2x_3 \quad 40$$
$$4x_1 \quad 2x_2 \quad x_3 \quad 24$$
$$x_1 \quad 5x_2 \quad 6x_3 \quad 2$$
$$x_1 \quad 0$$

2. Give computational procedure for simplex method for the solution of a maximization linear programming problem.

3. Obtain the dual of the following primal problem :

Minimize $Z \quad x_1 \quad 3x_2 \quad 2x_3$

subject to the constraints

$$3x_1 \quad x_2 \quad 2x_3 \quad 7$$
$$2x_1 \quad 4x_2 \quad 12$$
$$4x_1 \quad 3x_2 \quad 8x_3 \quad 10$$

$x_1 \quad 0$, $x_2 \quad 0$, x_3 is unrestricted.

4. What is the importance of integer programming problem (IPP)?

5. For what value of , the game with the following payoff matrix is strictly determinable?

	6	3
1		7
2	4	

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V/MAT (viii) (B)

2 0 1 5

(5th Semester)

MATHEMATICS

PAPER : MATH–354(B)

(Probability Theory)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Show that if an event A is independent of the events B, $B \quad C$ and $B \quad C$, then it is also independent of C. 5

(b) Prove that for any two events A and B

$$P(A \quad B) \quad P(A) \quad P(A \quad B) \quad P(A) \quad P(B)$$ 5

2. State and prove Bayes' theorem. 10

UNIT—II

3. For the binomial distribution $(q \quad p)^n$, prove that

$$_{r \; 1} \quad pq \quad nr \quad _{r \; 1} \quad \frac{d \quad _r}{dp}$$

where $_r$ is the rth central moment. Hence obtain $_2$, $_3$ and $_4$. Also find out $_1$ and $_2$. 10

4. *(a)* The mean and the variance of a binomial distribution are 4 and $\frac{4}{3}$ respectively. Find *(i)* the probability of two successes and *(ii)* the probability of more than two successes. 5

(b) The probability distribution function of a random variable X is

$$f(x) \quad \begin{matrix} x \;, & \text{for } 0 \quad x \quad 1 \\ 2 \quad x, & \text{for } 1 \quad x \quad 2 \\ 0 \;, & \text{for } \quad x \quad 2 \end{matrix}$$

Compute the cumulative distribution function of X. 5

UNIT—III

5. *(a)* For the following bivariate probability distribution of X and Y, find the following : 6

(i) $P(X \quad 1, Y \quad 2)$

(ii) $P(X \quad 1)$

(iii) $P(Y \quad 3)$

(iv) $P(Y \quad 3)$

(v) $P(X \quad 3, Y \quad 4)$

X \ Y	1	2	3	4	5	6
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
1	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$

(b) The joint density function of X, Y is given as

$$f(x, y) \quad 2, \quad 0 \quad x \quad y \quad 1$$
$$0, \quad \text{elsewhere}$$

Examine whether X and Y are independent or not. 4

6. *(a)* The joint probability density function of two random variables X and Y is given by

$$f(x, y) \quad \frac{9(1 \quad x \quad y)}{2(1 \quad x)^4 (1 \quad y)^4}, \quad \begin{matrix} 0 \quad x \\ 0 \quad y \end{matrix}$$

Obtain *(i)* marginal distribution of X and Y, and *(ii)* conditional distribution of Y for $X \quad x$. 6

(b) If X and Y are two independent continuous random variables, then find the probability distribution function of their product. 4

UNIT—IV

7. *(a)* A sample of 3 items is selected at random from a box containing 12 items of which 3 are defectives. Find the expected number of defective items. 7

(b) If X is a random variable, then prove that

$$\text{var}(X) \quad E(X^2) \quad [E(X)]^2$$ 3

8. Calculate the correlation coefficient for the following heights (in inches) of fathers (X) and their sons (Y) : 10

X	:	65	66	67	67	68	69	70	72
Y	:	67	68	65	68	72	72	69	71

UNIT—V

9. *(a)* Find the moment generating function of Poisson distribution. 3

(b) Find the moment generating function of exponential distribution. 3

(c) Define gamma distribution and then find the first four moments about the origin. 4

10. For a normal distribution, show that its even order central moments are given by the relation (about mean)

$$_{2n} \quad (2n \quad 1)(2n \quad 3)\ldots 3.1. \quad ^{2n}$$ 10

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Subject Code : V/MAT (viii) (B)

Booklet No. A

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Subject ....

Paper ....

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2015

(5th Semester)

MATHEMATICS

PAPER : MATH–354(B)

(Probability Theory)

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—A (Multiple choice)

(*Marks* : 10)

Each question carries 1 mark

Answer **all** questions

Put a Tick ☑ mark against the correct answer in the box provided : 1×10=10

1. The conditional probability of B given A is

(a) $\dfrac{P(A \quad B)}{P(B)}$ ☐

(b) $\dfrac{P(A \quad B)}{P(A)}$ ☐

(c) $\dfrac{P(A \quad B)}{P(A)}$ ☐

(d) $P(A)P(B)$ ☐

2. Let A and B be events with $P(A) \quad \frac{3}{8}$, $P(B) \quad \frac{1}{2}$ and $P(A \quad B) \quad \frac{1}{4}$, then $P(A \quad B)$ is

(a) $\frac{5}{8}$ ☐

(b) $\frac{1}{2}$ ☐

(c) $\frac{3}{8}$ ☐

(d) $\frac{3}{4}$ ☐

3. The parameters of a binomial distribution with mean 8 and variance 4 are

(a) $n \quad 2,\ p \quad \frac{1}{4}$ ☐

(b) $n \quad 16,\ p \quad \frac{1}{2}$ ☐

(c) $n \quad 32,\ p \quad \frac{1}{4}$ ☐

(d) None of the above ☐

4. The characteristic function of binomial distribution is

(a) $(q \quad pe^{it})^n$ ☐

(b) $(q \quad pe^{it})$ ☐

(c) $(qp \quad e^{it})^n$ ☐

(d) None of the above ☐

5. For two random variables X and Y, the relation $E(XY) \quad E(X)E(Y)$ holds good

(a) for all X and Y ☐

(b) if X and Y are identical ☐

(c) if X and Y are independent ☐

(d) None of the above ☐

6. If var $(X) \quad 2$, then var $(3X \quad 5)$ is equal to

(a) 13 ☐

(b) 18 ☐

(c) 5 ☐

(d) –1 ☐

7. The mean of a normal distribution is 50, its mode will be

(a) 25 ☐

(b) 40 ☐

(c) 50 ☐

(d) All of the above ☐

8. The mean and variance of exponential distribution with parameter are

(a) $\frac{1}{}, \frac{1}{}$ ☐

(b) $\frac{1}{}, \frac{1}{2}$ ☐

(c) $, ^{2}$ ☐

(d) None of the above ☐

9. The relationship between mean and variance of gamma distribution is

(a) mean = variance ☐

(b) mean = 2 variance ☐

(c) mean < variance ☐

(d) None of the above ☐

10. The mean and variance of the geometric distribution are

(a) $\frac{p}{q}, \frac{q}{p^2}$ ☐

(b) $\frac{p}{q}, \frac{p^2}{q}$ ☐

(c) $\frac{q}{p}, \frac{q}{p^2}$ ☐

(d) $\frac{q}{p}, \frac{p^2}{q}$ ☐

SECTION—B (Very short answer)

(*Marks* : 15)

Each question carries 3 marks

Answer **all** questions

1. If A and B are independent events, then $\overline{A}$ and $\overline{B}$ are also independent events. Prove it.

2. Is it possible to have a binomial distribution with mean 2 and variance 6?

3. Show that if X has Poisson distribution with parameter , then $E(X)$.

4. Find the moment generating function of geometric distribution.

5. Prove that the moment generating function of gamma distribution is

$$M_X(t) \quad (1 \quad t)^{\;n}, \quad |t| \quad 1$$

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