MATH/VI/09

2017

(6th Semester)

MATHEMATICS

Paper : MATH–361

(Modern Algebra)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Prove that, if H is a normal subgroup of a group G and K is a normal subgroup of G containing H, then

$$G/K \quad (G/H)/(K/H)$$ 8

(b) Show that every subgroup of an Abelian group is normal. 2

2. *(a)* If f is a homomorphism of a group G into a group G with kernel K, then prove that K is a normal subgroup of G. 5

(b) Prove that for an Abelian group the only inner automorphism is the identity mapping whereas for a non-Abelian group there exists non-trivial automorphisms. 5

UNIT—II

3. *(a)* Prove that every finite integral domain is a field. 7

(b) Prove that a skew field has no divisor of zero. 3

4. *(a)* Prove that a commutative ring with unity is a field, if it has no proper ideal. 4

(b) Prove that an ideal S of the ring of integers I is maximal if and only if S is generated by some prime integers. 6

UNIT—III

5. *(a)* Show that every field is a Euclidean ring. 6

(b) Find all the units of the integral domain of Gaussian integers. 4

6. *(a)* If a is a prime element of a unique factorization domain R and b, c are the elements of R, then prove that

$$a|bc \quad\quad a|b \text{ or } a|c$$ 5

(b) Let R be a Euclidean ring and a, b be two non-zero elements in R, then prove that, if b is not a unit in R, $d(ab) \quad d(a)$. 5

UNIT—IV

7. *(a)* Define basis of a finite dimensional vector space.

Let V is a finite dimensional vector space over the field F. Then show that any two bases of V have the same number of elements. 2+5=7

(b) Prove that if two vectors are linearly dependent, then one of them is a scalar multiple of the other. 3

8. *(a)* Prove that two finite dimensional vector spaces over the same field are isomorphic if and only if they are of the same dimension. 6

(b) In $V_3(R)$, where R is the field of real numbers, show that the set of vectors

$$\{(1, 2, 0), (0, 3, 1), (\;\; 1, 0, 1)\}$$

are linearly independent. 4

UNIT—V

9. *(a)* Show that two similar matrices A and B have the same characteristic polynomial and hence the same eigenvalues. 1½+1½=3

(b) Find the matrix representation of linear map $T : R^3 \quad R^3$ given by

$$T(x, y, z) \quad (z, y \quad z, x \quad y \quad z)$$

relative to the basis

$$[(1, 0, 1), (\;\; 1, 2, 1), (2, 1, 1)]$$ 7

10. *(a)* Prove that an $n \quad n$ matrix A over the field F is diagonalizable if and only if A has n linearly independent eigenvectors. 5

(b) Diagonalize the matrix

$$A \quad \begin{matrix} 1 & 3 & 3 \\ 3 & 5 & 3 \\ 6 & 6 & 4 \end{matrix}$$ 5

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Subject Code : MATH/VI/09

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

DEGREE 6th Semester
(Arts / Science / Commerce / ....) Exam., **2017**

Subject ....

Paper ....

INSTRUCTIONS TO CANDIDATES

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DEGREE 6th Semester
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Roll No. ....

Regn. No. ....

Subject ....

Paper ....

Descriptive Type
Booklet No. B ....

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/424

2017

(6th Semester)

MATHEMATICS

Paper : MATH–361

(Modern Algebra)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick (✓) mark against the correct answer in the brackets provided :

1. If a and b be two elements of a group G, then b is conjugate to a if

(a) $b \quad x^{\ 1}ax;\ x \quad G$ ( )

(b) $b \quad a^{\ 1}xa;\ x \quad G$ ( )

(c) $b \quad axa^{\ 1};\ x \quad G$ ( )

(d) $b \quad xax^{\ 1};\ x \quad G$ ( )

2. Which of the following statements is false?

(a) The centre Z of a group G is a normal subgroup of G. ( )

(b) The intersection of any two normal subgroups of a group is a normal subgroup. ( )

(c) A subgroup H of a group G is normal if and only if $x \ ^{1}Hx \quad H$. ( )

(d) A subgroup H of a group G is a normal subgroup of G if and only if $xH \quad Hx, \quad x \quad G$. ( )

3. In the ring of integers I, the maximal ideal is

(a) 12 ( )

(b) 5 ( )

(c) 9 ( )

(d) 15 ( )

4. The necessary and sufficient conditions for a non-empty subset K of a field F to be a subfield of F are

(a) $a \quad K, b \quad K \quad a \quad b \quad K$ and $ab \ ^{1} \quad K$ ( )

(b) $a \quad K, b \quad K \quad a \quad b \quad K$ and $ab \quad K$ ( )

(c) $a \quad K, b \quad K \quad a \quad b \quad K$ and $ab \ ^{1} \quad K$ ( )

(d) $a \quad K, b \quad K \quad a \quad b \quad K$ and $a \ ^{1}b \quad K$ ( )

5. The associates of a non-zero element $a \quad ib$ of the ring of Gaussian integers $D \quad \{a \quad ib,\ a,\ b \quad I\}$ are

(a) $a \quad ib,\ a \quad ib,\quad a \quad ib,\quad a \quad ib$ ( )

(b) $a \quad ib,\quad a \quad ib,\ b \quad ia,\ b \quad ia$ ( )

(c) $a \quad ib,\quad a \quad ib,\quad b \quad ia,\ b \quad ia$ ( )

(d) $a \quad ib,\quad a \quad ib,\quad b \quad ia,\ b \quad ia$ ( )

6. Let a and b be two non-zero elements in a Euclidean ring R. Then, if b is a unit in R,

(a) $d(ab) \quad d(a)$ ( )

(b) $d(ab) \quad d(a)$ ( )

(c) $d(a) \quad d(ab)$ ( )

(d) $d(a) \quad d(ab)$ ( )

7. Which of the following sets of vectors is linearly independent in $V_3(R)$?

(a) $\{(1,\ 2,\ 0),\ (0,\ 3,\ 1),\ (\ 1,\ 0,\ 1)\}$ ( )

(b) $\{(2,\ 1,\ 2),\ (8,\ 4,\ 8)\}$ ( )

(c) $\{(\ 1,\ 2,\ 1),\ (3,\ 0,\quad 1),\ (\ 5,\ 4,\ 3)\}$ ( )

(d) $\{(1,\ 2,\ 1),\ (3,\ 1,\ 5),\ (3,\quad 4,\ 7)\}$ ( )

8. Which of the following statements is false?

(a) Every superset of a linearly dependent set of vectors is linearly dependent. ( )

(b) There exists a basis for each finite dimensional vector space. ( )

(c) A system consisting of a single non-zero vector is always linearly independent. ( )

(d) Every linearly dependent subset of a finitely generated vector space $V(F)$ forms a part of a basis of V. ( )

9. The eigenvalues of a real skew symmetrix matrix are

(a) purely imaginary ( )

(b) all zero ( )

(c) purely imaginary or zero ( )

(d) all real ( )

10. Let $T : R^3 \quad R^2$ is a linear mapping of the matrix

$$\begin{matrix} 1 & 1 & 2 \\ 2 & 3 & 1 \end{matrix}$$

relative to the basis $\{(1, 1), (0, 1)\}$ of R^2 and

$$\{f_1 \quad (1, 1, 0), f_2 \quad (0, 1, 1), f_3 \quad (1, 0, 1)\}$$

of R^3 then

(a) $T(f_3) \quad (2, 3)$ ( )

(b) $T(f_3) \quad (1, 3)$ ( )

(c) $T(f_3) \quad (1, 4)$ ( )

(d) $T(f_3) \quad (2, 3, 1)$ ( )

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

Answer the following questions : 3×5=15

1. If H is the only subgroup of finite order m in the group G, then prove that H is a normal subgroup of G.

2. Show that the intersection of two ideals of a ring R is an ideal of R.

3. Prove that, if f is a homomorphism of a ring R into a ring R , with kernel S, then S is an ideal of R.

4. Prove that every superset of a linearly dependent set of vectors is linearly dependent.

5. If be an eigenvalue of a non-singular matrix A, then prove that ^{1} is an eigenvalue of A^{1}.

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2 0 1 7

(6th Semester)

MATHEMATICS

Paper : MATH–362

(Advanced Calculus)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* When do we say that a bounded real function f on $[a, b]$ be Riemann integrable? Show that every continuous function is Riemann integrable. 1+4=5

(b) A function f is bounded and integrable on $[a, b]$ and there exists a function F such that $F' = f$ on $[a, b]$, then prove that $\int_a^b f(x)\,dx = F(b) - F(a)$. 5

2. *(a)* If f_1 and f_2 are two bounded and integrable functions on $[a, b]$, then prove that $f = f_1 + f_2$ is also integrable on $[a, b]$ and $\int_a^b f\,dx = \int_a^b f_1 dx + \int_a^b f_2 dx.$ 5

(b) Compute the value of $\int_{-1}^{1} f\,dx$, where $f(x) = |x|$ by dividing the interval $[-1, 1]$ into $2n$ equal sub-intervals. 5

UNIT—II

3. *(a)* Prove that the improper integral

$$\int_a^b \frac{dx}{(x-a)^n}$$

converges if and only if $n < 1$. 5

(b) Examine the convergence of the following improper integrals : 2½+2½=5

(i) $\int_0^2 \frac{dx}{(1-x)^2}$

(ii) $\int \frac{dx}{(1+x^2)^2}$

4. *(a)* If is continuous in [0,) and

$$\lim_{x \quad 0} (x) \quad {}_0, \lim_{x} (x) \quad {}_1$$

then show that

$$_0 \frac{(ax) \quad (bx)}{x} dx \quad (\; _0 \quad _1) \log \frac{b}{a}$$ 4

(b) Show that

$$_0^1 x^{m \quad 1}(1 \quad x)^{n \quad 1} dx$$

exists if and only if m, n are both positive. 6

UNIT—III

5. *(a)* Prove that uniformly convergent improper integral of a continuous function is itself continuous. 4

(b) If $a \quad b$, then show that

$$_0^{\ /2} \log \frac{a \quad b\sin}{a \quad b\sin} \frac{d}{\sin} \qquad \sin^{\ 1} \frac{b}{a}$$ 6

6. *(a)* Examine the uniform convergence of the convergent improper integral

$$_{\ 1}^{1} \frac{\cos yx}{\sqrt{1 \quad x^2}} dx$$

in (,). 4

(b) If

$$f(x, y) \quad \frac{y^2}{x^2 \quad y^2} \text{ and } g(y) \quad _0^1 f(x, y)\, dx$$

then show that the right-hand and left-hand derivatives of g at $y \quad 0$ differ from each other and from $_0^1 f_y(x, 0)\, dx$. 6

UNIT—IV

7. *(a)* Evaluate $\quad x^2y^2 dx\, dy$ over the region bounded by $x \quad 0$, $y \quad 0$ and $x^2 \quad y^2 \quad 1$. 4

(b) Show that if $0 \quad h \quad 1$, then

$$_h^1 \; _h^1 f(x, y)\, dy \;\; dx \qquad _h^1 \; _h^1 f(x, y)\, dx \;\; dy \quad 0$$

but

$$_0^1 \; _0^1 f(x, y)\, dy \;\; dx \qquad _0^1 \; _0^1 f(x, y)\, dx \;\; dy$$

where

$$f(x, y) \quad \frac{y^2 \quad x^2}{(y^2 \quad x^2)^2}$$ 6

8. *(a)* Change the order of integration in the integral

$$\int_0^1 dx \int_0^{\sqrt{1-x^2}} \frac{1}{(1+e^y)\sqrt{1-x^2-y^2}}\, dy$$

and hence evaluate it. 5

(b) Show that

$$\int_0^1 \int_0^1 \frac{x-y}{(x+y)^2}\, dy\; dx \neq \int_0^1 \int_0^1 \frac{x-y}{(x+y)^2}\, dx\; dy$$ 5

UNIT—V

9. *(a)* Show that

$$f_n(x) = \frac{n}{x+n}$$

is uniformly convergent on $[0, k]$ whatever k may be but not uniformly convergent in $[0, \infty[$. 5

(b) Examine the term-by-term integration of the series whose sum to first n-terms is $n^2 x(1-x)^n$, $0 \le x \le 1$. 5

10. *(a)* Let $\{f_n\}$ be a sequence of function such that $\lim_{n \to \infty} f_n(x) = f(x)$, $x \in [a, b]$ and let

$$M_n = \sup_{x \in [a, b]} |f_n(x) - f(x)|$$

Then prove that the sequence $\{f_n\}$ converges uniformly to f on $[a, b]$ if and only if $M_n \to 0$ as $n \to \infty$. 5

(b) Examine whether the infinite series

$$\sum_{n=1}^{\infty} \frac{1}{n^3(1+nx^2)}$$

can be differentiated term-by-term between any finite limits. 5

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Subject Code : MATH/VI/10

Booklet No. A

Date Stamp ....

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To be filled in by the Candidate

DEGREE 6th Semester
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Subject ....
Paper ....

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Roll No. ....
Regn. No. ....

Subject ....
Paper ....

Descriptive Type
Booklet No. B ....

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MATH/VI/10

2 0 1 7

(6th Semester)

MATHEMATICS

Paper : MATH–362

(Advanced Calculus)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. For any two partitions P_1, P_2 on $[a, b]$ of a bounded function f, we have

(a) $L(P_1, f) \quad U(P_2, f)$ ☐

(b) $L(P_2, f) \quad U(P_1, f)$ ☐

(c) $U(P_2, f) \quad L(P_1, f)$ ☐

(d) $U(P_1, f) \quad U(P_2, f)$ ☐

2. If a bounded function f is integrable on $[a, b]$, then

(a) $\lim_{(P)} \quad S(P, f) \quad {}_a^b f\ dx$ ☐

(b) $\lim_{(P) \quad 0} S(P, f) \quad {}_a^b f\ dx$ ☐

(c) ${}_{\underline{a}}^{b} f\ dx \quad {}_a^{b} f\ dx$ ☐

(d) $L(P, f) \quad U(P, f) \quad S(P, f)$ ☐

where $L(P, f), U(P, f)$ and $S(P, f)$ are the lower Darboux, upper Darboux and Riemann sum of f corresponding to a partition P of $[a, b]$ with norm (P) .

3. Which of the following definite integrals is an improper integral?

(a) ${}_0^{/2} \sin x\, dx$ ☐

(b) ${}_1^{1} \dfrac{dx}{1 \quad x^2}$ ☐

(c) ${}_0^{4} \dfrac{dx}{(x \quad 2)(x \quad 3)}$ ☐

(d) ${}_0^{1} \dfrac{dx}{x(1 \quad x)}$ ☐

4. The improper integral $_{0}\ x^{n\ \ 1} e^{\ \ x} dx$ is convergent if and only if

(a) $n \quad 1$ ☐

(b) $n \quad 1$ ☐

(c) $n \quad 1$ ☐

(d) $n \quad 0$ ☐

5. The value of the improper integral

$$_{0}\ e^{\ \ x^2} dx$$

is

(a) $\frac{\sqrt{\ }}{2}$ ☐

(b) $\sqrt{\frac{\ }{2}}$ ☐

(c) $\frac{\ }{\sqrt{2}}$ ☐

(d) $\frac{\ }{2}$ ☐

6. The value of the improper integral

$$_0 \frac{dx}{a \quad b\cos x}$$

if a is positive and $|b| \quad a$, is

(a) $\dfrac{2}{(a^2 \quad b^2)^{1/2}}$ ☐

(b) $\dfrac{2}{(a^2 \quad b^2)^{3/2}}$ ☐

(c) $\dfrac{}{(a^2 \quad b^2)^{1/2}}$ ☐

(d) $\dfrac{}{(a^2 \quad b^2)^{3/2}}$ ☐

7. The value of the integral

$$_C \frac{dx}{x \quad y}$$

where C is the curve $x \quad at^2$, $y \quad 2at$, $0 \quad t \quad 2$ is

(a) $\dfrac{1}{6}$ ☐

(b) $\log 4$ ☐

(c) $\quad \dfrac{1}{6}$ ☐

(d) $\log 2$ ☐

8. The value of the double integral $\frac{x \quad y}{x \quad y}\, dx\, dy$

over $\frac{1}{2}, 1; \frac{1}{2}, 1$ is

(a) $\frac{1}{2}$ ☐

(b) $\frac{1}{2}$ ☐

(c) $\frac{}{2}$ ☐

(d) 0 ☐

9. With regards to uniform and point-wise convergence of sequences in [a, b], which of the following is true?

(a) Point-wise convergence Uniform convergence ☐

(b) Uniform convergence Point-wise convergence ☐

(c) Uniform limit Point-wise limit ☐

(d) All of the above ☐

10. The sequence of function $f_n(x) \quad nxe^{\quad nx^2}$ is point-wise, but not uniformly convergent on

(a) $[0, k]$, where $k \quad 1$ ☐

(b) [0,] ☐

(c) [0,) ☐

(d) (0,) ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

1. For the integral $\int_0^1 x\,dx$, find the upper Riemann integral corresponding to the division of [0, 1] into 6 equal interval.

2. Examine the convergence of the improper integral

$$_0 \frac{x^2}{\sqrt{x^5 \quad 1}}\, dx$$

3. Given that

$$\int_0^\infty \frac{\cos mx}{a^2+x^2}\,dx = \frac{\pi}{2a}e^{-ma}$$

then prove that

$$\int_0^\infty \frac{x\sin mx}{1+x^2}\,dx = \frac{\pi}{2}e^{-m}$$

4. Show that

$$\int_C (x^2 + y^2)\,dy = \frac{46}{15}a^3$$

where C is the arc of the parabola $y^2 = 4ax$ between $(0, 0)$ and $(a, 2a)$.

5. Show that

$$f_n(x) \quad \frac{nx}{1 \quad n^2 x^2}$$

is not uniformly convergent in any interval containing zero.

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MATH/VI/11

2017

(6th Semester)

MATHEMATICS

Paper : Math–363

(Mechanics)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Forces equal to $3P$, $7P$ and $5P$ act along the side AB, BC and CA of an equilateral triangle ABC. Find the magnitude, direction and line of action of the resultant. 5

(b) Determine how high can a particle rest inside a rough hollow sphere of radius a, if the coefficient of friction is $\frac{1}{\sqrt{3}}$. 5

2. *(a)* A uniform rod rests within a fixed vertical subtending an angle 2   at the centre, its upper end is smooth and its lower end is rough. Show that the angle which the rod makes with horizon cannot be greater than   , where

$$\tan \qquad \frac{\sin}{\cos \qquad \cos(\quad 2 \quad)}$$ 5

(b) A hemisphere of radius a and weight W is placed with its curved surface on a smooth table and a string of length $l(\quad a)$ is attached to a point on its rim and a point on the table. Find the position of equilibrium and prove that the tension of the string is

$$\frac{3W}{8} \frac{a \quad l}{\sqrt{2al \quad l^2}}$$ 5

UNIT—II

3. *(a)* State and prove perpendicular axes theorem on moments of inertia. 5

(b) A thin uniform wire is bent into the form of a triangle ABC and heavy particles of weights P, Q, R are placed at the angular points. If the centre of mass of the particles coincides with that of the wire, then prove that

$$\frac{P}{b \quad c} \quad \frac{Q}{c \quad a} \quad \frac{R}{a \quad b}$$ 5

4. *(a)* If a piece of wire is bent into a shape of an isosceles triangle, whose equal sides are a and b, show that the distance of the CG from the base is

$$\frac{a}{2}\sqrt{\frac{2a \quad b}{2a \quad b}}$$ 5

(b) From a circular disc of radius r, a circle is cut out such that its diameter is the radius of the disc. Find the CG of the remaining disc. 5

UNIT—III

5. *(a)* Prove that if the tangential and normal component of acceleration of a particle describing a plane curve be constant throughout the motion, the angle through which the direction of motion turns in time t is given by

$$A \log(1 \quad Bt)$$

where A and B are constants. 5

(b) A curve is described by a particle having constant acceleration in a direction inclined at a constant angle to the tangent. Show that the curve is an equiangular spiral. 5

6. *(a)* A train travels a distance s in t second. It starts from rest and ends at rest. In the first part of the journey it moves with a constant acceleration f and in the second part with constant retardation f . Show that if s is the distance between the two stations, then

$$t \quad \sqrt{2s \quad \frac{1}{f} \quad \frac{1}{f}}$$ 5

(b) A particle rests in equilibrium under the attraction of two centres of forces which attract directly as the distance between the two centres, their intensities being , . The particle is displaced slightly towards one of them. Show that the time of small oscillation is

$$\frac{2}{\quad}$$ 5

UNIT—IV

7. *(a)* If t be the time in which a projectile reaches a point P in its path and t the time from P till it reaches the horizontal plane through the point of projection, show that the height of P above the horizontal plane is $\frac{1}{2}gtt$. Also prove that the greatest height of the projectile is $\frac{1}{8}g(t \quad t)^2$. 5

(b) A body is projected at an angle to the horizon, so as just to clear two walls of equal height a at a distance $2a$ from each other. Show that the range is equal to

$$2a\cot\frac{\quad}{2}$$ 5

8. *(a)* From a point in a given inclined plane, two bodies are projected with the same velocity in the same vertical plane at right angles to one another. Show that the difference of their ranges is constant. 5

(b) A particle of mass m is falling under the influence of gravity through a medium whose resistance equals times the velocity. If the particle is released from rest, then show that the distance fallen through in time t is

$$g\frac{m^2}{{}^2}\quad e^{\frac{t}{m}}\quad 1\quad \frac{t}{m}$$ 5

UNIT—V

9. *(a)* A shot of mass is projected from a gun of mass M by an explosion which generates a kinetic energy E. Show that the gun recoil with a velocity

$$\sqrt{\frac{2mE}{M(M\quad m)}}$$

and the initial velocity of the shot is

$$\sqrt{\frac{2mE}{m(M\quad m)}}$$ 5

(b) The earth's attraction on a particle varies inversely as the square of its distance from the earth's centre. A particle whose weight on the surface of the earth is W, falls to the surface of the earth from a height $5a$ above it. Show that the work done by the earth's attraction is

$$\frac{5aW}{6}$$

where a is the radius of the earth. 5

10. *(a)* A smooth sphere of mass m, travelling with a velocity u, impinges obliquely on a smooth sphere of mass M at rest, the original line of motion of the first sphere making an angle with the line of centres at the moment of the impact. If the coefficient of restitution be e, show that the impinging sphere will be deflected through a right angle, if

$$\tan^2 \quad \frac{eM\quad m}{M\quad m}$$

and that its velocity perpendicular to its original line of motion will be obtained, if $45°$. 5

(b) A sphere impinges directly on an equal sphere which is at rest. Show that a fraction

$$\frac{1}{2}(1 \quad e^2)$$

of the original kinetic energy is lost during the impact. 5

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Subject Code : MATH/VI/11

Booklet No. A

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DEGREE 6th Semester
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Subject ....
Paper ....

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Roll No. ....
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Paper ....

Descriptive Type
Booklet No. B ....

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/426

2 0 1 7

(6th Semester)

MATHEMATICS

Paper : Math–363

(Mechanics)

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Put a Tick ☑ mark against the correct answer in the box provided : 1×10=10

1. The least force P required to pull a body down an inclined plane inclined at an angle to the horizontal is attained, when (where is the angle of friction and is the angle made by the force P to the inclined plane)

(a) ☐

(b) ☐

(c) ☐

(d) ☐

2. Suppose that a system of forces acts at different points of a rigid body is in equilibrium, then

(a) the resultant R must vanish ☐

(b) the moment of all the forces w.r.t. each of three collinear points is zero ☐

(c) the couple G must vanish ☐

(d) the resultant R and the couple G must separately vanish ☐

3. The centre of gravity of three uniform rods forming a triangle is at

(a) the incenter of the triangle ☐

(b) the centroid of the triangle ☐

(c) the orthocenter of the triangle ☐

(d) None of the above ☐

4. The moment of inertia of a plane distribution with respect to any normal axis

(a) is equal to its moment of inertia ☐

(b) cannot be determined ☐

(c) is equal to the sum of the moments of inertia ☐

(d) is equal to the product of inertia ☐

5. If the position of a moving particle at time t referred to rectangular axes is given by $x \quad at$, $y \quad bt \quad ct^2$, where a, b and c are constants, then its acceleration at time t is

(a) along the x-axis ☐

(b) $a \quad b \quad c$ along the x-axis ☐

(c) $\sqrt{a^2 \quad b^2}$ along the y-axis ☐

(d) $2c$ along the y-axis ☐

6. If a particle moves so that its normal acceleration is always zero, then its path is

(a) a circle ☐

(b) a parabola ☐

(c) a straight line ☐

(d) None of the above ☐

7. If a particle moves along the x-axis under an attraction towards the origin O, varying inversely as the square of the distance from it, then the equation of motion is

(a) $x \quad \frac{}{x^2}$ ☐

(b) $x \quad \frac{}{x^2}$ ☐

(c) $x \quad x^2$ ☐

(d) $x \quad x^2$ ☐

8. If a particle is projected with a velocity u from the ground at an angle with the horizontal, then the velocity of the particle at height h is

(a) $\sqrt{u^2 \quad 2gh}$ ☐

(b) $u^2 \quad 2gh$ ☐

(c) $\sqrt{u \quad h \sin}$ ☐

(d) None of the above ☐

9. A smooth sphere of mass m strikes a plane normally and is rebounded. If e is the coefficient of restitution, then the loss of kinetic energy is

(a) $\frac{1}{2}me^2u^2$ ☐

(b) $\frac{1}{2}m(1 \quad e^2)u^2$ ☐

(c) $\frac{1}{2}m(1 \quad e^2)u^2$ ☐

(d) None of the above ☐

10. A sphere falling vertically from a height h impinges on a horizontal fixed table and rebounds to a height h_1. If e is the coefficient of restitution between the sphere and the plane, then

(a) $h_1 \quad e^2h$ ☐

(b) $h_1 \quad 2e^2h$ ☐

(c) $h_1 \quad \sqrt{eh}$ ☐

(d) None of the above ☐

SECTION—B

(*Marks* : 15)

Answer the following questions : 3×5=15

1. Prove that if three forces, acting in one plane upon a rigid body, keep it in equilibrium, then they must either meet in a point or be parallel.

2. Prove that the centre of gravity of a triangular area coincides with that of three equal particles placed at the middle points of its sides.

3. A particle moves in a plane with constant speed. Prove that its acceleration is perpendicular to its velocity.

4. Find the greatest height attained by the projectile thrown with a velocity u at an angle   with the horizontal.

5. A billiard ball impinges directly on another equal ball at rest. If e is the coefficient of restitution, prove that their velocities after impact are in the ratio $(1 \quad e):(1 \quad e)$.

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2017

(6th Semester)

MATHEMATICS

Paper : Math–364 (A)

(Computer Programming in C)

Full Marks : 55

Time : 2½ hours

(PART : B—DESCRIPTIVE)

(*Marks* : 35)

The figures in the margin indicate full marks for the questions

1. *(a)* Write a simple C program and explain the structure of C program. 4

(b) Explain formatted and unformatted input/output functions in brief. 3

Or

Differentiate between operators and operands. Discuss the commonly used operators in C programming. 7

2. Write a program to differentiate break and continue statements. Explain briefly. 7

Or

What is nested for loop? Write a program to illustrate nested for loop and explain it. 7

3. Explain and differentiate call by value and call by reference with examples. 7

Or

(a) Write a program for insertion sort. 3

(b) What is recursive function? Write a C program of factorial by using recursive function. 1+3=4

4. Write a C program implementing pointer and function. 7

Or

Write a C program of function for concatenation of two strings, comparing two strings. 7

5. What is structure within a structure? Write a C program to explain structure within a structure and explain the program. 7

Or

Explain the function fprintf ( ) and fscanf ( ) and write complete example of file management. 7

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Subject Code : MATH/VI/12 (a)

Booklet No. A

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2 0 1 7

(6th Semester)

MATHEMATICS

Paper : Math–364 (A)

(Computer Programming in C)

(PART : A—OBJECTIVE)

(*Marks* : 20)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

1. Put a Tick (✓) mark against the correct answer in the brackets provided : 1×10=10

(a) What will be the output of the following arithmetic expression?

5 3 2%10 8 6

(i) 27 ( )

(ii) 42 ( )

(iii) 32 ( )

(iv) 37 ( )

(b) What will be the output of the code

$b \qquad c \qquad c$

if *c* is 10 initially?

(i) 20 ( )

(ii) 21 ( )

(iii) 22 ( )

(iv) 23 ( )

(c) In case of ordinary integer variable

(i) the leftmost bit is reserved for sign ( )

(ii) the rightmost bit is reserved for sign ( )

(iii) no bit is reserved for sign ( )

(iv) all bits are reserved for sign ( )

(d) Which key word is used for skipping part of the loop?

(i) Skip ( )

(ii) Continue ( )

(iii) Break ( )

(iv) Jump ( )

(e) What is the only function all C programs must contain?

(i) start( ) ( )

(ii) system( ) ( )

(iii) main( ) ( )

(iv) include( ) ( )

(f) The declaration void function (int) indicates function which

(i) return but no argument ( )

(ii) return nothing but argument ( )

(iii) no return no argument ( )

(iv) Both *(i)* and *(ii)* ( )

(g) Which of the following is not logical operator?

(i) & ( )

(ii) && ( )

(iii) | | ( )

(iv) ! ( )

(h) In C, if you pass an array as an argument to a function, what actually gets passed?

(i) Value of elements in the array ( )

(ii) First element of the array ( )

(iii) Base address of the array ( )

(iv) Address of the last element of the array ( )

(i) If the two strings are identical, then strcmp ( ) function returns

(i) –1 ( )

(ii) 1 ( )

(iii) 0 ( )

(iv) True ( )

(j) What is the similarity among structure, union and enumeration?

(i) All of them let you define new data types ( )

(ii) All of them let you define new values ( )

(iii) All of them let you define new pointers ( )

(iv) All of them let you define new structures ( )

SECTION—B

(*Marks* : 10)

2. Answer the following questions : 2×5=10

(a) Differentiate between while loop and do-while loop.

(b) What is multidimensional array?

(c) Mention two advantages and disadvantages of pointer.

(d) Briefly explain union.

(e) Write on differential diversion and iteration.

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2 0 1 7

(6th Semester)

MATHEMATICS

Paper : MATH–364 (B)

(Computer Programming in FORTRAN)

Full Marks : 55

Time : 2½ hours

(PART : B—DESCRIPTIVE)

(*Marks* : 35)

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. (*a*) Write a flowchart to find the largest of three numbers. 3

(*b*) What is the final value of J calculated in the following? 2+2=4

(*i*) J=B*B/2.+B*4./A–B+A**3 (A=3.0, B=2.0)

(*ii*) J=K/2*4+15/4+K**2 (K=4)

2. (*a*) Write a FORTRAN program to find the factorial of a positive integer. 4

(*b*) Write an algorithm to find the perimeter of a rectangle. 3

UNIT—II

3. (*a*) Write the general form of DATA statement. What will be the value of A and B from the following statements? 1+3=4

(*i*) DATA A,B/25.2,19.8/

(*ii*) DATA A,B/2*3.7/

(*b*) If I = J = 1; what values the following logical expressions have? 3

((I.GT.0).AND.(J.LT.0)).OR.(.NOT.(J.GT.0).AND..NOT.(I.LT.0))

4. (*a*) Write short notes on any *two* of the following : 2×2=4

(*i*) Complex variable

(*ii*) Double-precision statement

(*iii*) Logical variable

(*b*) Write a program to find the distance between two points (x_1, y_1) and (x_2, y_2). 3

UNIT—III

5. *(a)* Write the general form of IF-THEN-ELSE statement. 2

(b) N is said to be a prime number if its only divisors are 1 and itself. Write a FORTRAN program using 'DO loop' that reads an integer N > 2 and determines if N is a prime by testing if N is divisible by any of the integers 2, 3,...N/2. 5

6. *(a)* Write a program to find the sum of digit of a five-digit number using DO loop. 4

(b) We give below a program to count and print out the number of zeros out of N integers input by the programmer. Find the errors in the program. Indicate whether the errors are 'syntax errors' or 'logical errors'. Also correct the program if possible : 3

```
      INTEGER ZEROD, S, D
      READ (*,*) N
      ZEROD = 0
      DO 25 S = 1, N
   2  READ (*,*) D
      IF (D.NE.O) GOTO 2
      ZEROD = ZEROD + 1 25
      WRITE (*,*) ZEROD
      END
```


UNIT—IV

7. *(a)* Write a program to arrange the numbers in ascending orders. 3

(b) Write a program to find the product of two matrices. 4

8. *(a)* Write a program to find the trace of an $m \quad n$ matrix. 3

(b) Write a function subprogram to evaluate : 4

$$f(x) \quad 2x^2 \quad 3x \quad 4 \qquad \text{for} \quad x \quad 2$$
$$f(x) \quad 0 \qquad\qquad\qquad \text{for} \quad x \quad 2$$
$$f(x) \quad\; 2x^2 \quad 3x \quad 4 \quad \text{for} \quad x \quad 2$$

UNIT—V

9. Write a function subprogram to find the factorial of *n*. Also write a main program to call this function and evaluate ${}^nC_r$, where

$$ {}^nC_r \quad \frac{n!}{r!(n \quad r)!} $$ 3+4=7

10. *(a)* Write a subroutine to find the roots of a quadratic equation $ax^2 \quad bx \quad c \quad 0.$ 5

(b) Write a note on COMMON statement in FORTRAN. 2

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Subject Code : MATH/VI/12 (b)

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2 0 1 7

(6th Semester)

MATHEMATICS

Paper : MATH–364 (B)

(Computer Programming in FORTRAN)

(PART : A—OBJECTIVE)

(*Marks* : 20)

Answer **all** questions

SECTION—A

(*Marks* : 5)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. A step-by-step procedure to solve a problem is called

(a) flowchart ☐

(b) algorithm ☐

(c) array ☐

(d) loop ☐

2. The general form of computed GOTO statement is

(a) GOTO n ☐

(b) GOTO(n1,n2,n3,...,nm) i ☐

(c) GOTO(n1,n2,n3,...,nm), i ☐

(d) GOTO i, (n1,n2,n3,...,nm) ☐

3. Which one of the following is valid FORTRAN real variable name?

(a) GOTO ☐

(b) ABACUSA ☐

(c) ABRAHAM ☐

(d) AIZAWL ☐

4. Choose the invalid FORTRAN statement :

(a) IF(N.LT.0) Y=2.3 ☐

(b) DO 10 J=1.13 ☐

(c) IF (D) 11,22,33 ☐

(d) WRITE (*,*)“123=X” ☐

5. Which one of the following is invalid real constant in exponent form?

(a) 625E10 ☐

(b) 567.0E11 ☐

(c) 123.E+8 ☐

(d) 234.E+07 ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

1. Convert the following algebraic expression into FORTRAN expression :

$$\sqrt{\sin \frac{3ax \quad 3}{c^2}}$$

2. Trace through the following logical expressions step-by-step, illustrating the hierarchy of operations. For each problem, assume that X= –5.1, Y=3, Z=10.0 :

(X.GE.Z.AND..NOT.((Z*Y.LE.X).OR..NOT.(X.EQ.Y)))

3. Write the general form of arithmetic IF statement.

4. Write any two FORTRAN library functions with suitable examples.

5. Write the arithmetic statement function to find the area of a circle.

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2017

(6th Semester)

MATHEMATICS

Paper : MATH–364 (C)

(Astronomy)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* Prove that the sides and the angles of a polar triangle are respectively supplements of the angles and sides of primitive triangle. 5

(b) If $B \quad C$, show that

$$\sin 2B \quad \sin 2C \quad 0$$ 5

2. *(a)* In a spherical triangle ABC, prove that

$$\sin\frac{A}{2} \quad \sqrt{\frac{\sin(s \quad b)\sin(s \quad c)}{\sin b \sin c}}$$

where $2s \quad a \quad b \quad c$. 5

(b) In an equilateral spherical triangle ABC, show that

$$1 \quad 2\cos a \quad \cot^2\frac{A}{2}$$ 5

UNIT—II

3. *(a)* Given the right ascension , the declination and the obliquity of the ecliptic of a star, show that its latitude and longitude can be calculated from the formulas—

(i) sin cos sin sin cos sin ;

(ii) $\tan \quad \frac{\sin \tan \quad \cos \sin}{\cos}$ 3+3=6

(b) If H is the hour angle of a star at rising, then show that

$$\tan^2\frac{H}{2} \quad \frac{\cos(\quad)}{\cos(\quad)}$$ 4

4. If the zenith distance z of a star is less than the colatitude C, prove that

$$C \quad x \quad \cos^{\;1}(\cos z \;\; \sec y)$$

where

$$\tan x \quad \cot \quad \cos H \quad \text{and} \quad \sin y \quad \cos \quad \sin H$$

H being the hour angle. 10

UNIT—III

5. *(a)* Prove that if the declination of a star is unaffected by refraction at a given moment, azimuth is then a maximum. 4

(b) Find the effect of aberration on right ascension of a star. 6

6. If ⊙ is the longitude of the Sun and   is the right ascension, then show that the greatest value of   ⊙ occurs when $\tan\odot \quad (\sin \quad)^{1/2}$ and $\tan \quad (\cos \quad)^{1/2}$, where   is the obliquity of the ecliptic. 10

UNIT—IV

7. Assuming the Venus and the Earth describe circular orbits in the ecliptic, show that Venus will appear the brightest at an elongation   given by

$$\cos \quad \frac{2}{3}\{(3 \quad a^2)^{1/2} \quad a\}$$

where a is the heliocentric distance of Venus in astronomical unit. 10

8. If a and b are radii of the orbits of the Earth (E) and a superior planet P ; u and v their respective linear velocities, then prove that the square of the velocity of P, relative to E at a stationary point is

$$\frac{(u^2 \quad v^2)(bu \quad av)}{(bu \quad av)}$$

10

UNIT—V

9. *(a)* Deduce Kepler's third law from Newton's law of universal gravitation. 5

(b) Prove that the dip of the visible horizon at a height h above the earth's surface is $\sqrt{\frac{2h}{a}}$, a is the radius of the earth. 5

10. Show that in a place of latitude , the sunrise will be accelerated by

$$t \quad \frac{12}{\quad}\sqrt{\frac{2h}{a}} \quad \frac{1}{\sqrt{\cos^2 \quad \cos^2}} \quad \text{hr}$$

at a top of a mountain of height h,   being the declination of the Sun and a the radius of the earth. 10

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Subject Code : MATH/VI/12 (c)

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2 0 1 7

(6th Semester)

MATHEMATICS

Paper : MATH–364 (C)

(Astronomy)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. The section of a sphere by a plane is

(a) a parabola ☐

(b) a circle ☐

(c) an ellipse ☐

(d) a hyperbola ☐

2. One of the four parts formula in a spherical triangle *ABC* is

(a) $\sin C \sin B \quad \cos c \cot a \quad \sin B \cot A$ ☐

(b) $\sin C \sin B \quad \sin c \cot a \quad \cos B \cot A$ ☐

(c) $\cos C \cos B \quad \sin c \cot a \quad \sin A \cot B$ ☐

(d) $\cos C \cos B \quad \sin c \cot a \quad \sin B \cot A$ ☐

3. Which one of the following statements is not true?

(a) The altitude of the pole is equal to the latitude of that place. ☐

(b) The angle between the equator and the ecliptic is known as obliquity of ecliptic. ☐

(c) The angular distance of the star from the ecliptic measure along the secondary to the ecliptic through the star is called the longitude of the star. ☐

(d) If the declination and the longitude of a star are equal, its right ascension and latitude will also be equal. ☐

4. The evening twilight ceases when the Sun's zenith distance has become

(a) 18 ☐ *(b)* 72 ☐

(c) 90 ☐ *(d)* 108 ☐

5. The angle between real direction of the star and the direction of the earth's motion is called

(a) parallax ☐

(b) aberration ☐

(c) earth's way ☐

(d) None of the above ☐

6. The equation of time arises due to

(a) variable motion of the Sun along the ecliptic ☐

(b) obliquity of the ecliptic ☐

(c) Both *(a)* and *(b)* ☐

(d) None of the above ☐

7. If S is the geocentric longitude of the planet, then the planet's motion is said to be stationary when ds / dt is

(a) positive ☐

(b) negative ☐

(c) 0 ☐

(d) None of the above ☐

8. If T_1 and T_2 be the periodic times of inferior and superior planets and S be the synodic period, then

(a) $\frac{1}{S} \quad \frac{1}{T_1} \quad \frac{1}{T_2}$ ☐

(b) $\frac{1}{S} \quad \frac{1}{T_1} \quad \frac{1}{T_2}$ ☐

(c) $\frac{1}{S} \qquad \frac{1}{T_1} \quad \frac{1}{T_2}$ ☐

(d) $S \quad T_1 \quad T_2$ ☐

9. The boundary of the earth's surface visible from any position is called the

(a) imaginary horizon ☐

(b) visible horizon ☐

(c) dip of horizon ☐

(d) celestial horizon ☐

10. Which of the following statements is true?

(a) Velocity of a planet is greatest when it is nearest to the Sun. ☐

(b) Velocity of the planet is least when it is nearest to the Sun. ☐

(c) Both *(a)* and *(b)* are true ☐

(d) Neither *(a)* nor *(b)* is true ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

1. In a right-angled polar triangle ABC, $C = \frac{\pi}{2}$, then prove that $\sin a = \sin A \sin C$.

2. Define altitude and zenith distance of a star and find the relation between them.

3. Where must be a star situated so as to have no displacement due to annual parallax?

4. If the line joining two planets to one another subtends an angle 60° at the Sun, when the planets appear to each other to be stationary, then show that

$$a^2 \quad b^2 \quad 7ab$$

where a and b are the distances of planets from the Sun.

5. State Kepler's laws of planetary motion.

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