2017

(5th Semester)

MATHEMATICS

Paper : MATH–351

(Computer-oriented Numerical Analysis)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Define factorial polynomial. Express $f(x) \quad x^3 \quad 3x^2 \quad 5x \quad 7$ in terms of factorial polynomial taking $h \quad 2$ and find its differences. 1+3+1=5

(b) Express any value of y in terms of y_n and the backward differences of y_n. Find the value of $y(\quad 1)$ if $y(0) \quad 2$, $y(1) \quad 9$, $y(2) \quad 28$, $y(3) \quad 65$, $y(4) \quad 126$ and $y(5) \quad 217$. 2+3=5

2. *(a)* Find a real root of the equation $e^x \quad 3x \quad 0$ correct up to four decimal places by the method of successive iteration. 5

(b) Find the positive real roots of the equation $x\log_{10} x \quad 1 \; 2 \quad 0$ by regula-falsi method. 5

UNIT—II

3. *(a)* Obtain Newton's backward interpolation formula with equal intervals of the argument. 6

(b) Find a polynomial of degree four which takes the values : 4

x	2	4	6	8	10
y	0	0	1	0	0

4. *(a)* Obtain Lagrange's interpolation formula for unequal intervals. 6

(b) Using Newton's divided difference formula, find the values of $f(2)$ and $f(8)$ from the following table : 4

x	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

UNIT—III

5. *(a)* Solve the following system of equations by Gaussian elimination method : 5

$$x \quad 2y \quad z \quad 3$$
$$2x \quad 3y \quad 3z \quad 10$$
$$3x \quad y \quad 2z \quad 13$$

(b) Explain the difference between Gauss elimination and Gauss-Jordan elimination methods. Write an algorithm for Gauss-Jordan elimination method. 2+3=5

6. *(a)* Solve, by Gauss-Seidel method, the following system of equations: 4

$$2x \quad y \quad 3$$
$$2x \quad 3y \quad 5$$

(b) By Routh's method, solve the following system of simultaneous equations : 6

$$x \quad y \quad z \quad 3$$
$$2x \quad y \quad 3z \quad 16$$
$$3x \quad y \quad z \quad 3$$

UNIT—IV

7. *(a)* Using Newton's forward difference interpolation formula, find the derivatives of $y \quad f(x)$ passing through $n \quad 1$ points up to third-order derivatives. 5

(b) Evaluate

$$\int_0^1 \frac{dx}{1 \quad x^2}$$

using trapezoidal rule with $h \quad 0\ 2$. Hence, obtain an approximate value of . 5

8. *(a)* Find the first two derivatives of $y \quad x^{1/3}$ at $x \quad 50$ and $x \quad 56$ from the following table : 5

x	50	51	52	53	54	55	56
$y \quad \sqrt[3]{x}$	3·6840	3·7084	3·7325	3·7563	3·7798	3·8030	3·8259

(b) Evaluate

$$\int_0^{/2} \sqrt{\sin \quad} \, d$$

using Simpson's one-third rule taking ten equal intervals. 5

UNIT—V

9. *(a)* Using Taylor series method, find the values of $y(1 \quad 1)$ and $y(1 \quad 2)$ correct up to four decimal places given below : 5

$$\frac{dy}{dx} \quad xy^{1/3}, \; y(1) \quad 1$$

(b) Solve

$$\frac{dy}{dx} \quad x \quad y$$

given $y(0) \quad 1$. Obtain the values of $y(0 \;\; 1)$ and $y(0 \;\; 2)$ using Picard's method. 5

10. *(a)* Solve the differential equation

$$\frac{dy}{dx} \quad 1 \quad y$$

given $y(0) \quad 0$. Using modified Euler's method, find $y(0 \;\; 1)$ and $y(0 \;\; 2)$. 4

(b) Apply the fourth-order Runge-Kutta method to find $y(0 \;\; 3)$, given that

$$\frac{dy}{dx} \quad y \quad xy^2 \quad 0, \; y(0) \quad 1$$

by taking $h \quad 0 \;\; 1$ (correct up to four decimal places). 6

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Subject Code : MATH/V/05

Booklet No. A

Date Stamp ....

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DEGREE 5th Semester
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Subject ....

Paper ....

INSTRUCTIONS TO CANDIDATES

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Roll No. ....

Regn. No. ....

Subject ....

Paper ....

Descriptive Type
Booklet No. B ....

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Signature of Examiner(s)

Signature of Invigilator(s)

MATH/V/05

2017

(5th Semester)

MATHEMATICS

Paper : MATH–351

(Computer-oriented Numerical Analysis)

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. Which of the following relations is true?

(a) E 1 ☐

(b) E 1 ☐

(c) E 1 ☐

(d) E 1 ☐

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2. A polynomial of odd degree with real coefficients always has

(a) at least no real roots ☐

(b) at least one real root ☐

(c) at least two real roots ☐

(d) at least one real root and one imaginary root ☐

3. A forward difference table contains n arguments, then the highest-order differences in the table is

(a) $n \quad 1$ ☐

(b) n ☐

(c) $n \quad 1$ ☐

(d) None of the above ☐

4. Lagrange's interpolation polynomial for the data $f(0) \quad 0$, $f(1) \quad 0$ and $f(3) \quad 6$ is

(a) $(x \quad 1)(x \quad 1)$ ☐

(b) $x^2 \quad 1$ ☐

(c) $x(x \quad 1)$ ☐

(d) $x(x \quad 1)$ ☐

5. If A is the coefficient matrix obtained from a system of simultaneous equations, then the system has a single and unique solution if and only if the determinant of A is

(a) equal to zero ☐

(b) not equal to zero ☐

(c) greater than zero ☐

(d) smaller than zero ☐

6. Gauss-Jordan elimination method reduces the coefficient matrix of the simultaneous equations into

(a) column matrix ☐

(b) lower triangular matrix ☐

(c) upper triangular matrix ☐

(d) identity matrix ☐

7. If $f(3)$ 0, $f(4)$ 2 and $f(5)$ 6, then f (3) is equal to

(a) 0 ☐

(b) 1 ☐

(c) 2 ☐

(d) 3 ☐

8. The value of

$$^{/3}_{/6} \sin x \, dx$$

using trapezoidal rule by taking $h \quad \overline{6}$ is

(a) $\frac{1 \quad \sqrt{3}}{24}$ ☐

(b) $\frac{1 \quad \sqrt{3}}{12}$ ☐

(c) $\frac{(1 \quad \sqrt{3})}{24}$ ☐

(d) $\frac{(1 \quad \sqrt{3})}{12}$ ☐

9. Given $y \quad y^2$ and $y(0) \quad 1$. The value of y at $x \quad 1$ taking $h \quad 1$ using Euler's method is

(a) 0 ☐

(b) –2 ☐

(c) 1 ☐

(d) 2 ☐

10. The Runge-Kutta method of second order is exactly similar to

(a) Picard's method ☐

(b) modified Euler's method ☐

(c) Euler's method ☐

(d) Taylor series method ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

1. Establish the relation between operators E and .

2. Construct the divided difference table of $f(x) \quad x^3 \quad x \quad 2$ for the arguments 1, 3, 6, 11.

3. Solve the given equations by Gauss-Jordan method :

$$2x \quad y \quad 3$$

$$7x \quad 3y \quad 4$$

4. Evaluate

$$\int_1^3 x^4 \, dx$$

using Simpson's one-third rule by dividing the range into four equal intervals.

5. Using Milne's method, find $y(4\ \ 4)$, given $5xy \quad y^2 \quad 2 \quad 0$, $y_1 \quad 0\ \ 0493$, $y_2 \quad 0\ \ 0467$ and $y_3 \quad 0\ \ 0452$.

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2017

(5th Semester)

MATHEMATICS

Paper : MATH–352

(Real Analysis)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Prove that every open cover of a compact set admits of finite subcover. 6

(b) Show that a set is closed if and only if its complement is open. 4

2. State and prove the Bolzano-Weierrstrass theorem for the subsets of R^n. 2+8=10

UNIT—II

3. *(a)* State and prove intermediate value theorem of a real valued function of several real variables. 1+5=6

(b) Prove that the function

$$\frac{1}{1!} \quad \frac{1}{2!} \quad \cdots \quad \frac{1}{n!}, \quad n \quad N$$

is not continuous at (0, 0). 4

4. *(a)* Define limits of functions of several variables. Let

$$\lim_{x \quad a} f(x) \quad b \text{ and } b \quad (b_1, b_2, \ldots, b_n),$$

$$f \quad (f_1, f_2, \ldots, f_m)$$

then show that $\lim_{x \quad a} f_i(x) \quad b_i, \ 1 \quad i \quad m.$ 2+5=7

(b) Show that the function

$$f(x, y) \quad \frac{x^2 \quad y^2}{x^2 \quad y^2}$$

has repeated limits. 3

UNIT—III

5. *(a)* Given

$$f(x, y) \quad \begin{cases} \dfrac{xy}{\sqrt{x^2 \quad y^2}}, & (x, y) \quad (0, 0) \\ 0, & (x, y) \quad (0, 0) \end{cases}$$

Show that f is continuous, possesses partial derivatives but is not differentiable at (0, 0). 6

(b) Let

$$f(x, y) \quad \begin{cases} \dfrac{x^2 y}{x^4 \quad y^2}, & (x, y) \quad (0, 0) \\ 0, & (x, y) \quad (0, 0) \end{cases}$$

Show that f has a directional derivative at (0, 0) in any direction (l, m), $l^2 \quad m^2 \quad 1$ but f is discontinuous at (0, 0). 4

6. *(a)* Prove that a function which is differentiable at a point admits of partial derivatives at the point. 5

(b) Show that the functions $u \quad x \quad y \quad z$, $v \quad x \quad y \quad z$, $w \quad x^2 \quad y^2 \quad z^2 \quad 2yz$ are not independent. Find the relation among them. 5

UNIT—IV

7. State and prove Young's theorem. 2+8=10

8. *(a)* If

$$f(x, y) \quad \begin{cases} (x^2 \quad y^2)\tan^{\ 1}\dfrac{y}{x}, & x \quad 0 \\ \dfrac{}{2}y^2, & x \quad 0 \end{cases}$$

then show that $f_{xy}(0, 0) \quad f_{yx}(0, 0)$. 5

(b) Show that the function

$$f(x, y) \quad 2x^4 \quad 3x^2 y \quad y^2$$

has neither a maxima nor a minima at (0, 0). 5

UNIT—V

9. *(a)* Let X be the set of all sequences of complex numbers. Show that the function d defined by

$$d(x, y) \quad \frac{1}{2^n}\frac{|x_n \quad y_n|}{(1 \quad |x_n \quad y_n|)},$$

$$x \quad \{x_n\},\ y \quad \{y_n\} \quad X$$

is a metric space. 5

(b) Prove that every compact subset A of a metric space (X, d) is bounded. 5

10. Define complete metric space. Let X be the set of all continuous real valued functions defined on [0, 1] and let

$$d(x, y) \quad {}_{0}^{1}|x(t) \quad y(t)|\, dt, \quad x, y \quad X$$

Show that (X, d) is not complete. 2+8=10

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Subject Code : MATH/V/06

Booklet No. A

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DEGREE 5th Semester
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Paper ....

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Paper ....

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MATH/V/06

2 0 1 7

(5th Semester)

MATHEMATICS

Paper : MATH–352

(Real Analysis)

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. In R^2, the limit point of the set $Q^2 \quad \{(x, y) : x \quad Q,\ y \quad Q\}$ is

(a) (0, 0) ☐

(b) (1, 1) ☐

(c) every point of R^2 ☐

(d) None of the above ☐

2. If every limit point of a set belongs to the set, then the set is

(a) closed ☐

(b) open ☐

(c) derived ☐

(d) None of the above ☐

3. A function $f(x, y)$ is said to be continuous if

(a) it is continuous at isolated point only ☐

(b) it is continuous at each point of its domain ☐

(c) it is continuous at some deleted neighbourhood of domain ☐

(d) None of the above ☐

4. The range of a function continuous on a compact set is

(a) cover ☐

(b) subcover ☐

(c) compact ☐

(d) None of the above ☐

5. If a real valued function f defined in an open set $\quad R^n$ possesses first order partial derivatives at each point of D and the functions $D_1 f$, $D_2 f$, ... $D_n f$ are all continuous, then f is

(a) derivable in ☐

(b) partially derivable in ☐

(c) continuously derivable in ☐

(d) not derivable in ☐

6. $\lim_{h \quad 0} \frac{f(x \quad h, y) \quad f(x, y)}{h}$, if exist is called the partial derivative of f with respect to

(a) x at (a, b) ☐

(b) x at (x, y) ☐

(c) y at (a, b) ☐

(d) y at (x, y) ☐

7. If (a, b) be a point of the domain contained in R^2 of a function f such that f_x and f_y are both differentiable at (a, b), then

(a) $f_{xy}(a, b) \quad f_{yx}(a, b)$ ☐

(b) $f_{xy}(a, b) \quad f_{yx}(a, b)$ ☐

(c) f_{xy} and f_{yx} do not exist at (a, b) ☐

(d) None of the above ☐

8. When

$$f_{xx}(a, b)\ \ f_{yy}(a, b)\ \ \{f_{xy}(a, b)\}^2$$
$$0,\ f_{xx}(a, b)\quad 0,$$
$$f_{yy}(a, b)\quad 0$$

then f is

(a) minimum at (a, b) ☐

(b) maximum or minimum at (a, b) ☐

(c) maximum at (a, b) ☐

(d) None of the above ☐

9. The subset $S\quad \{(x, y): x^2\quad y^2\quad 1,\ \ x, y\quad R\}$ of R^2 with the Euclidean metric is

(a) open ☐

(b) closed ☐

(c) bounded ☐

(d) compact ☐

10. If every Cauchy sequence of X converges to a point of X, then a metric space (X, d) is

(a) compact ☐

(b) interior ☐

(c) complete ☐

(d) closure ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

Answer the following :

1. Prove that the union of a finite number of open sets in R is an open set.

2. Define convex set and uniform continuity.

3. If

$$u_1 \quad \frac{x_2 x_3}{x_1}, \;\; u_2 \quad \frac{x_1 x_3}{x_2}, \;\; u_3 \quad \frac{x_1 x_2}{x_3}$$

then prove that $J(u_1, u_2, u_3) \quad 4$.

4. State Taylor's theorem for a real valued function in R^n.

5. Let A be any subset of a metric space (X, d). Then prove that $A \quad \overline{A}$ if and only if A is closed. ($\overline{A}$ is closure of A)

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MATH/V/07

2017

(5th Semester)

MATHEMATICS

SEVENTH PAPER (MATH–353)

(Complex Analysis)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Show that the modulus of sum of two complex numbers is always less than or equal to the sum of their moduli. 5

(b) Show that arg $\dfrac{z_1 \quad z_2}{z_3 \quad z_4}$ is the angle between the lines joining z_2 to z_1 and z_4 to z_3 on the Argand plane. Also find the conditions if two lines are perpendicular or parallel. 5

2. *(a)* If z_1 and z_2 are two complex numbers, then prove that $|z_1 \quad z_2|^2 \quad |z_1|^2 \quad |z_2|^2$ if and only if $z_1\bar{z}_2$ is purely imaginary. 5

(b) If z_1, z_2, z_3 are the vertices of an isosceles triangle, right angled at the vertex z_2, then prove that

$$z_1^2 \quad 2z_2^2 \quad z_3^2 \quad 2z_2(z_1 \quad z_3)$$ 5

UNIT—II

3. *(a)* Show that the continuity is a necessary but not the sufficient condition for the existence of a finite derivative. 5

(b) Show that the function $f(z) \quad \sqrt{|xy|}$ where, $z \quad x \quad iy$ is not analytic at the origin, although the Cauchy-Riemann equations are satisfied at that point. 5

4. *(a)* If $u \quad x^3 \quad 3xy^2 \quad 3x^2 \quad 3y^2 \quad 1$, then determine harmonic conjugate function and find the corresponding analytic function in terms of *z*. 5

(b) For what value of *z*, the function *w* defined by the following equations ceases to be analytic?

$$z \quad \log \quad i\,;\, w \quad (\cos \quad i\sin \quad)$$ 5

UNIT—III

5. *(a)* State and prove Cauchy-Hadamard formula for the radius of convergence. 5

(b) Find the domain of convergence of the series $\left(\frac{iz \quad 1}{2 \quad i}\right)^n$. 5

6. *(a)* Find the radiis of convergence of the following power series : 2×3=6

(i) $\frac{z^n}{2^n \quad 1}$

(ii) $1 \quad \frac{a.b}{1.c} z \quad \frac{a.(a \quad 1).b.(b \quad 1)}{1.2.c.(c \quad 1)} z^2 \quad \cdots$

(b) If R_1 and R_2 are the radii of convergence of the power series $a_n z^n$ and $b_n z^n$ respectively, then find the radius of the convergence of the power series $a_n b_n z^n$. 4

UNIT—IV

7. *(a)* Find the value of the integral—

$$\int_0^{1 \quad i} (x \quad y \quad ix^2)\, dz$$

(i) along the straight line from $z \quad 0$ to $z \quad 1 \quad i$;

(ii) along the real axis from $z \quad 0$ to $z \quad 1$ and then along a line parallel to the imaginary axis from $z \quad 1$ to $z \quad 1 \quad i$. 5

(b) Show that if a function $f(z)$ is analytic in a region D, then its derivative at any point $z \quad a$ of D is also analytic in D, and is given by

$$f \quad (a) \quad \frac{1}{2 \quad i} \,_C \frac{f(z)\, dz}{(z \quad a)^2}$$

where C is any closed contour in D surrounding the point $z \quad a$. 5

8. *(a)* Evaluate by Cauchy integral formula

$$_C \frac{z\, dz}{(9 \quad z^2)(z \quad i)}$$

where C is the circle $|z| \quad 2$. 5

(b) If $f(z)$ is a continuous function in a domain D and if for every closed contour C in the domain D $\,_C f(z)\, dz \quad 0$, then prove that $f(z)$ is analytic within D. 5

UNIT—V

9. *(a)* Obtain the Laurent's series which represents the function

$$\frac{1}{(1 \quad z^2)(z \quad 2)}$$

in the region $|z| \quad 2$. 5

(b) Define singularity of a complex function. With suitable example, explain the terms isolated and non-isolated singularities. 5

10. *(a)* Find the singularities of the following functions : 4

(i) $\sin \dfrac{1}{1 \quad z}$ at $z \quad 1$

(ii) $\operatorname{cosec} \dfrac{1}{z}$ at $z \quad 0$

(b) State and prove the maximum modulus theorem. 6

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Subject Code : MATH/V/07

Booklet No. A

Date Stamp ....

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DEGREE 5th Semester
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Subject ....
Paper ....

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DEGREE 5th Semester
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Roll No. ....
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Subject ....
Paper ....

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MATH/V/07

2 0 1 7

(5th Semester)

MATHEMATICS

SEVENTH PAPER (MATH–353)

(Complex Analysis)

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. The modulus of $\exp(1 \quad i)$ is

(a) e ☐

(b) $1/e$ ☐

(c) 1 ☐

(d) $\quad e$ ☐

2. If $\bar{z}$ is the conjugate of z, then

(a) $|z| \quad |\bar{z}|$ ☐

(b) $|z| \quad |\bar{z}|$ ☐

(c) $|z| \quad |\bar{z}|$ ☐

(d) $|z| \qquad |\bar{z}|$ ☐

3. The derivative of $e^{x}(\cos y \quad i \sin y)$ is

(a) e^{x} ☐

(b) e^{y} ☐

(c) $e^{x \quad iy}$ ☐

(d) $e^{x \quad iy}$ ☐

4. The analytic function whose imaginary part is $e^{x} \cos y$ is

(a) e^{z} ☐

(b) ie^{z} ☐

(c) $ie^{\quad z}$ ☐

(d) $e^{\quad z}$ ☐

5. If $\lim_{n} \quad |u_n|^{1/n} \quad l$, then the series $\quad u_n$ is convergent if

(a) $l \quad 1$ ☐

(b) $l \quad 1$ ☐

(c) $l \quad 1$ ☐

(d) $l \quad 1$ ☐

6. The power series $\quad n!z^n$ will converge

(a) if $z \quad 0$ ☐

(b) if $|z| \quad 1$ ☐

(c) if $|z| \quad 1$ ☐

(d) for all real values of z ☐

7. If C is given by the equation $|z \quad a| \quad R$, then the value of $\;_{C}\dfrac{dz}{z \quad a}$ is

(a) $2 \; i$ ☐

(b) $\quad i$ ☐

(c) $\quad 2 \; i$ ☐

(d) $\quad i$ ☐

8. A continuous arc without multiple points is called a

(a) Jordan arc ☐

(b) continous arc ☐

(c) contour ☐

(d) rectifiable arc ☐

9. The function $\frac{\sin(z \quad a)}{(z \quad a)}$ at $z \quad a$ has

(a) removable singularity ☐

(b) non-isolated singularity ☐

(c) isolated singularity ☐

(d) a pole ☐

10. Zeros of the function $\frac{z^2 \quad 4}{e^z}$ at z are

(a) $2i$ ☐

(b) $2i$ ☐

(c) $2i$ ☐

(d) z ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

Answer **all** questions

Answer the following :

1. If $a^2 \quad b^2 \quad 1$, then find the value of

$$\frac{1 \quad a \quad ib}{1 \quad a \quad ib}$$

2. If $w \quad f(z)$ is an analytic function, $z \quad x \quad iy$, show that

$$\frac{dw}{dz} \quad (\cos \quad i \sin \ ) \frac{w}{r}$$

where $x \quad r\cos \ , y \quad r\sin \ .$

3. Examine the convergence of the series z^n.

4. Evaluate $\int_{2\ \ i}^{5\ \ 3i} z^3 dz$

5. Give the statements of Taylor's and Laurent's theorems.

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2 0 1 7

(5th Semester)

MATHEMATICS

Paper : MATH–354(A)

(Operations Research)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The questions are of equal value

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. The manager of an oil refinery must decide on the optimal mix of two possible blending processes of which the inputs and outputs per production run are as follows :

Process	*Input*		*Output*	
	Crude A	*Crude B*	*Gasoline X*	*Gasoline Y*
1	5	3	5	8
2	4	5	4	4

The maximum amounts available in crude *A* and crude *B* are 200 units and 150 units respectively. Market requirements show that at least 100 units of gasoline *X* and 80 units of gasoline *Y* must be produced. The profits per production run from process 1 and process 2 are ₹ 300 and ₹ 400 respectively. Formulate an LPP and solve it for maximization of profit.

2. The standard weight of a special purpose brick is 5 kg and it contains two ingredients *A* and *B*. *A* costs ₹ 5 per kg and *B* costs ₹ 8 per kg. Strength considerations dictate that the brick contains not more than 4 kg of *A* and a minimum of 2 kg of *B*. Since the demand for the product is likely to be related to the price of the brick, find the minimum cost of the brick satisfying the above conditions. Use the graphical method.

UNIT—II

3. Solve the following LPP using simplex method :

Maximize $Z \quad 5x_1 \quad 7x_2$

subject to the constraints

$$x_1 \quad x_2 \quad 4$$
$$3x_1 \quad 8x_2 \quad 24$$
$$10x_1 \quad 7x_2 \quad 35$$
$$x_1, x_2 \quad 0$$

4. Solve the following LPP by Big-M method :

Maximize $Z \quad x_1 \quad x_2 \quad x_3$

subject to the constraints

$x_1 \quad 4x_2 \quad 2x_3 \quad 5$

$3x_1 \quad x_2 \quad 2x_3 \quad 4$

and $x_1, x_2, x_3 \quad 0$

UNIT—III

5. Use duality to solve the following LPP :

Maximize $Z \quad 3x_1 \quad x_2$

subject to the constraints

$2x_1 \quad 3x_2 \quad 2$

$x_1 \quad x_2 \quad 1$

$x_1, x_2 \quad 0$

6. Given x_{13} 50 units, x_{14} 20 units, x_{21} 55 units, x_{31} 30 units, x_{32} 35 units and x_{34} 25 units. With the cost of transportation as

	D_1	D_2	D_3	D_4	Supply
S_1	6	1	9	3	70
S_2	11	5	2	8	55
S_3	10	12	4	7	90
Demand	85	35	50	45	

is it an optimum solution to the transportation problem? If not, find the optimum solution.

UNIT—IV

7. Use Gomory's cutting plane method to solve the following LPP :

Maximize $Z \quad 3x_1 \quad 4x_2$

subject to the constraints

$3x_1 \quad 2x_2 \quad 8$

$x_1 \quad 4x_2 \quad 10$

$x_1, x_2 \quad 0$ and integers

8. Use the branch and bound technique to solve the following LPP :

Maximize $Z \quad x_1 \quad x_2$

subject to the constraints

$2x_1 \quad 5x_2 \quad 16$

$6x_1 \quad 5x_2 \quad 30$

$x_1, x_2 \quad 0$ and x_2 integer

UNIT—V

9. Using dominance rule and graphical method, solve the two-person zero-sum game, whose payoff matrix is given below :

Player B

Player A	1	2	3	1
	2	2	1	5
	3	1	0	2
	4	3	2	6

10. Reduce the following two-person zero-sum game to an LPP and then solve it using simplex method :

Player B

	3	2	4	0
	3	4	2	4
Player A	4	2	4	0
	0	4	0	8

★★★

2017

(5th Semester)

MATHEMATICS

Paper : MATH–354(B)

(Probability Theory)

Full Marks : 75

Time : 3 hours

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* If A, B and C are random events in a sample space where A, B and C are pair-wise independent and A is independent of $B \quad C$, then prove that A, B and C are mutually independent. 5

(b) A speaks the truth in 60% and B in 75% of the cases. In what percentage of the cases are they likely to contradict each other in starting the same fact? 5

2. *(a)* Prove that for any two events A and B,

$$P(A \quad B) \quad P(A) \quad P(A \quad B) \quad P(A) \quad P(B)$$ 5

(b) From a vessel containing 3 white and 5 black balls, 4 balls are transferred into an empty vessel. From this vessel a ball is drawn and is found to be white. What is the probability that out of the four balls transferred 3 are white and 1 is black? 5

UNIT—II

3. *(a)* Two dice are rolled. Let X denote the random variable which counts the total number of points on the upturned faces. Construct a table giving the non-zero values of probability mass function. Also, find the distribution function of X. 5

(b) Let X be a continuous random variable with probability density function

$$f(x) \quad \begin{matrix} ax, & 0 \quad x \quad 1 \\ a, & 1 \quad x \quad 2 \\ ax \quad 3a, & 2 \quad x \quad 3 \\ 0, & \text{elsewhere} \end{matrix}$$

(i) Determine the constant a.

(ii) Compute $P(X \quad 1 \; 5)$. 5

4. For a binomial variate x with parameters n and p, prove that

$$_{r\ 1} \quad pq \;\; nr \quad _{r\ 1} \quad \frac{d \quad _r}{dp}$$

where $_r$ is the rth central moment. Hence obtain $_2$, $_3$ and $_4$. 10

UNIT—III

5. *(a)* Prove that two random variables X and Y with joint probability density function $f(x, y)$ are stochastically independent if and only if $f_{XY}(x, y)$ can be expressed as the product of a non-negative function of x alone and a non-negative function of y alone, that is, if $f_{XY}(x, y) \quad h_X(x) \; k_Y(y)$, where $h(\ ) \quad 0$ and $k(\ ) \quad 0$. 6

(b) A random observation on a bivariate population (X, Y) can yield one of the following pairs of values with probabilities noted against them :

For each observation pair	*Probability*
(1, 1); (2, 1); (3, 3); (4, 3)	1/20
(3, 1); (4, 1); (1, 2); (2, 2); (3, 2); (4, 2); (1, 3); (2, 3)	1/10

Examine if the two events $X \quad 4$ and $Y \quad 2$ are independent. 4

6. *(a)* The joint probability distribution of two random variables X and Y given below :

Y \ X	0	1	2
0	3/28	9/28	3/28
1	3/14	3/14	0
2	1/28	0	0

Find—

(i) the marginal probability distribution of X and Y;

(ii) the conditional distribution of X, given the value of $Y \quad 0$. 5

(b) If X and Y are two random variables having joint density function

$$f(x, y) \quad \begin{matrix} \frac{1}{8}(6 \quad x \quad y), & 0 \quad x \quad 2, 2 \quad y \quad 4 \\ 0, & \text{otherwise} \end{matrix}$$

find—

(i) $P(X \quad 1 \quad Y \quad 3)$;

(ii) $P(X \quad Y \quad 3)$;

(iii) $P(X \quad 1 \,|\, Y \quad 3)$. 5

UNIT—IV

7. Two unbiased coins are thrown. If X is the sum of the numbers showing up, prove that

$$P(|X \quad 7| \quad 3) \quad \frac{35}{54}$$

by using Chebychev's inequality. 10

8. Prove that correlation coefficient is independent of the change of origin and scale. 10

UNIT—V

9. *(a)* Define geometric distribution for a random variable X. Find its mean and variance. 7

(b) Find the moment generating function of exponential distribution. 3

10. *(a)* If X and Y are independent Poisson variates, calculate the conditional distribution of X, given $X \quad Y$ is binomial. 5

(b) For a normal distribution, prove that mean = median. 5

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Subject Code : MATH/V/08b

Booklet No. A

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DEGREE 5th Semester
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Subject ....

Paper ....

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2 0 1 7

(5th Semester)

MATHEMATICS

Paper : MATH–354(B)

(Probability Theory)

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided : 1×10=10

1. If $p_1 \quad P(A)$, $p_2 \quad P(B)$, $p_3 \quad P(A \quad B)$, then $P(\overline{A \quad B})$ in terms of p_1, p_2, p_3 is

(a) $1 \quad p_1 \quad p_2 \quad p_3$ ☐

(b) $1 \quad p_1 \quad p_2 \quad p_3$ ☐

(c) $1 \quad p_1 \quad p_3$ ☐

(d) $1 \quad p_1 \quad p_3$ ☐

2. A coin is tossed three times in succession, the number of sample points in sample space is

(a) 6 ☐

(b) 8 ☐

(c) 3 ☐

(d) 9 ☐

3. Let X be a random variable having discrete uniform distribution over the range $[1, n]$. Then the variance $V(X)$ is given by

(a) $\dfrac{(n \quad 1)}{2}$ ☐

(b) $\dfrac{(n \quad 1)(2n \quad 1)}{6}$ ☐

(c) $\dfrac{(n \quad 1)(n \quad 1)}{12}$ ☐

(d) $\dfrac{(n \quad 1)(2n \quad 1)}{6}$ ☐

4. For the probability mass function $f(x) \quad cx^2(1 \quad x)$, $0 \quad x \quad 1$, the value of the constant c is

(a) 4 ☐

(b) 8 ☐

(c) 12 ☐

(d) 0 ☐

5. The marginal probability function of a continuous random variable X is defined as

(a) $f_X(x) \quad \quad p_{XY}(x, y)$ ☐
$\quad y$

(b) $f_X(x) \quad \quad p_{XY}(x, y)$ ☐
$\quad x$

(c) $f_X(x) \quad \quad f_{XY}(x, y)\, dx$ ☐

(d) $f_X(x) \quad \quad f_{XY}(x, y)\, dy$ ☐

6. If X and Y are two random variables, the covariance between them is defined as

(a) $\text{Cov}(X, Y) \quad E(XY) \quad E(X)E(Y)$ ☐

(b) $\text{Cov}(X, Y) \quad E(XY) \quad E(X)E(Y)$ ☐

(c) $\text{Cov}(X, Y) \quad E(X) \quad E(Y) \quad E(XY)$ ☐

(d) $\text{Cov}(X, Y) \quad E(X)E(Y) \quad E(XY)$ ☐

7. If X is a random variable, then

(a) $\text{Var}(X) \quad \{E(X)\}^2 \quad E(X^2)$ ☐

(b) $\text{Var}(X) \quad E(X^2) \quad \{E(X)\}^2$ ☐

(c) $\text{Var}(X) \quad E(X) \quad E(X^2)$ ☐

(d) $\text{Var}(X) \quad E(X^2) \quad E(X)$ ☐

8. If X and Y are independent random variables, then

(a) $\text{Cov}(X, Y) \quad E(XY) \quad E(X)E(Y)$ ☐

(b) $\text{Cov}(X, Y) \quad E(X)E(Y) \quad E(XY)$ ☐

(c) $\text{Cov}(X, Y) \quad 1$ ☐

(d) $\text{Cov}(X, Y) \quad 0$ ☐

9. Normal curve is

(a) very flat ☐

(b) bell-shaped symmetrical about mean ☐

(c) very peaked ☐

(d) smooth ☐

10. The relationship between mean and variance of Gamma distribution is

(a) mean = variance ☐

(b) mean = 2 variance ☐

(c) mean < variance ☐

(d) mean > variance ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

1. A letter is known to have come either from TATANAGAR or from CALCUTTA. On the envelope just two consecutive letters TA are visible. What is the probability that the letter came from CALCUTTA?

2. A continuous random variable X has a probability density function $f(x) \quad 3x^2, 0 \quad x \quad 1$. Find a and b such that

(a) $P(X \quad a) \quad P(X \quad a)$

(b) $P(X \quad b) \quad 0 \;\, 05$

3. The joint probability density function of a two-dimensional random variable (X, Y) is given by

$$f(x, y) \quad \begin{matrix} 2, & 0 \quad x \quad 1, \quad 0 \quad y \quad x \\ 0, & \text{elsewhere} \end{matrix}$$

Find the marginal density functions of X and Y.

4. Let X be a random variable with the following probability distribution :

x	:	-3	6	9
$P(X \quad x)$	:	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find $E(X)$ and $E(X^2)$ and using the laws of expectation, evaluate $E(2X \quad 1)^2$.

5. Prove that the sum of two independent Poisson variates is again a Poisson variate.

★★★

Subject Code : MATH/V/08a

Booklet No. A

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2 0 1 7

(5th Semester)

MATHEMATICS

Paper : MATH–354(A)

(Operations Research)

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. A constraint in an LPP is expressed as

(a) an equation with = sign ☐

(b) an inequality with sign ☐

(c) an inequality with sign ☐

(d) None of the above ☐

2. A feasible solution to an LPP should satisfy

(a) all the constraints and x_i 0 ☐

(b) all the constraints and x_i 0 ☐

(c) all the constraints only ☐

(d) None of the above ☐

3. In an LPP with m constraints and n unknowns (m n), the number of basic variables will be

(a) n ☐

(b) m ☐

(c) m n ☐

(d) None of the above ☐

4. If there is a negative value in solution values (x_B) of the simplex method, then

(a) the basic solution is optimum ☐

(b) the basic solution is infeasible ☐

(c) the basic solution is unbounded ☐

(d) None of the above ☐

5. The right-hand side constant of a constraint in a primal problem appears in the corresponding dual as

(a) a coefficient in the objective function ☐

(b) a right-hand side constant of a constraint ☐

(c) a value of the objective function ☐

(d) None of the above ☐

6. If there are n workers and n jobs in an assignment problem, there would be

(a) n solutions ☐

(b) n^2 solutions ☐

(c) $n!$ solutions ☐

(d) None of the above ☐

7. In cutting plane algorithm, each cut involves the introduction of

(a) an equality constraint ☐

(b) a less than or equal to constraint ☐

(c) an artificial variable ☐

(d) a greater than or equal to constraint ☐

8. Branch and bound method divides the feasible solution space into smaller parts by

(a) enumerating ☐

(b) bounding ☐

(c) branching ☐

(d) None of the above ☐

9. A game is said to be strictly determinable, if

(a) minimax value is greater than maximin value ☐

(b) minimax value is equal to maximin value ☐

(c) minimax value is less then maximin value ☐

(d) None of the above ☐

10. The size of a payoff matrix of a game can be reduced by using the principle of

(a) dominance ☐

(b) rotation reduction ☐

(c) game inversion ☐

(d) None of the above ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

1. Solve the following LPP by graphical method :

Maximize $Z \quad 2x_1 \quad x_2$

subject to

$$3x_1 \quad 5x_2 \quad 15$$
$$3x_1 \quad 4x_2 \quad 12$$
$$x_1, \ x_2 \quad 0$$

2. Using simplex method, solve the following simultaneous equations :

$$5x \quad y \quad 11$$
$$2x \quad 3y \quad 1$$

3. Find the optimal assignment to find the minimum cost for the following problem :

	J_1	J_2
W_1	3	9
W_2	2	11

4. Use cutting plane method to solve the following integer linear programming problem :

$$\text{Maximize } Z \quad 2x_1 \quad x_2$$

subject to

$$x_1 \quad 8x_2 \quad 24$$

$$x_1 \quad 4$$

$$x_1, x_2 \quad 0 \text{ and integers}$$

5. For what value of , the game with the following payoff matrix is strictly determinable?

B

		6	2
A	− 1		0
	2	4	

★★★