

2025

(NEP—2020)

(5th Semester)

MATHEMATICS (MAJOR2)**(Analytical Geometry)***Full Marks : 75**Time : 3 hours**The figures in the margin indicate full marks for the questions***(SECTION : A—OBJECTIVE)***(Marks : 10)*

Tick (✓) the correct answer in the brackets provided :

1×10=10

1. If the pair of lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angle between the other pair, then

(a) $p + q = 1$ ()

(b) $p - q = 1$ ()

(c) $pq = -1$ ()

(d) $pq = 1$ ()

2. The common tangent of the circle $x^2 + y^2 - 4ax = 0$ and the parabola $y^2 = 4ax$ is

(a) $x = a$ ()

(b) $x = 0$ ()

(c) $x = 4a$ ()

(d) $x + a = 0$ ()

3. The gradients m_1 and m_2 of the conjugate diameter of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are related by

(a) $m_1 \cdot m_2 = \frac{a^2}{b^2}$ ()

(b) $m_1 \cdot m_2 = -\frac{b^2}{a^2}$ ()

(c) $m_1 + m_2 = -\frac{a^2}{b^2}$ ()

(d) $m_1 + m_2 = \frac{b^2}{a^2}$ ()

4. The conic $\frac{3}{r} = 4 + 4 \cos \theta$ represents

(a) parabola ()

(b) ellipse ()

(c) hyperbola ()

(d) rectangular hyperbola ()

5. The distance of the point (4, 3, 5) from XZ -plane is

(a) 3 ()

(b) 5 ()

(c) 7 ()

(d) $5\sqrt{2}$ ()

6. For what value of k the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = y-5 = \frac{z-6}{-5}$ will be perpendicular?

(a) 8 ()

(b) $\frac{2}{3}$ ()

(c) $-\frac{10}{7}$ ()

(d) $\frac{1}{2}$ ()

7. The distance of the point (3, 2, 1) from the line $\frac{x-1}{3} = \frac{y}{4} = \frac{z-3}{1}$ is

(a) $\sqrt{17}$ ()

(b) $\sqrt{\frac{5}{2}}$ ()

(c) 8 ()

(d) 13 ()

8. The equation of sphere passing through origin and having intercept parts a , b and c with coordinate axes is

(a) $x^2 + y^2 + z^2 + ax + by + cz = 0$ ()

(b) $x^2 + y^2 + z^2 + 2ax + 2by + 2cz = 0$ ()

(c) $x^2 + y^2 + z^2 = a + b + c$ ()

(d) $x^2 + y^2 + z^2 - ax - by - cz = 0$ ()

9. The axis of the cylinder $f(x, y) = 0$ whose guiding curve is $f(x, y) = 0, z = 0$ is

(a) X-axis ()

(b) Y-axis ()

(c) Z-axis ()

(d) not defined ()

10. The cone $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ will have three mutually perpendicular tangent planes, if

(a) $a^2 + b^2 + c^2 - fg - gh - hf = 0$ ()

(b) $f^2 + g^2 + h^2 - ab - bc - ac = 0$ ()

(c) $a^2 + b^2 + c^2 = 3fgh$ ()

(d) $f^2 + g^2 + h^2 = 3abc$ ()

(SECTION : B—SHORT ANSWERS)

(Marks : 15)

Answer *five* questions, taking at least *one* from each Unit :

3×5=15

UNIT—I

1. Find the angle through which the axes are to be rotated so that the equation $3x^2 + 2xy + 3y^2 - 1 = 0$ may be reduced to the form $Ax^2 + By^2 = 1$. Also find A and B .
2. Find the equation of the pair of straight lines through the origin and perpendicular to the pair of straight lines given by $2x^2 + 5xy + 2y^2 + 10x + 5y = 0$.

UNIT—II

3. Find the locus of middle points of chords of the parabola $y^2 = 4ax$ which are passing through the focus.
4. Find the polar form of the left branch of the hyperbola $9x^2 - 16y^2 = 144$.

UNIT—III

5. Find the plane that bisects the acute angle between the planes $3x - 6y - 2z + 5 = 0$ and $4x - 12y + 3z - 3 = 0$.
6. Find the plane which contains the straight line $x = \frac{y-3}{2} = \frac{z-5}{3}$ and perpendicular to the plane $2x + 7y + 3z = 1$.

UNIT—IV

7. Obtain the equation of the circle lying on the sphere

$$x^2 + y^2 + z^2 - 2x + 2y - 4z + 3 = 0$$

and having its centre at the point $(2, 2, -3)$.

8. Find the equation of the right circular cylinder of radius 2 and whose axis is the line $\frac{x-1}{2} = \frac{y}{3} = \frac{z-3}{1}$.

(SECTION : C—DESCRIPTIVE)

(Marks : 50)

Answer five questions, taking at least one from each Unit :

10×5=50

UNIT—I

1. (a) Prove that the general equation of second degree

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

will represent two parallel straight lines, if $h^2 = ab$ and $bg^2 = af^2$, and

the distance between them is $2\sqrt{\frac{g^2 - ac}{a(a+b)}}$.

6

- (b) Reduce the equation $7x^2 - 2xy + 7y^2 - 16x + 16y - 8 = 0$ to the standard form.

4

2. (a) Find the locus of the point of intersection of tangents of the ellipse which meet at right angle.

5

- (b) If the normal to the rectangular hyperbola $xy = c^2$ at $\left(ct, \frac{c}{t}\right)$ meets the curve at $\left(ct_1, \frac{c}{t_1}\right)$, then show that $t^3t_1 + 1 = 0$.

5

UNIT—II

3. (a) If a pair of conjugate diameters cut the hyperbola and its conjugate at P, P' and D, D' respectively, then show that $CP^2 - CD^2 = a^2 - b^2$, where C is the centre of the hyperbola.

5

- (b) If PSP' and QSQ' are two perpendicular focal chords of a conic, then prove that $\frac{1}{SP \cdot SP'} + \frac{1}{SQ \cdot SQ'}$ is constant.

5

4. (a) Find the equation of tangent to a conic in polar form.

5

- (b) If the tangent at any point P of the conic $\frac{l}{r} = 1 - e \cos \theta$ meets the directrix at K , then show that the angle PSK is a right angle, where S is the focus of the conic. 5

UNIT—III

5. (a) If a plane meets the axes at A, B, C and the centroid of ΔABC is (α, β, γ) , then show that the equation of the plane is $\frac{x}{\alpha} + \frac{y}{\beta} + \frac{z}{\gamma} = 3$. 5
- (b) Find the equation of the plane which passes through $(2, -3, 1)$ and is perpendicular to the line joining the points $(3, 4, -1)$ and $(2, -1, 5)$. 5
6. (a) Find the equation of the line through the point $(2, 3, 1)$ and parallel to the planes $2x + 3y + 4z = 5$ and $3x + 5y + 4z = 6$. 5
- (b) Find the magnitude and position of the shortest distance between the lines $\frac{x}{4} = \frac{y+1}{3} = \frac{z-2}{2}$ and $5x - 2y - 3z + 6 = 0 = x - 3y + 2z - 3$. 5

UNIT—IV

7. (a) Find the equation of a sphere which touches the sphere $x^2 + y^2 + z^2 + 2x - 6y + 1 = 0$ at $(1, 2, -2)$ and passes through the origin. 5
- (b) Find the equation of the right circular cone with vertex at $(2, 1, -3)$, semi-vertical angle is 45° and axis is parallel to OY . 5
8. (a) Find the angle between the lines of intersection of the plane $x - 3y + z = 0$ and the cone $x^2 - 5y^2 + z^2 = 0$. 5
- (b) The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ meets the coordinate axes at A, B, C . Prove that the equation of the cone generated by the line drawn from origin to meet the circle ABC is $yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0$. 5
