2025

(NEP—2020)

(3rd Semester)

MATHEMATICS (MAJOR/MINOR)

(Differential Equation)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. The order and degree of the differential equation

$$\frac{d^2y}{dx^2} = \sqrt{1+\left(\frac{dy}{dx}\right)^3}$$

are

(a) 2, 3 (   )

(b) 1, 2 (   )

(c) 2, 2 (   )

(d) 1, 1 (   )

2. Choose the correct statement :

A. The differential equation always has one and unique solution.

B. Particular solutions are obtained by giving particular values to arbitrary constant in the general solution.

(a) A is correct and B is false ( )

(b) A is false and B is correct ( )

(c) Both A and B are correct ( )

(d) Neither A nor B is correct ( )

3. The integrating factor of the differential equation

$$\frac{dy}{dx}(x\log x) + y = 2\log x$$

is given by

(a) e^x ( )

(b) $\log(\log x)$ ( )

(c) x ( )

(d) $\log x$ ( )

4. The solution of the differential equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$ is

(a) $y = (A + B)e^{-2x}$ ( )

(b) $y = Ax + Be^{-2x}$ ( )

(c) $y = (A + Bx)e^{-2x}$ ( )

(d) $y = (A + Bx)e^{2x}$ ( )

5. The general solution of $(D^2 - m^2)y = 0$ is

(a) $y = (c_1 + c_2 x)e^{mx}$ ( )

(b) $y = c_1 e^{mx} + c_2 e^{mx}$ ( )

(c) $y = c_1 \sin mx + c_2 \cos mx$ ( )

(d) None of the above ( )

6. The orthogonal trajectories of a system of concurrent straight lines are

(a) a parabola ( )

(b) an ellipse ( )

(c) the concentric circle ( )

(d) the straight line itself ( )

7. In the linear differential equation of second order

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0$$

where P and Q are functions of x only or constant, if $P + Qx = 0$, then $y =$

(a) x ( )

(b) e^x ( )

(c) x^2 ( )

(d) x^m ( )

8. In order to remove the first derivative from the second-order linear differential equation $y_2 + Py_1 + Qy = R$, where $y_1 = dy/dx$ and so on, we choose

(a) $u = e^{\frac{1}{2}\int Q dx}$ ( )

(b) $u = e^{-\frac{1}{2}\int P dx}$ ( )

(c) $u = e^{-\frac{1}{2}\int Q dx}$ ( )

(d) $u = e^{\frac{1}{2}\int P dx}$ ( )

9. The PDE obtained from $z = (x-a)^2 + (y-b)^2$ by removing a and b is

(a) $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 4z = 0$ ( )

(b) $\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2 + 4z = 0$ ( )

(c) $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 - 4z = 0$ ( )

(d) None of the above ( )

10. The complete integral of the equation $pq = xy$ is

(a) $2az = a^2x^2 + y^2 + 2ab$ ( )

(b) $2az = a^2x^2 + y^2 - 2ab$ ( )

(c) $az = a^2x^2 + y^2 + ab$ ( )

(d) None of the above ( )

(SECTION : B—SHORT ANSWERS)

(*Marks* : 15)

Answer *five* questions, taking at least *one* from each Unit : 3×5=15

UNIT—I

1. Find the differential equation of the family of circle $x^2 + y^2 = a^2$, where a is an arbitrary constant.

2. Test the exactness and solve the differential equation

$$\left(1 + 3e^{\frac{x}{y}}\right)dx + 3e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$$

UNIT—II

3. Find the complete solution of $(D^2 + D + 1)y = e^{-x}$, where $D \equiv d / dx$.

4. Find the general and singular solution of

$$y = px + \frac{a}{p}$$

where $p = \frac{dy}{dx}$.

UNIT—III

5. Solve the linear homogeneous differential equation

$$x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = x \log x$$

6. Solve the total differential equation

$$(yz + 2x)dx + (zx + 2y)dy + (xy + 2z)dz = 0$$

UNIT—IV

7. Solve the partial differential equation

$$(x + 2z)p + (4zx - y)q = x^2 + y$$

8. Show that the equations $p = 6x - 3y$ and $q = 3x - 4y$ are compatible and solve it.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer *five* questions, taking at least *one* from each Unit : 10×5=50

UNIT—I

1. (a) Reduce the equation

$$\frac{dy}{dx} + \frac{1}{x}\sin 2y = x^3 \cos^2 y$$

to a linear differential equation and solve it. 5

(b) Reduce the equation $xdy + ydx = -xy^2 dx$ to exact form and solve it. 5

2. (a) Solve the differential equation

$$\frac{dy}{dx} = \sin(x+y)$$

5

(b) Solve the differential equation

$$(x+y)^2 \frac{dy}{dx} = a^2$$

5

UNIT—II

3. Solve the differential equations (any *two*) : 5+5=10

(a) $(D^2 + 2D + 1)y = e^{-x} + x^2$

(b) $(D^3 - D^2 - 6D)y = 1 + x^2$

(c) $(D^2 + 4)y = x\cos x$

where $D \equiv d/dx$.

4. (a) Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$, λ being a parameter. 5

(b) Solve (any *two*) : 2½+2½=5

(i) $yp^2 - (x^2 - y^2)p - xy = 0$

(ii) $y = yp^2 + 2px$

(iii) $y + px = p^2 x^4$

where $p = \frac{dy}{dx}$.

UNIT—III

5. *(a)* Show that the equation

$$(2x^2 + 3x)\frac{d^2y}{dx^2} + (6x+5)\frac{dy}{dx} + 2y = (x+1)e^x$$

is exact and solve it. 5

(b) Solve : 5

$$(x\sin x + \cos x)\frac{d^2y}{dx^2} - x\cos x\frac{dy}{dx} + y\cos x = \sin x(x\sin x + \cos x)^2$$

6. *(a)* Apply the method of variation of parameters to solve the following equation : 5

$$(1-x)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = (x-1)^2$$

(b) Solve the following simultaneous differential equations : 5

$$\frac{dx}{dt} - 7x + y = 0 \text{ and } \frac{dy}{dt} - 2x - 5y = 0$$

UNIT—IV

7. *(a)* Solve the following PDE by Lagrange's method : 5

$$(y^3x - 2x^4)p + (2y^4 - x^3y)q = 9z(x^3 - y^3)$$

(b) Find the general integral of the partial differential equation $(2xy-1)p + (z-2x^2)q = 2(x-yz)$ and also the particular integral which passes through the line $x=1$, $y=0$. 5

8. *(a)* Find the complete integral of $x(1+y)p = y(1+x)q$. 4

(b) Using Charpit's method, find the complete integral of the following partial differential equation : 6

$$p^2 + q^2 - 2px - 2qy + 2xy = 0$$

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