

2025

(NEP—2020)

(3rd Semester)

MATHEMATICS (MAJOR)

(Modern Algebra—I)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(Marks : 10)

Put a Tick (✓) mark against the correct answer in the brackets provided :

1×10=10

1. The inverse of the element a of the group of all integers under the operation defined by $a * b = a + b + 1$ is

(a) $2 - a$ ()

(b) $-2 - a$ ()

(c) $-2 + a$ ()

(d) $a + 2$ ()

2. The number of commutative binary operations on a finite set A having n elements is

(a) n^2 ()

(b) $\frac{n^2}{2}$ ()

(c) $\frac{n^2+1}{2}$ ()

(d) n^{n^2} ()

3. In multiplicative group of rational numbers, the order of 7 is

(a) 0 ()

(b) 1 ()

(c) 2 ()

(d) infinite ()

4. Let G be the additive group of integers and $H = \{3x : x \in G\}$. Then the number of distinct right cosets is

(a) 2 ()

(b) 3 ()

(c) 1 ()

(d) infinite ()

5. The set of all permutations on the set $\{1, 2, 3\}$ is the group of order

(a) 3 ()

(b) 1 ()

(c) 6 ()

(d) 9 ()

6. Let $T = \{\pm 1, \pm i, \pm j, \pm k\}$ be a quaternion group. Then

(a) T is non-Abelian group of order 4 ()

(b) T is non-Abelian group of order 8 ()

(c) T is Abelian group of order 4 ()

(d) T is Abelian group of order 8 ()

7. The number of generators of the cyclic group of order 8 is

(a) 4 ()

(b) 3 ()

(c) 8 ()

(d) 7 ()

8. The special linear group $SL(n, R)$ is of all $n \times n$ matrices with determinant equal to

- (a) 0 ()
- (b) 1 ()
- (c) -1 ()
- (d) None of the above ()

9. The generator(s) of the group $G = (\{0, 1, 2, 3, 4, 5\} \oplus_6)$ are/is

- (a) 0 and 1 ()
- (b) 1 and 5 ()
- (c) 0 and 5 ()
- (d) only 5 ()

10. In S_3 , $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ is a cycle of length

- (a) 3 ()
- (b) 2 ()
- (c) 1 ()
- (d) None of the above ()

Answer

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2. Pro

3. Sho

4. If t
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5. If c
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6. In
no

7. If c
of

8. Fir

(SECTION : B—SHORT ANSWERS)

(Marks : 15)

3×5=15

Answer *five* questions, taking at least *one* from each Unit :

UNIT—1

1. Show that the matrix multiplication is associative but not commutative binary operation on $M_2(R)$, the set of all 2×2 matrices over R .
2. Prove that the inverse of each element of a group is unique.

UNIT—2

3. Show that the union of two subgroups is not necessarily a subgroup.
4. If the elements of a group G is of order n , then show that $a^m = e$ if and only if n is a divisor of m .

UNIT—3

5. If a and b are any two elements of a group G , then show that $(ab)^2 = a^2b^2$ if and only if G is abelian.
6. In a dihedral group of D_3 , calculate the total number of subgroups and normal subgroups of D_3 .

UNIT—4

7. If a is a generator of a cyclic group G , then show that a^{-1} is also a generator of G .
8. Find the remainder when 99^{20} is divided by 25.

(SECTION : C—DESCRIPTIVE)

(Marks : 50)

10×5=50

Answer five questions, selecting at least one from each Unit :

UNIT—1

1. (a) Prove that the set $G = \{0, 1, 2, 3, 4, 5\}$ is a finite Abelian group with respect to addition modulo 6. 5
- (b) Show that the set $G = \{1, \omega, \omega^2\}$, where ω is an imaginary cube root of unity, is a group with respect to multiplication. 5
2. (a) If a and b are any two elements of a group G , then show that the equations $ax = b$ and $ya = b$ have unique solution in G . 5
- (b) Prove that the inverse of the product of two elements of a group G is the product of inverse taken in reverse order. 5

UNIT—2

3. (a) Let G be the additive group of integers. Then prove that the set of all multiples of integers by a fixed integer m is a subgroup of G . 5
- (b) Show that the union of two subgroups is a subgroup if and only if one is contained in the other. 5
4. (a) Prove that the order of an element of a group is the same as that of its inverse. 4
- (b) Prove that the order of each subgroup of a finite group is a divisor of the group. 6

UNIT—3

5. (a) If a group G has four elements, then show that it must be abelian. 5
- (b) Show that if every element of a group G is its own inverse, then G is abelian. 5

6. (a) Define special linear group $SL(n, R)$. Show that $SL(n, R)$ is a non-Abelian group under matrix multiplication for $n \geq 2$. 2+3=5
- (b) Prove that a group G is abelian if every element of G except identity element is of order 2. 5

UNIT—4

7. (a) Define cyclic group. Prove that every infinite cyclic group has two and only two generators. 1+4=5
- (b) Prove that the normalizer $N(a)$ of $a \in G$ is a subgroup of G . 5
8. (a) State and prove Cayley's theorem. 6
- (b) Write down all the permutations on four symbols 1, 2, 3, 4. Which of these permutations are even? 4
