MATH101 (MAJOR/MINOR) **Student's Copy**

2025

(NEP–2020)

(1st Semester)

MATHEMATICS (MAJOR/MINOR)

(Calculus)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. The graph of the function $f(x) = |x^2 - 1|$ and $f(x) = x^2 - 1$ never touch in the interval

(a) $(-\infty, 0)$ ( )

(b) $(-1, 1)$ ( )

(c) $[-1, 1]$ ( )

(d) $(0, \infty)$ ( )

2. Choose the correct statement :

I. A continuous real-valued function is bounded in a closed interval.

II. A continuous real-valued function in $[a, b]$ attains its extreme values on $[a, b]$.

(a) Only I is correct ( )

(b) Only II is correct ( )

(c) Both I and II are correct ( )

(d) Neither I nor II is correct ( )

3. The function $f(x) = |x - 2|$ is

(a) not bounded at $x = 2$ ( )

(b) not defined at $x = 2$ ( )

(c) not continuous at $x = 2$ ( )

(d) not differentiable at $x = 2$ ( )

4. Choose the correct statement :

The Cauchy's mean value theorem is valid for the functions

I. $f(x) = 5x^{4/5}$

II. $f(x) = x^3$

in the interval $[-1, 1]$.

(a) Only I is correct ( )

(b) Only II is correct ( )

(c) Both I and II are correct ( )

(d) Neither I nor II is correct ( )

5. The function $f(x) = \log x$ can be expanded in the power of $(x-1)$ by using

(a) Maclaurin's theorem ( )

(b) Taylor's theorem ( )

(c) Both *(a)* and *(b)* ( )

(d) Neither *(a)* nor *(b)* ( )

6. The integral $\int e^{x}[f(x) + f'(x)]dx$ equals

(a) $e^{x} f(x)$ ( )

(b) $e^{x} f'(x)$ ( )

(c) $e^{-x} f(x)$ ( )

(d) $e^{-x} f'(x)$ ( )

7. The formula for $\int \sec x\, dx$ is

(a) $\log\left[\tan\left(\frac{\pi}{4}+\frac{x}{4}\right)\right]+C$ ( )

(b) $\log\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]+C$ ( )

(c) $\log\left[\tan\left(\frac{\pi}{4}-\frac{x}{4}\right)\right]+C$ ( )

(d) $\log\left[\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]+C$ ( )

8. Let $f(x, y) = \frac{x + y}{x^2 + y}$, when $x^2 + y \neq 0$. Then $\lim_{(x, y) \to (-1, 1)} f(x, y)$ along the line $x - 1 = 0$ is equal to

(a) -1 ( )

(b) 0 ( )

(c) 1 ( )

(d) Does not exist ( )

9. Choose the correct statement :

I. Existence of double limit does not follow repeated limit.

II. Existence of repeated limit always follow double limit.

(a) Only I is correct ( )

(b) Only II is correct ( )

(c) Both I and II are correct ( )

(d) Neither I nor II is correct ( )

10. The sequence $\left\{\frac{1}{n!}\right\}$ is

(a) bounded below but not above ( )

(b) bounded above but not below ( )

(c) monotonic decreasing ( )

(d) monotonic increasing ( )

(SECTION : B—SHORT ANSWERS)

(*Marks* : 15)

Answer *five* questions, taking at least *one* from each Unit : 3×5=15

UNIT—I

1. Discuss the derivability of $f(x) = |x|$ at the origin.

2. Find $\dfrac{d^n f(x)}{dx^n}$ for $f(x) = x^2 e^{\lambda x}$, where λ is constant.

UNIT—II

3. Expand e^x in the power of $x+5$.

4. State Cauchy's mean value theorem.

UNIT—III

5. Find the reduction formula for $\int \tan^n x \, dx$.

6. Using Leibniz's rule find $\dfrac{df(x)}{dx}$ for $f(x) = \int_x^{x^2} e^t \, dt$.

UNIT—IV

7. Show that the sequence $\{f_n\}$ defined by $f_n = 1 + \dfrac{1}{2} + \dfrac{1}{3} + \cdots + \dfrac{1}{n}$ cannot be converge.

8. Show that the series $1 + \dfrac{1}{2} + \dfrac{1}{3} + \cdots$ is not convergent.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer *five* questions, taking at least one from each Unit : 10×5=50

UNIT—I

1. (*a*) Evaluate : 4

(*i*) $\lim_{x\to\infty} \frac{x^4}{e^x}$

(*ii*) $\frac{dy}{dx}$ for $x = \cos t$ and $y = \sin t$

(*b*) Draw the graph of the function $f(x) = |1-x| + |1+x|$. Discuss the continuity and derivability at $x = 1$ and $x = -1$. 6

2. (*a*) Using ε-δ definition, show that $\lim_{x\to 0} (1+e^{-1/x})^{-1} = 1$. 5

(*b*) Show that a function which is derivable at a point is necessarily continuous but not conversely. 5

UNIT—II

3. (*a*) State and prove Rolle's theorem. 7

(*b*) Verify the validity of Rolle's theorem for $f(x) = \tan x$ in the interval $[0, \pi]$. 3

4. (*a*) State and prove Taylor's theorem with remainder in Cauchy's form. 7

(*b*) Expand e^x in an infinite series in power of x. 3

UNIT—III

5. (*a*) Evaluate : (*i*) $\int_0^1 x^2 dx$ as a limit of sum and (*ii*) $\int_{-1}^{1} |x| dx$. 4

(*b*) Obtain reduction formula for $\int_0^{\pi/2} \sin x \, dx$. 6

6. If $I_n = \int_0^{\pi/4} \tan^n x \, dx$, prove that $n(I_{n-1} + I_{n+1}) = 1$. Using this relation, evaluate $\int_0^{\pi/4} \tan^8 x \, dx$. 10

UNIT—IV

7. *(a)* Let $u = f(x, y)$ be a homogeneous function of n-degree, then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu$$ 5

(b) Define harmonic function. Prove that $u(x, y) = e^x \cos y$ is harmonic in x and y. 5

8. *(a)* Prove that if a sequence is convergent, then it converges to unique limit. 5

(b) Prove that a positive term series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ is convergent if and only if $p > 1$, and the series diverges for $p \le 1$. 5

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