2025

(NEP—2020)

(1st Semester)

MATHEMATICS (MAJOR)

(Vector Analysis)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. If $\vec{F} = t^2\hat{i} + \sin t\hat{j} + e^t\hat{k}$, then the value of $\frac{d\vec{F}}{dt}$ at $t = 0$ is

(a) $\hat{k}$ ( )

(b) $\hat{j} + \hat{k}$ ( )

(c) $2\hat{i} + \hat{j} + e^t\hat{k}$ ( )

(d) $\hat{i}$ ( )

2. The derivative of a constant vector field $\vec{A}$ is

(a) zero vector ( )

(b) magnitude of the vector field ( )

(c) a new vector field dependent on time ( )

(d) undefined ( )

3. For a vector field $\vec{A}(x, y, z)$, the partial derivative $\frac{\partial \vec{A}}{\partial x}$ applies to

(a) the x-component only ( )

(b) only components not dependent on x ( )

(c) the magnitude of the vector field ( )

(d) all components that depend on x ( )

4. The divergence of a vector field $\vec{F}$ is

(a) a scalar field ( )

(b) a vector field ( )

(c) a constant ( )

(d) None of the above ( )

5. The directional derivative of $\phi(x, y) = x^2 + y^2$ at (2, 1) in the direction of $\hat{j}$ is

(a) 0 ( )

(b) 1 ( )

(c) 2 ( )

(d) 3 ( )

6. If ϕ is a scalar field, then the value of $\vec{\nabla} \times (\vec{\nabla}\phi)$ is

(a) $\nabla^2\phi$ ( )

(b) 0 ( )

(c) $\vec{\nabla}(\vec{\nabla}\phi) \cdot \vec{\nabla}$ ( )

(d) $\phi(\vec{\nabla}\phi)$ ( )

7. The surface integral $\iint_S \vec{F} \cdot d\vec{S}$ computes

(a) the flux of $\vec{F}$ through the surface S ( )

(b) the circulation of $\vec{F}$ along the boundary of S ( )

(c) the divergence of $\vec{F}$ ( )

(d) the curl of $\vec{F}$ ( )

8. For a conservative vector field $\vec{F}$, the scalar potential function ϕ satisfies

(a) $\vec{F} = \vec{\nabla} \cdot \phi$ ( )

(b) $\vec{F} = \vec{\nabla}\phi$ ( )

(c) $\vec{\nabla} \times \vec{F} = \phi$ ( )

(d) $\vec{\nabla} \cdot \vec{F} = 0$ ( )

9. What does Gauss divergence theorem relate?

(a) Line integrals and surface integrals ( )

(b) Volume integrals and line integrals ( )

(c) Surface integrals and volume integrals ( )

(d) Surface integrals and curl ( )

10. Stokes' theorem is applicable when the surface is

(a) closed ( )

(b) infinite ( )

(c) defined in three dimensions ( )

(d) open with a boundary ( )

(SECTION : B—SHORT ANSWERS)

(*Marks* : 15)

Answer *five* questions, taking at least *one* from each Unit : 3×5=15

UNIT—I

1. If $\vec{F}(x, y) = (e^x)\hat{i} + (x^2y + \cos y)\hat{j}$, then find the values of $\dfrac{\partial \vec{F}}{\partial x}$ and $\dfrac{\partial \vec{F}}{\partial y}$.

2. If $\vec{A} = (t+1)\hat{i} + (t^2 + t + 1)\hat{j} + (t^3 + t^2 + 2)\hat{k}$, then find the value of $\dfrac{d^2\vec{A}}{dt^2}$.

UNIT—II

3. Show that $\vec{\nabla} \cdot (\vec{r} / r^2) = r^{-2}$.

4. Suppose that $\vec{A}$ and $\vec{B}$ are irrotational. Prove that $\vec{A} \times \vec{B}$ is solenoidal.

UNIT—III

5. Evaluate the volume integral $\iiint_V x^2 dV$, where V is the cube defined by

$$0 \le x \le 2,\ 0 \le y \le 3,\ 0 \le z \le 4$$

6. Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where C is the path along a straight line joining (0, 0) to (1, 1) and $\vec{F} = (3x^2 + 6y)\hat{i} - 14y\hat{j}$.

UNIT—IV

7. If S is any closed surface enclosing a volume V and $\vec{A} = ax\hat{i} + by\hat{j} + cz\hat{k}$, then prove that $\iint_S \vec{A} \cdot \hat{n} dS = (a + b + c)V$.

8. Find the area of the circle $x^2 + y^2 = r^2$, using Green's theorem.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer *five* questions, taking at least *one* from each Unit : 10×5=50

UNIT—I

1. (a) A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is time. Find the scalar and vector components of its velocity and acceleration at time $t = 1$ in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$. 5

   (b) If $\phi(x, y, z) = xy^2z$ and $\vec{A} = xz\hat{i} - xy^2\hat{j} + yz^2\hat{k}$, then find $\frac{\partial^3}{\partial x^2 \partial z}(\phi\vec{A})$ at the point $(2, -1, 1)$. 5

2. (a) If $\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}$ and $\vec{B} = z\hat{i} + x\hat{j} + y\hat{k}$, then evaluate the differential of $(\vec{A} \times \vec{B})$. 5

   (b) Prove that a vector function $\vec{F}(t)$ is of constant length if and only if

$$\vec{F} \cdot \frac{d\vec{F}}{dt} = 0$$

5

UNIT—II

3. (a) Find div$\vec{F}$ and curl$\vec{F}$, where $\vec{F} = \text{grad}\,(x^2 + y^2 + z^2 - 3xyz)$. 3

   (b) Find the directional derivatives of $\phi = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ, where Q has coordinates $(5, 0, 4)$. 3

   (c) Show that $\nabla^2(\log r) = \frac{1}{r^2}$. 4

4. *(a)* Find the direction along which the directional derivative of the function

$$x(y+z)-y(z-x)+z(x+y)$$

at the point $(0, -1, 2)$. Also find the greatest directional derivative. 5

(b) If $\vec{A}=(y^2+z^3,\ 2xy-5z,\ 3xz^2-5y)$, then show that $\text{curl}\,\vec{A}=0$ and find scalar function $\phi(x, y, z)$ such that $\vec{A}=\text{grad}\,\phi$. 5

UNIT—III

5. *(a)* If $\vec{F}=(2x+y)\hat{i}+(3y-x)\hat{j}$, then evaluate $\int_C \vec{F}\cdot d\vec{r}$, where C is the curve in the xy-plane consisting of straight line from $(0, 0)$ to $(2, 0)$ and then to $(3, 2)$. 5

(b) Find the total work done in moving a particle in a force field $\vec{F}=3xy\hat{i}-5z\hat{j}+10x\hat{k}$ along the curve $x=t^2+1$, $y=2t^2$, $z=t^3$ from $t=1$ to $t=2$. 5

6. *(a)* Evaluate $\iint_S \vec{A}\cdot\hat{n}\,dS$ where $\vec{A}=(x+y^2)\hat{i}-2x\hat{j}+2yz\hat{k}$ and S is the surface of the plane $2x+y+2z=6$ in the first octant. 5

(b) Calculate $\iiint_V \vec{F}\cdot dV$ where $\vec{F}=x^2\hat{i}+y\hat{j}-z\hat{k}$, and V is the region bounded by the surface $x=0$, $y=0$, $x=1$, $y=x$, $z=0$, $z=1-x$. 5

UNIT—IV

7. *(a)* Verify Green's theorem for the integral

$$\oint_C (xy+y^2)\,dx+x^2\,dy$$

where C is the closed curve of the region R bounded by $y=x$; $y=x^2$. 5

(b) Using divergence theorem, evaluate $\iint_S \vec{A} \cdot \hat{n} dS$ where

$$\vec{A} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$$

and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. 5

8. (a) Verify Stokes' theorem for

$$\vec{F} = (2x - y)\hat{i} - (yz^2)\hat{j} + (y^2 z)\hat{k}$$

where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. 5

(b) Use divergence theorem to evaluate

$$\iint_S (x\,dy\,dz + y\,dz\,dx + z\,dx\,dy)$$

where S is the portion of the plane $x + 2y + 3z = 6$ which lies in the first quadrant. 5

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