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(CBCS)

(6th Semester)

MATHEMATICS

TWELFTH (A) PAPER

(Astronomy)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—A

(*Marks* : 10)

Answer **all** questions

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. If the plane cutting the sphere does not pass through the centre of the sphere, then the corresponding section is called

(*a*) a small circle ☐

(*b*) a great circle ☐

(*c*) an ellipse ☐

(*d*) None of the above ☐

2. Let A, B, C be angles of a spherical triangle, then

(a) $A \quad B \quad C$ ☐

(b) $A \quad B \quad C$ ☐

(c) $A \quad B \quad C \quad \frac{}{2}$ ☐

(d) None of the above ☐

3. The right ascension of the sun at the vernal equinox is

(a) 90° ☐

(b) 30° ☐

(c) 60° ☐

(d) 0° ☐

4. A star whose declination is will not set or rise at the place of latitude , when

(a) 90° ☐

(b) 90° ☐

(c) ☐

(d) 90° ☐

5. The angle between real direction of the star and the direction of the earth's motion is called

(a) earth's way ☐

(b) parallax ☐

(c) aberration ☐

(d) None of the above ☐

6. Lunar eclipse can happen when the moon is

(a) in opposition ☐

(b) in conjunction ☐

(c) Both (a) and (b) ☐

(d) None of the above ☐

7. If S is the geometric longitude of the planet, then the planet's motion is direct when $\frac{dS}{dt}$ is

(a) negative ☐

(b) 0 ☐

(c) positive ☐

(d) None of the above ☐

8. The points where direct motion changes to retrograde motion and vice-versa are called

(a) elongation points ☐

(b) geocentric points ☐

(c) sidereal points ☐

(d) stationary points ☐

9. The planets which revolve outside the earth's orbit are called

(a) inferior planets ☐

(b) superior planets ☐

(c) satellite planets ☐

(d) None of the above ☐

10. By Kepler's third law, the angular velocity of a planet

(a) varies directly as the square of its distance from the sun ☐

(b) varies directly as its distance from the sun ☐

(c) varies inversely as the square of its distance from the sun ☐

(d) None of the above ☐

SECTION—B

(*Marks* : 15)

Answer **all** questions

Each question carries 3 marks

1. In a spherical triangle ABC, show that

$$\cot a \sin b \quad \cot A \sin C \quad \cos b \cos C$$

OR

Show that in a special triangle ABC, where $AB \quad c$, $BC \quad a$, $CA \quad b$, $\sin c \cos B \quad \sin a \cos b \quad \cos a \sin b \cos C$.

2. The mid-night depression below the horizon of the mid-summer sun is $15^{\circ}27$. Find the latitude of the place.

OR

A ship starts from a point on the equator and sails in great circle, cutting the equator at an angle of 45°. Find how much it has changed its longitude when it has reached a latitude of $\tan^{1}\frac{1}{2}$.

3. Show that the path of the star described on account of parallax is ellipse.

OR

The horizontal parallax of the moon is 75 and her angular diameter is 31 5 . Find the diameter of the moon in kilometers assuming the radius of the earth is 6400 km.

4. Define direct and retrograde motions of planets.

OR

If the line joining two planets to one another subtends an angle of 60° at the sun when the planets appear to each other to be stationery, then show that $a^2 \quad b^2 \quad 7ab$, where a and b are the distances of the planets from the sun.

5. Find the geographical position of the sun at GMT 10h30m48s, given that its declination is 10°40 and that the equation of time is –4m18s.

OR

If T is the orbital period of a planet, using Kepler's third law, show that a small increase $\quad a$ in the semi-major axis a will produce an increase of $\frac{3T}{2a} \quad a$ in the period.

(Part : B—Descriptive)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

Unit—I

1. *(a)* In a spherical triangle ABC, prove that

$$\frac{\sin A}{\sin a} \quad \frac{\sin B}{\sin b} \quad \frac{\sin C}{\sin c}$$

where the symbols carry their usual meaning. 6

(b) In a spherical triangle ABC, if D be the mid-point of AB, then show that

$$\cos AC \quad \cos BC \quad 2\cos\frac{1}{2}AB\cos CD$$ 4

2. *(a)* In an spherical equilateral triangle ABC, show that

$$\sec A \quad 1 \quad \sec a$$ 4

(b) In a spherical triangle ABC, prove that

$$\tan\frac{A \quad B}{2} \quad \frac{\cos\frac{a \quad b}{2}}{\cos\frac{a \quad b}{2}}\cot\frac{C}{2}$$ 6

UNIT—II

3. *(a)* If H is the hour angle of the sun at rising, then show that

$$2\cos^2\frac{H}{2} \quad \sec \quad \sec \quad \cos(\qquad)$$

where and stand for latitude and declination respectively. 5

(b) Show that in 45° latitude, the interval between the time at which any star passes due east and the time of setting is constant and is equal to half sidereal day. 5

4. Two stars ($_1$, $_1$) and ($_2$, $_2$) have the same longitude. Then prove that

$$\sin(\;_1 \quad _2) \quad \tan \quad (\cos \;_1 \tan \;_2 \quad \cos \;_2 \tan \;_1)$$

where be the obliquity of ecliptic. 10

UNIT—III

5. Derive Cassini's formula for atmospheric refraction. 10

6. *(a)* Find the effect of parallax on declination. 4

(b) Prove that the equation of time due to obliquity of ecliptic is maximum, when the longitude $\odot$ of the sun is given by

$$\sin\odot \quad \frac{1}{\sqrt{2}}\sec\frac{\;}{2}$$ 6

UNIT—IV

7. If and are the velocities of two planets in circular and coplanar orbits, then show that the period of direct motion is to the period of retrograde motion as 180 : , where

$$\cos \quad \frac{}{\;^2 \qquad \;^2}$$ 10

8. *(a)* Show that the elongation of Venus V when it is brightest is given by the equation $3\cos^2 \quad 4k\cos \quad 4 \quad 0$, where k is the ratio of its distance from the sun to that of the earth. 5

(b) If be the angle subtended at the earth by the sun and a stationary point of a planet's orbit and be the greatest elongation of the planet, then prove that

$$2\cot \qquad \sec\frac{}{2} \quad \operatorname{cosec}\frac{}{2}$$ 5

UNIT—V

9. State and derive Kepler's first law from Newton's law of gravitation. 10

10. (a) Prove that the dip of the visible horizon at a height h above the earth's surface is $\sqrt{\frac{2h}{a}}$, where a is the radius of the earth. 5

(b) Prove that at either equinox, in latitude , a mountain whose height is $\frac{1}{n}$ of earth's radius will catch the sun's rays in the morning $\frac{12}{\cos}\sqrt{\frac{2}{n}}$. 5

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