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(CBCS)

(6th Semester)

MATHEMATICS

ELEVENTH PAPER

(Mechanics)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. Three coplanar forces acting on a rigid body are in equilibrium, if

(a) two of them form a couple ( )

(b) the resultant R vanishes ( )

(c) all three meet at a point ( )

(d) two of them meet at a point ( )

2. In case of limiting equilibrium of a body on a rough surface, if F be the limiting friction at the point of contact, R the normal reaction between the bodies and the coefficient of friction, then

(a) $\frac{F}{R}$ ( ) *(b)* $\frac{R}{F}$ ( )

(c) $\frac{F}{R}$ 1 ( ) *(d)* $\frac{R}{F}$ 1 ( )

3. The moment of inertia of a rectangular lamina about a line through its centre and parallel to one of its edges is

(a) $\frac{M}{13}a$ ( )

(b) $\frac{M}{13}a^2$ ( )

(c) $\frac{M}{12}a$ ( )

(d) $\frac{M}{12}a^2$ ( )

4. The centre of gravity of three uniform rods forming a triangle is at

(a) the centroid of the triangle ( )

(b) the orthocentre of the triangle ( )

(c) the incentre of the triangle ( )

(d) None of the above ( )

5. If time t be regarded as a function of velocity v, then the rate of decrease of acceleration is given by

(a) $f^4 \frac{d^2t}{dv^2}$ ( )

(b) $f^3 \frac{d^2t}{dv^2}$ ( )

(c) $f^2 \frac{d^2t}{dv^2}$ ( )

(d) $f \frac{d^2t}{dv^2}$ ( )

6. If a particle moves in a plane with constant speed, then

(a) its acceleration is perpendicular to its velocity ( )

(b) its acceleration is parallel to its velocity ( )

(c) its acceleration is zero ( )

(d) None of the above ( )

7. The path of a projectile is known as

(a) range ( )

(b) curvature ( )

(c) course ( )

(d) trajectory ( )

8. The maximum range down an inclined plane is

(a) $\frac{u^2}{g(1 \quad \cos \ )}$ ( )

(b) $\frac{u^2}{g(1 \quad \cos \ )}$ ( )

(c) $\frac{u^2}{g(1 \quad \sin \ )}$ ( )

(d) $\frac{u^2}{g(1 \quad \sin \ )}$ ( )

9. If a particle moves so that its normal acceleration is always zero, then the path is

(a) a circle ( )

(b) a straight line ( )

(c) a parabola ( )

(d) a hyperbola ( )

10. If e be the coefficient of restitution of collision of two inelastic bodies, then

(a) $e \quad 1$ ( )

(b) $e \quad 1$ ( )

(c) $e \quad \frac{1}{2}$ ( )

(d) $e \quad 0$ ( )

SECTION—B

(*Marks* : 15)

Answer the following questions : $3 \times 5 = 15$

1. *(a)* A uniform ladder rests in limiting equilibrium with its upper end against a smooth wall. If be the inclination of the ladder to the vertical, prove that tan 2 , where is the coefficient of friction.

OR

(b) Forces proportional to 1, 2, 3 and 4 act along the sides *AB*, *BC*, *AD* and *DC* respectively of a square *ABCD*, the length of each side is 2 feet. Find the magnitude and line of action of their resultant.

2. *(a)* Find the centre of gravity of a uniform semi-circular wire of radius a unit.

OR

(b) Prove that the centre of gravity of a triangular area coincides with that of three equal particles placed at the middle points of its sides.

3. *(a)* If the angular velocity of a point moving in a plane curve be constant about a fixed origin, show that its transverse acceleration varies as its radial velocity.

OR

(b) Find the total energy of a body of mass m executing simple harmonic motion of period $\frac{2}{\quad}$ and amplitude a.

4. *(a)* If h and h be the greatest heights in the two paths of a projectile with a given velocity for a given range R, then show that $R \quad 4\sqrt{hh}$.

OR

(b) If a particle is projected with velocity u from the ground at an angle with the horizontal, find the time to reach the greatest height.

5. *(a)* The earth's attraction on a particle varies inversely as the square of its distance from the earth's centre. A particle whose weight on the surface of the earth is W, falls to the surface of the earth from a height $5a$ above it. Show that the work done by the earth's attraction is $\frac{5aW}{6}$, where a is the radius of the earth.

OR

(b) A sphere impinges directly on an equal sphere at rest. If the coefficient of restitution be e, show that their velocities after impact are as $(1 \quad e) : (1 \quad e)$.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* If a system of coplanar forces reduces to a single couple whose moment is G, such that when each force is turned round its point of application through a right angle, it reduces to a couple H. Prove that when each force is turned through an angle , the system is equivalent to a couple whose moment is $G\cos \quad H\sin$. 5

(b) A beam whose centre of gravity divides it into two portions a and b, is placed inside a smooth sphere. If be the inclination to the horizon in the position of equilibrium and 2 be the angle subtended by the beam at the centre of the sphere, then show that

$$\tan \quad \frac{b \quad a}{b \quad a}\tan$$ 5

2. *(a)* The altitude of a cone is h and the radius of its base is r, a string is fastened to the vertex and to a point on the circumference of the circular base, and is then put over a smooth peg. Show that if the cone rests with its axis horizontal, the length of the string must be $\sqrt{h^2 \quad 4r^2}$. 5

(b) A uniform rod rests in limiting equilibrium within a rough hollow sphere. If the rod subtends an angle 2 at the centre of the sphere and if be the angle of friction, show that the angle of inclination of the rod to the horizon is

$$\tan^{1} \frac{\sin 2}{2\cos(\qquad)\cos(\qquad)}$$ 5

UNIT—II

3. *(a)* A thin uniform wire is bent into the form of a triangle *ABC* and heavy particles of weight *P*, *Q*, *R* are placed at the angular points. If the centre of mass of the particles coincides with that of the wire, prove that

$$\frac{P}{b \quad c} \qquad \frac{Q}{c \quad a} \qquad \frac{R}{a \quad b}$$ 5

(b) A square hole is punched out of a circular lamina, the diagonal of the square being the radius of the circle. Show that the centre of gravity of the remainder is at a distance $\frac{a}{8 \quad 4}$ from the centre of the circle. 5

4. *(a)* Find the centroid of the area formed by the curve $y \quad \sin x$ and $y \quad 0$, where $0 \quad x \quad$. 5

(b) State and prove perpendicular axes theorem on moments of inertia. 5

UNIT—III

5. *(a)* The velocity of a train increases at a constant rate f_1 from rest to v, then remains constant for an interval and finally decreases to zero at the constant rate f_2. If x be the total distance described, prove that the total time taken is

$$\frac{x}{v} \quad \frac{v}{2} \frac{1}{f_1} \quad \frac{1}{f_2}$$ 5

(b) A body moving in a straight line *OAB* with simple harmonic motion has zero velocity when at the points *A* and *B* whose distances from *O* are *a* and *b* respectively, and has a velocity *v* when halfway between them. Show that the complete period is

$$\frac{(b \quad a)}{v}$$ 5

6. *(a)* A particle moves in a straight line. Its acceleration directed towards a fixed point *O* in the line, is equal to $\frac{a^5}{x^2} \quad ^{1/3}$, when it is at a distance x from *O*. If it starts from rest at a distance *a* from *O*, show that it will arrive at *O* with a velocity $a\sqrt{6}$ after time $\frac{8}{15}\sqrt{\frac{6}{}}$. 5

(b) A particle if projected vertically upward with a velocity v against a resistance proportional to the square of the velocity. If V is the terminal velocity of the body and m its mass, show that, when the body has fallen back to the point of projection, the loss of KE is

$$\frac{1}{2} mu^2 \quad \frac{u^2}{V^2 \quad u^2}$$ 5

UNIT—IV

7. (a) Two bodies are projected from the same point in direction making angles $_1$, $_2$ with the horizontal and strike at the same point in the horizontal plane through the point of projection. If t_1, t_2 be their times of flight, prove that

$$\frac{t_1^2 \quad t_2^2}{t_1^2 \quad t_2^2} \quad \frac{\sin(\, _1 \qquad _2)}{\sin(\, _1 \qquad _2)}$$ 5

(b) Particles are projected from the same point in a vertical plane with velocity $\sqrt{2gk}$. Prove that the locus of the vertices of their paths is the ellipse $x^2 \quad 4y(y \quad k) \quad 0$. 5

8. (a) Two particles are projected simultaneously in the same vertical plane from the same point with velocities u and v at angles and to the horizontal. Prove that the time that elapses between their transits through the other common point is

$$\frac{2uv \sin(\qquad)}{g(u \cos \qquad v \cos \;)}$$ 5

(b) A particle is projected vertically upwards with a velocity v in a medium whose resistance is kv^2 per unit mass. Show that the greatest height attained by the particle is

$$\frac{1}{2k} \log \; 1 \quad \frac{kv^2}{g}$$ 5

UNIT—V

9. *(a)* A uniform string of mass M and length $2a$, is placed symmetrically over a small smooth peg and has two particles of masses m, m attached to its extremities. Show that when the string runs off the peg, its velocity is

$$\sqrt{\frac{M \quad 2(m \quad m\ )}{M \quad m \quad m} ag}$$ 5

(b) A shot of mass m is projected from a gun of mass M by an explosion which generates a kinetic energy E. Show that the gun recoils with a velocity $\sqrt{\frac{2mE}{M(M \quad m)}}$ and the initial velocity of the shot is $\sqrt{\frac{2ME}{M(M \quad m)}}$. 5

10. *(a)* Two spheres of masses M and m impinge directly when moving in opposite directions with velocity u and v respectively. If the sphere of mass m is brought to rest by the collision, show that $v(m \quad eM) \quad M(1 \quad e)u$. 5

(b) A smooth sphere impinges on an equal sphere at rest. Before impact, the first sphere was moving in a direction making an angle   with the line of centres at the moment of impact. If the direction of motion of the first sphere is turned through an angle   by the impact, show that

$$\tan \quad \frac{(1 \quad e)\tan}{1 \quad e \quad 2\tan^2}$$ 5

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