2 0 1 8

(CBCS)

(5th Semester)

MATHEMATICS

EIGHTH (A) PAPER

[MATH–354 (A)]

(Operations Research)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. Which of the following statements is correct?

(a) If an LPP has two optimal solutions, then it has infinitely many solutions ☐

(b) Every LPP has a unique optimal solution ☐

(c) An LPP can have only two decision variables ☐

(d) Every LPP has at least one optimal solution ☐

2. If two constraints do not intersect in the positive quadrant of the graph, then

(a) one of the constraints is redundant ☐

(b) the solution is infeasible ☐

(c) the solution is unbounded ☐

(d) None of the above ☐

3. For maximization LPP, the objective function coefficient for an artificial variable is

(a) M ☐

(b) zero ☐

(c) 1 ☐

(d) M ☐

4. If an optimum solution is degenerate, then

(a) the solution is infeasible ☐

(b) there are alternative optimum solutions ☐

(c) the solution is of no use to the decision maker ☐

(d) None of the above ☐

5. If dual has an unbounded solution, primal has

(a) an unbounded solution ☐

(b) an infeasible solution ☐

(c) a feasible solution ☐

(d) a bounded solution ☐

6. The transportation problem deals with the transportation of

(a) a single-product from several sources to a destination ☐

(b) a multi-product from several sources to several destinations ☐

(c) a single-product from several sources to several destinations ☐

(d) a single-product from a source to several destinations ☐

7. In a mixed integer programming problem

(a) different objective functions are mixed together ☐

(b) all of the decision variables require integer solution ☐

(c) more of the decision variables require integer solutions ☐

(d) only few of the decision variables require integer solutions ☐

8. The use of cutting plane method

(a) requires the use of standard linear programming approach between each cutting plane application ☐

(b) yields better value of the objective function ☐

(c) reduces the number of constraints in the given problem ☐

(d) Both *(b)* and *(c)* ☐

9. A game is said to be strictly determinable, if

(a) minimax value is greater than maximin value ☐

(b) minimax value is equal to maximin value ☐

(c) minimax value is less than maximin value ☐

(d) maximin value is greater than or equal to minimax value ☐

10. In a two-person zero-sum game, which of the following characteristics is correct?

(a) Only two players are not involved ☐

(b) Each player has an infinite number of strategies to use ☐

(c) Total payoff to the players at the end of each play is zero ☐

(d) Some specific strategy results in a payoff ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

Answer the following :

1. *(a)* Express the following LPP into standard form :

Maximize $Z \quad 3x_1 \quad 2x_2 \quad 5x_3$

subject to

$$2x_1 \quad 3x_2 \quad 3$$
$$x_1 \quad 2x_2 \quad 3x_3 \quad 5$$
$$3x_1 \quad 2x_3 \quad 2$$

and $x_1, x_2 \quad 0$

OR

(b) Write down the mathematical formulation of a given LPP.

2. *(a)* Use simplex method to solve the following problem :

Maximize $Z \quad 20x \quad 30y$

subject to

$$3x \quad 3y \quad 36$$
$$5x \quad 2y \quad 50$$
$$2x \quad 6y \quad 60$$

and $x, y \quad 0$

OR

(b) Write down the algorithm of a minimization case using simplex method.

3. *(a)* Write the dual of the following problem :

Minimize $Z \quad 2x_2 \quad 5x_3$

subject to the constraints

$$\begin{array}{rrrr} x_1 & x_2 & & 2 \\ 2x_1 & x_2 & 6x_3 & 6 \\ x_1 & x_2 & 3x_3 & 4 \end{array}$$

and $x_1, x_2, x_3 \quad 0$

OR

(b) Solve the following minimal assignment problem :

		Man			
		1	*2*	*3*	*4*
Job	*I*	12	30	21	15
	II	18	33	9	31
	III	44	25	24	21
	IV	23	30	28	14

4. *(a)* Write the systematic step-by-step solution of IPP by branch and bound techniques.

OR

(b) Define pure and mixed integer programming problems with suitable example.

5. *(a)* Solve the game whose payoff matrix is

	Player B		
	15	2	3
Player A	6	5	7
	7	4	0

OR

(b) Consider the given game of the following payoff matrix :

		Player B	
Player A		2	6
		2	

(i) Show that *G* is strictly determinable, whatever may be.

(ii) Determine the value of *G*.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The questions are of equal value

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. A firm manufactures two types of product *A* and *B* and sells them at a profit of ₹ 2 on type *A* and ₹ 3 on type *B*. Each product is processed on two machines *M*1 and *M*2. Type *A* requires one minute of processing time on *M*1 and two minutes on *M*2; type *B* requires one minute on *M*1 and one minute on *M*2. The machine *M*1 is available for 10 hours during any working day. Formulate the problem as linear programming problem and find how may products of each type should the firm produce each day in order to get maximum profit by graphical method.

2. A manufacturer of a line of patent medicines is preparing a production plant on medicines *A* and *B*. There are sufficient ingredients available to make 20000 bottles of *A* and 40000 bottles of *B* but there are only 45000 bottles into which either of the medicines can be put. Furthermore, it takes 3 hours to prepare enough material to fill 1000 bottles of *A*, it takes 1 hour to prepare enough material to fill 1000 bottles of *B* and there are 66 hours available for this operation. The profit is ₹ 8 per bottle for *A* and ₹ 7 per bottle of *B*.

(a) Formulate this problem as a linear programming problem.

(b) How should the manufacture schedule production in order to maximize his profit?

UNIT—II

3. Solve the given LPP by using simplex method :

Minimize $Z \quad x_1 \quad 3x_2 \quad 2x_3$

subject to

$$3x_1 \quad x_2 \quad 2x_3 \quad 7$$
$$2x_1 \quad 4x_2 \quad 12$$
$$4x_1 \quad 3x_2 \quad 8x_3 \quad 10$$

and $x_1, x_2, x_3 \quad 0$

4. Food *X* contains 6 units of vitamin *A* per gram and 7 units of vitamin *B* per gram, and costs 12 paise per gram. Food *Y* contains 8 units of vitamin *A* per gram and 12 units of vitamin *B* per gram, and costs 20 paise per gram. The daily minimum requirement of vitamin *A* and vitamin *B* is 100 units and 120 units respectively. Find the minimum cost of product mix by Big-M method.

UNIT—III

5. Solve the transportation problem :

Source	*D*1	*D*2	*D*3	*Supply*
*S*1	9	8	5	25
*S*2	6	8	4	35
*S*3	7	6	9	40
Demand	30	25	45	100

Find the initial solutions by using—

(a) North-West corner method;

(b) least cost method;

(c) Vogel's approximations method.

6. A company has four jobs *A*, *B*, *C* and *D* to be done on four machines *W*, *X*, *Y* and *Z*. Each job must be done on one and only one machine. The cost (in ₹) of each job on each machine is given in the following cost table :

Cost Table

Job	*W*	*X*	*Y*	*Z*
A	7	9	8	13
B	16	16	15	11
C	16	19	10	15
D	16	17	14	16

Using Hungarian method of assignment, determine the job assignments to the machine so as to minimize the total cost.

UNIT—IV

7. Use Gomory's cutting plane algorithm to solve the following LPP :

Maximize $Z \quad 5x_1 \quad 7x_2$

subject to the constraint

$$2x_1 \quad 3x_2 \quad 6$$
$$6x_1 \quad x_2 \quad 30$$

$x_1, x_2 \quad 0$ and are integers

8. Solve the following mixed integer problem by branch and bound method :

Maximize $Z \quad x_1 \quad x_2$

subject to the constraint

$$2x_1 \quad 5x_2 \quad 16$$
$$6x_1 \quad 5x_2 \quad 30$$

$x_1, x_2 \quad 0$ and are integers

UNIT—V

9. Solve the following game by using the principle of dominance :

	Player B					
	4	2	0	2	1	1
	4	3	1	3	2	2
Player A	4	3	7	5	1	2
	4	3	4	1	2	2
	4	3	3	2	2	2

10. Solve the following 2 4 game by graphical method :

	Player B			
Player A	3	3	4	0
	5	4	3	7

★ ★ ★