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(CBCS)

(5th Semester)

MATHEMATICS

EIGHTH (B) PAPER : MATH–354 (B)

(Probability Theory)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Tick ☑ the correct answer in the box provided : 1×10=10

1. The chance that a leap year selected at random will contain 53 Sundays is

(a) $\frac{3}{7}$ ☐

(b) $\frac{2}{7}$ ☐

(c) $\frac{1}{7}$ ☐

(d) $\frac{5}{7}$ ☐

2. If A and B are two events such that $P(A) \quad \frac{1}{3}$, $P(B) \quad \frac{1}{4}$ and $P(A \quad B) \quad \frac{1}{2}$, then $P(A / \overline{B})$ is

(a) $\frac{1}{3}$ ☐ (b) $\frac{2}{3}$ ☐

(c) $\frac{1}{5}$ ☐ (d) None of the above ☐

3. For the probability density function $f(x) \quad cx^2(1 \quad x)$, $0 \quad x \quad 1$, the value of the constant c is

(a) 0 ☐ (b) 8 ☐

(c) 12 ☐ (d) $\frac{3}{5}$ ☐

4. For the binomial distribution, the first moment about origin is

(a) np ☐ (b) npq ☐

(c) $npq \quad n^2 p^2$ ☐ (d) $npq(q \quad p)$ ☐

5. The random variables X and Y with joint probability distribution $f(x, y)$ and marginal distribution $g(x)$ and $h(y)$ respectively are independent, iff

(a) $f(x, y) \quad g(x) \quad h(y)$ ☐ (b) $f(x, y) \quad g(x)h(y)$ ☐

(c) $f(x, y) \quad g(x) / h(y)$ ☐ (d) $f(x, y) \quad g(x) \quad h(y)$ ☐

6. If X and Y are two random variables, the covariance between them is defined as

(a) $\text{cov}(X, Y) \quad E(XY) \quad E(X)E(Y)$ ☐

(b) $\text{cov}(X, Y) \quad E(XY) \quad E(X)E(Y)$ ☐

(c) $\text{cov}(X, Y) \quad E(X) \quad E(Y) \quad E(XY)$ ☐

(d) $\text{cov}(X, Y) \quad E(X)E(Y) \quad E(XY)$ ☐

7. The expected value of the number of points obtained in a single throw with an ordinary dice is

(a) $\frac{7}{8}$ ☐ (b) $\frac{7}{2}$ ☐

(c) $\frac{7}{3}$ ☐ (d) None of the above ☐

8. The mean and variance of exponential distribution with parameter are

(a) $\frac{1}{\ }, \frac{1}{\ }$ ☐ (b) $\frac{1}{\ }, \frac{1}{\ ^{2}}$ ☐

(c) $\ , \ ^{2}$ ☐ (d) None of the above ☐

9. The relationship between mean and variance of gamma distribution is

(a) mean = 2 variance ☐ (b) mean < variance ☐

(c) mean = variance ☐ (d) None of the above ☐

10. Let X and Y be independent random variables with $Z \quad X \quad Y$. Let $M_X(t)$, $M_Y(t)$ and $M_Z(t)$ be the moment generating functions of X, Y and Z respectively, then

(a) $M_Z(t) \quad M_X(t) \quad M_Y(t)$ ☐ (b) $M_Z(t) \quad M_X(t) M_Y(t)$ ☐

(c) $M_Z(t) \quad M_X(t) / M_Y(t)$ ☐ (d) $M_Z(t) \quad M_X(t) \quad M_Y(t)$ ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

1. A bag contains 7 red, 12 white and 4 blue balls. What is the probability that three balls drawn at random are one of each colour?

OR

A speaks the truth in 60% and B in 75% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact?

2. If X is uniformly distributed over the interval $[a,\ b]$, prove that $E(X) \quad \frac{a \quad b}{2}$.

OR

If $\quad \frac{1}{36}$ and $\ _2 \quad \frac{35}{12}$, then find the corresponding binomial distribution.

3. If X and Y are random variables having joint density function

$$f(x,\ y) \qquad \begin{matrix} \frac{1}{8}(6 \quad x \quad y), & 0 \quad x \quad 2,\ 2 \quad y \quad 4 \\ 0, & \text{otherwise} \end{matrix}$$

find $P(X \quad 1 \quad Y \quad 3)$.

OR

Two random variables have the joint density function

$$f(x,\ y) \qquad \begin{matrix} 4xy, & 0 \quad x \quad 1,\ 0 \quad y \quad 1 \\ 0, & \text{otherwise} \end{matrix}$$

Find $P\ 0 \quad X \quad \frac{3}{4}, \frac{1}{8} \quad Y \quad \frac{1}{2}$.

4. Two random variables X and Y have the following joint probability density function :

$$f(x,\ y) \qquad \begin{matrix} 2 \quad x \quad y, & 0 \quad x \quad 1,\ 0 \quad y \quad 1 \\ 0, & \text{otherwise} \end{matrix}$$

Find $V(X)$ and $V(Y)$.

OR

If n dice are tossed and X denotes sum of the numbers on them, find $E(X)$.

5. Suppose X has a Poisson distribution. If $P(X \quad 2) \quad \frac{2}{3}P(X \quad 1)$, then find $P(X \quad 0)$.

OR

Prove that the moment generating function of gamma distribution is $M_X(t) \quad (1 \quad t)^{\ n},\ |t| \quad 1$.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* For any three events A, B and C, prove that

$$P(A \quad \overline{B}/C) \quad P(A \quad B/C) \quad P(A/C)$$ 5

(b) A problem in statistics is given to 3 students A, B and C where chances of solving it are $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{1}{4}$ respectively. What is the probability that exactly one of them will solve the problem? 5

2. *(a)* If A and B are mutually exclusive and $P(A \quad B) \quad 0$, then prove that $P(A/A \quad B) \quad \frac{P(A)}{P(A) \quad P(B)}$. 5

(b) In a certain college, 4% of the men and 1% of the women are taller than 1·8 m. Furthermore, 60% of the students are women. Now if a student is selected at random and is taller than 1·8 m, what is the probability that the student is a woman? 5

UNIT—II

3. *(a)* For the binomial distribution, prove that

$$_1 \quad \frac{1 \quad 2p)^2}{\sqrt{npq}} \text{ and } _2 \quad 3 \quad \frac{1 \quad 6pq}{npq}$$

Hence obtain $_1$ and $_2$. 5

(b) Ten coins are thrown simultaneously. Find the probability of getting at least eight tails. 5

4. *(a)* A continuous distribution of a variable X in the range (3, 3) is defined by

$$f(x) \quad \begin{cases} \frac{1}{16}(3 \quad x)^2, & 3 \quad x \quad 1 \\ \frac{1}{16}(6 \quad 2x^2), & 1 \quad x \quad 1 \\ \frac{1}{16}(3 \quad x)^2, & 1 \quad x \quad 3 \end{cases}$$

Find the mean of the above distribution. 5

(b) From a lot of 10 items containing 3 defectives, a sample of 4 items is drawn at random without replacement.

(i) Find the probability distribution of X, the number of defectives.

(ii) Find $P(X \quad 1)$, $P(X \quad 1)$ and $P(0 \quad X \quad 2)$. 5

UNIT—III

5. *(a)* For the following bivariate probability distribution of X and Y, find the following :

Y / X	1	2	3	4	5	6
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$

(i) $P(X \quad 1, Y \quad 2)$

(ii) $P(X \quad 1)$

(iii) $P(Y \quad 3)$ 5

(b) Let X and Y be two random variables, then prove that X and Y are independent, if $f(x, y) \quad 4xye^{\ (x^2 \quad y^2)}$; $x \quad 0$, $y \quad 0$. Also find that the conditional density of X given $Y \quad y$. 5

6. *(a)* For the joint probability density function

$$f(x, y) \quad \frac{1}{24}(x^3 y \quad xy^3); \; 0 \quad x \quad 2, \; 0 \quad y \quad 2$$

find the marginal and conditional densities of X and Y. 4

(b) The joint probability density function of two random variables X and Y is given by

$$f(x, y) \quad \frac{9(1 \quad x \quad y)}{2(1 \quad x)^4 (1 \quad y)^4}; \; 0 \quad x \quad , \; 0 \quad y$$

Obtain *(i)* marginal distributions of X and Y and *(ii)* conditional distribution of Y for $X = x$. 6

UNIT—IV

7. Two unbiased dice are thrown. If X is the sum of the numbers showing up, prove that $P[|X \quad 7| \quad 3] \quad \frac{35}{54}$. Also obtain the actual probability. 10

8. *(a)* A sample of 3 items is selected at random from a box containing 12 items of which 3 are defective. Find the expected number of defective items. 6

(b) Two random variables X and Y have the following joint probability density function :

$$f(x, y) \quad \begin{matrix} 2 \quad x \quad y; & 0 \quad x \quad 1, \; 0 \quad y \quad 1 \\ 0; & \text{otherwise} \end{matrix}$$

Find $V(X)$ and $\text{cov}(X, Y)$. 4

UNIT—V

9. *(a)* If X is a Poission variate with parameter and $_r$ as the rth central moment, then prove that $_{r\;1} \quad r \quad _{r\;1} \quad \frac{d \; _r}{d}$. 5

(b) For a certain normal distribution, the first moment about 10 is 40 and the fourth moment about 50 is 48. What are the arithmetic mean and variance of the normal distribution? 5

10. *(a)* Find the moment generating functions of exponential distribution and Gamma distribution. 5

(b) Define geometric distribution for a random variable X. Find its mean and variance. 5

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