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(CBCS)

(5th Semester)

MATHEMATICS

EIGHTH (B) PAPER : MATH–354 (B)

(Probability Theory)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Tick ☑ the correct answer in the box provided : 1×10=10

1. If $p_1 \quad P(A)$, $p_2 \quad P(B)$, $p_3 \quad P(A \quad B)$, then $P(A \quad \overline{B})$ in terms of p_1, p_2, p_3 is

(a) $1 \quad p_1 \quad p_2$ ☐

(b) $p_1 \quad p_3$ ☐

(c) $p_1 \quad p_2$ ☐

(d) $1 \quad p_2 \quad p_3$ ☐

2. If $P(A)$ 0 25, $P(B)$ 0 15, $P(A \quad B)$ 0 10, then $P(A \quad B)$ is

(a) $\frac{2}{3}$ ☐

(b) 0 10 ☐

(c) 0 30 ☐

(d) None of the above ☐

3. Let X be a random variable having discrete uniform distribution over the range $[1, n]$. Then the variance $V(X)$ is given by

(a) $\frac{(n \quad 1)(2n \quad 1)}{6}$ ☐

(b) $\frac{(n \quad 1)(2n \quad 1)}{12}$ ☐

(c) $\frac{(n \quad 1)}{2}$ ☐

(d) $\frac{(n \quad 1)(n \quad 1)}{12}$ ☐

4. For the binomial distribution

(a) mean < variance ☐

(b) variance < mean ☐

(c) mean = variance ☐

(d) None of the above ☐

5. The marginal probability function of Y for the discrete random variable is defined as

(a) $f_Y(y) \quad \underset{y}{\quad} p_{XY}(x, y)$ ☐

(b) $f_Y(y) \quad \underset{x}{\quad} p_{XY}(x, y)$ ☐

(c) $f_Y(y) \qquad f_{XY}(x, y)dx$ ☐

(d) $f_Y(y) \qquad f_{XY}(x, y)dy$ ☐

6. Let X and Y be two independent random variables. Then var(X Y) is equal to

(a) var(X) var(Y) ☐

(b) var(X) var(Y) ☐

(c) var(X) var(Y) 2 cov(X, Y) ☐

(d) var(X) var(Y) 2 cov(X, Y) ☐

7. If n dice are tossed and X denotes the sum of the numbers on them, then $E(X)$ is

(a) $\frac{7n}{2}$ ☐

(b) $\frac{3n}{2}$ ☐

(c) $\frac{5n}{4}$ ☐

(d) $\frac{7n}{3}$ ☐

8. Two unbiased dice are thrown. If X is the sum of the numbers showing up, then $P[|X \quad 7| \quad 3]$ is

(a) $\frac{7}{3}$ ☐

(b) $\frac{1}{3}$ ☐

(c) $\frac{5}{3}$ ☐

(d) None of the above ☐

9. The relationship between mean and variance of exponential distribution is

(a) $\frac{1}{\ }, \frac{1}{\ }$ ☐

(b) $\frac{1}{\ }, \frac{1}{\ ^2}$ ☐

(c) $, \ ^2$ ☐

(d) None of the above ☐

10. Let X and Y be independent random variables with $Z \quad X \quad Y$. Let $M_X(t)$, $M_Y(t)$ and $M_Z(t)$ be the moment generating functions of X, Y and Z respectively. Then

(a) $M_Z(t) \quad M_X(t) \quad M_Y(t)$ ☐

(b) $M_Z(t) \quad M_X(t) \quad M_Y(t)$ ☐

(c) $M_Z(t) \quad M_X(t) M_Y(t)$ ☐

(d) $M_Z(t) \quad \dfrac{M_X(t)}{M_Y(t)}$ ☐

SECTION—B

(*Marks* : 15)

Each question carries *3* marks

1. *(a)* If A and B are independent events, then $\overline{A}$ and $\overline{B}$ are also independent events. Prove it.

OR

(b) A speaks the truth in 60% and B in 75% of the cases. In what percentage of the cases are they likely to contradict each other in starting the same fact?

2. *(a)* If X is uniformly distributed over the interval $[a, b]$, then prove that $E(X) \quad \dfrac{a \quad b}{2}$.

OR

(b) Determine the binomial distribution for which the mean is 4 and the variance is 3.

3. *(a)* If

$$f(x, y) \quad \begin{matrix} \frac{2}{5}(2x \quad 3y), & 0 \quad x \quad 1, 0 \quad y \quad 1 \\ 0 & \text{otherwise} \end{matrix}$$

Then prove that $f(x, y)$ is a joint probability density function.

OR

(b) The joint density function of X, Y is given as

$$f(x, y) \quad \begin{matrix} 2, & 0 \quad x \quad y \quad 1 \\ 0 & \text{otherwise} \end{matrix}$$

Prove that X and Y are not independent random variables.

4. *(a)* If we have the probability density function

$$f(x, y) \quad \begin{matrix} \dfrac{20000}{x^3}, & x \quad 100 \\ 0 & \text{elsewhere} \end{matrix}$$

Then find the value of $E(X)$.

OR

(b) Two unbiased dice are thrown. If X is the sum of the numbers showing up, then find the value of $E(X)$.

5. *(a)* Prove that the moment generating function of gamma distribution is $M_X(t) \quad (1 \quad t)^{\quad n}, \; |t| \quad 1$.

OR

(b) For a certain normal distribution, the first moment about 10 is 40 and the fourth moment about 50 is 48. What are the arithmetic mean and variance of the normal distribution?

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* If A and B be events with $P(A) \quad \frac{3}{8}$, $P(B) \quad \frac{1}{2}$ and $P(A \quad B) \quad \frac{1}{4}$, find *(i)* $P(A \quad B)$, *(ii)* $P(\overline{A})$ and $P(\overline{B})$, *(iii)* $P(\overline{A} \quad \overline{B})$, *(iv)* $P(\overline{A} \quad \overline{B})$ and *(v)* $P(A \quad \overline{B})$. 5

(b) A box contains 6 red, 4 white and 5 black balls. A person draws 4 balls from the box at random. Find the probability that among the balls drawn there is at least one ball of each colour. 5

2. *(a)* Prove that if an event A is independent of the events B, $B \quad C$ and $B \quad C$ then it is also independent of C. 5

(b) A problem in statistics is given to 3 students A, B and C whose chances of solving it are $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{1}{4}$ respectively. Find the probability that exactly one of them will solve the problem. 5

UNIT—II

3. For the binomial distribution $(q \quad p)^n$, prove that

$$_{r \; 1} \quad pq \;\; nr \;\; _{r \; 1} \quad \frac{d \;\; _r}{dp}$$

where $_r$ is the rth central moment. Hence obtain $_2$, $_3$ and $_4$. Also find out $_1$ and $_2$. 10

4. *(a)* The probability distribution function of a random variable X is

$$f(x) \quad \begin{matrix} x, & \text{for } 0 \quad x \quad 1 \\ 2 \quad x, & \text{for } 1 \quad x \quad 2 \\ 0, & \text{for } \quad x \quad 2 \end{matrix}$$

Compute the cumulative distribution function of X. 5

(b) A random variable X has the probability density function as follows :

$$f(x) \quad \begin{matrix} \frac{1}{4} & 2 \quad x \quad 2 \\ 0 & \text{otherwise} \end{matrix}$$

Obtain the values of *(i)* $P(X \quad 1)$, *(ii)* $P(|X| \quad 1)$ and *(iii)* $P[(2X \quad 3) \quad 5]$. 5

UNIT—III

5. *(a)* For the following bivariate probability distribution of X and Y, find the following :

(i) $P(X \quad 1)$

(ii) $P(Y \quad 3)$

(iii) $P(X \quad 1, Y \quad 3)$

(iv) $P(X \quad 1 / Y \quad 3)$

(v) $P(Y \quad 3 / X \quad 1)$ 6

Y / X	1	2	3	4	5	6	$p(x)$
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$	$\frac{16}{64}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{40}{64}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$	$\frac{8}{64}$
$p(y)$	$\frac{6}{64}$	$\frac{6}{64}$	$\frac{11}{64}$	$\frac{13}{64}$	$\frac{12}{64}$	$\frac{16}{64}$	1

(b) If $f(x, y) \quad 4(1 \quad x)(1 \quad y)$; $0 \quad x, y \quad 1$, then prove that $f(x, y)$ to be a joint probability density function. 4

6. (a) The joint distribution of X and Y are given by $f(x, y) \quad 4xye^{\,(x^2 \quad y^2)}$; $x \quad 0, y \quad 0$. Prove that X and Y are independent. Also find the conditional density of X given $Y \quad y6$. 6

(b) The joint density function of X and Y is given by $f(x, y) \quad e^{\,(x \quad y)}$; $x \quad 0, y \quad 0$. Find the marginal distribution of X and Y. 4

UNIT—IV

7. State and prove Chebyshev's inequality. 10

8. (a) If X is a random variable, then prove that $\text{var}(X) \quad E(X^2) \quad [E(X)]^2$. 3

(b) The probability functions of random variables X and Y are given as follows :

x_i	1	2	3	4
$f(x_i)$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{8}$

y_i	2	4	6	8
$f(y_i)$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{8}$

Find *(i)* $\text{cov}(X, Y)$ and *(ii)* $_{xy}$ (correlation coefficient between X and Y). 7

UNIT—V

9. *(a)* If X is a Poisson variate such that $P(X \quad 2) \quad 9P(X \quad 4) \quad 90P(X \quad 6)$, find the value of the parameter . 5

(b) Define exponential distribution. Also find the moment generating function of exponential distribution. 5

10. *(a)* Define geometric distribution for a random variable X. Find its mean and variance. 5

(b) Find the mean and variance of normal distribution. 5

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