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(CBCS)

(5th Semester)

MATHEMATICS

SEVENTH PAPER (Math–353)

(Complex Analysis)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Tick (✓) the correct answer in the brackets provided :

1. The value of $(1 \quad i)^{5404}$ is

(a) 2^{2702} ( )

(b) $\quad 2^{2702}$ ( )

(c) 0 ( )

(d) 1 ( )

2. The equation of ellipse in complex form whose major axis is 7 and foci at (2, 0) and (2, 0) is

(a) $z\,|z \quad 2| \quad 4$ ( )

(b) $z(z \quad 4) \quad 7$ ( )

(c) $|z \quad 2| \quad |z \quad 2| \quad 7$ ( )

(d) $z^2 \quad 11$ ( )

3. The corresponding analytic function for the function $u \quad e^x(x\cos y \quad y\sin y)$ is

(a) e^z ( )

(b) $e^{\quad z}$ ( )

(c) ze^z ( )

(d) $e^z(z \quad 1)$ ( )

4. The derivative of the function $r^n\{\cos n \quad i\sin n \;\}$ is

(a) $nr^{n \quad 1}\{\cos(n \quad 1) \quad i\sin(n \quad 1) \;\}$ ( )

(b) $n(n \quad 1)\{\;\sin(n \quad 1) \quad i\cos(n \quad 1) \;\}$ ( )

(c) $n(n \quad 1)^2\{\;\sin(n \quad 1) \quad i\cos(n \quad 1) \;\}$ ( )

(d) $(n \quad 2)r^{n \quad 2}\{\;\sin(n \quad 2) \quad i\cos(n \quad 2) \;\}$ ( )

5. The radius of convergence of the series $\frac{1}{n^n}z^n$ is

(a) 1 ( )

(b) e ( )

(c) ( )

(d) 0 ( )

6. The series ${}_{0}z^{n}$ is divergent for

(a) $| z |$ 1 (   )

(b) $| z |$ 1 (   )

(c) $| z |$ 1 (   )

(d) $| z |$ 0 (   )

7. The value of $\frac{1}{z}dz$ along a semi-circular arc $| z |$ 1 from 1 to 1 is

(a) i (   )

(b) i (   )

(c) i (   )

(d) i (   )

8. The value of ${}_{C}z\,dz$ (where C is a curve from z a to z b) is

(a) $b \quad a$ (   )

(b) $b^{2} \quad a^{2}$ (   )

(c) $\frac{b^{2} \quad a^{2}}{2}$ (   )

(d) $\frac{b \quad a}{2}$ (   )

9. The function $\log z$ has a singularity at origin

(a) as removable singularity (   )

(b) as pole (   )

(c) as essential singularity (   )

(d) as non-isolated singularity (   )

10. If $f(z)$ be analytic within and on a simple closed contour C, then the point giving the maximum of $|f(z)|$ can be

(a) within C ( )

(b) outside C ( )

(c) on the boundary of C and not within it ( )

(d) on the boundary of C and not within C ( )

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

Answer the following questions :

1. Find the value of $|z|$ from $iz^2 \quad \bar{z} \quad 0$.

OR

Find the inverse of the point $(1 \quad i)$ with respect to the circle $|z| \quad 1$.

2. Show that $f(z) \quad |z|^2$ is not differentiable for any non-zero value of z.

OR

For what value of z the function w defined by $z \quad e^{\;v}(\cos u \quad i \sin u)$, where $w \quad u \quad iv$ ceases to be analytic?

3. For what value of z the power series $\;!n\, z^n$ will converge and diverge?

OR

Find the radius of convergence of the series $\;_{n \;\; 2} \dfrac{z^n}{n(\log n)^2}$.

4. Evaluate $\displaystyle _{L}\frac{z \quad 2}{z}dz$ where L is semi-circle $z \quad 2e^{it}$, $0 \quad t \quad$.

OR

Evaluate $_{C}|\,z\,|dz$ where C is circle $|z \quad 1| \quad 1$ described in the positive sense.

5. Define isolated singularity with example.

OR

Find the nature and location of the singularity of the function $f(z) \quad \dfrac{1}{z(e^{z} \quad 1)}$.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Show that $|\,z_1 \quad z_2 \quad z_3\,| \quad |\,z_1\,| \quad |\,z_2\,| \quad |\,z_3\,|$. 5

(b) Find the locus of the point z satisfying the condition $|\,z^2 \quad 1| \quad 1$. 5

2. *(a)* Find the equation of the circle through the points $3 \quad i$, $2 \quad 2i$, $1 \quad i$. Also find its centre and radius. 6

(b) Prove that $|z_1 \quad z_2|^2 \quad |z_1 \quad z_2|^2 \quad 2(|z_1|^2 \quad |z_2|^2)$ where z_1 and z_2 are any complex numbers. 4

UNIT—II

3. *(a)* For an analytic complex function $f(z) \quad u(r, \quad) \quad iv(r, \quad)$, show that

$$\frac{u}{r} \quad \frac{1}{r}\frac{v}{} \text{ and } \frac{u}{} \qquad r\frac{v}{r}$$ 5

(b) If $u \quad x^3 \quad 3xy^2$, then show that there exists a function $v(x, y)$ such that $w \quad u \quad iv$ is analytic in any finite region. 5

4. *(a)* If $f(z) \quad u \quad iv$ is an analytic function and $u \quad v \quad (x \quad y)(x^2 \quad 4xy \quad y^2)$, then find $f(z)$ in term of z. 5

(b) If $f(z) \quad u \quad iv$ is an analytical function of $z \quad x \quad iy$ and is any function of x and y, then show that

$$\frac{2}{x^2} \quad \frac{2}{y^2} \qquad \frac{2}{u^2} \quad \frac{2}{v^2} \quad |f \ (z)|^2$$ 5

UNIT—III

5. *(a)* If the power series $a_n z^n$ converge for a particular value z_0 of z, then prove that it converges absolutely for all values of z for which $|z| \quad |z_0|$. 5

(b) Find the domain of convergence of the series $\left(\frac{z^2 \quad 1}{1 \quad i}\right)^n n^2$. 5

6. *(a)* Find the radius of convergence of the power series $f(z) \qquad \frac{z^n}{2^n \quad 1}$ and prove that $\{(2 \quad z) f(z) \quad 2\} \qquad 0$ as $z \qquad 2$. 5

(b) Find the domain of convergence of the power series $\left(\frac{2i}{z \quad i \quad 1}\right)^n$. 5

UNIT—IV

7. *(a)* Show that

$$\left| \;_{L} f(z)\, dz \right| \quad _{L} |f(z)|\, |dz|$$

where L is the rectifiable curve of $f(z)$. 4

(b) Let $f(z)$ is an analytic function in a simple connected domain D bounded by a rectifiable curve C and is continuous in C. If a be any point of D, then show that

$$f(a) \quad \frac{1}{2 \quad i} \; _{C} \frac{f(z)}{(z \quad a)} dz$$ 6

8. *(a)* If a function $f(z)$ satisfying the inequality $|f(z)| \quad M$ for all values of z, where M is a positive constant, then show that $f(z)$ is constant function. 5

(b) Evaluate by Cauchy integral formula $_{C} \dfrac{dz}{z(z \quad\quad i)}$ where C is $|z \quad 3i| \quad 4$. 5

UNIT—V

9. *(a)* Expand $\dfrac{1}{z(z^2 \quad 3z \quad 2)}$ for the region $1 \quad |z| \quad 2$. 5

(b) Examine the singularity of the function—

(i) $\dfrac{\cot \quad z}{(z \quad a)^2}$ at $z \quad a$ and $z \quad\quad$;

(ii) $\operatorname{cosec} \dfrac{1}{z}$ at $z \quad 0$. 5

10. State and prove maximum modulus theorem. 10

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