2 0 1 8

(CBCS)

(5th Semester)

MATHEMATICS

FIFTH PAPER (Math–351)

(Computer-oriented Numerical Analysis)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Tick ☑ the correct answer in the box provided : 1×10=10

1. Which of the following identities is true?

(a) E 1 ☐ *(b)* E 1 ☐

(c) E 1 ☐ *(d)* E^{1} 1 ☐

2. The best method among the following for finding the solution of algebraic and trancendental equations is

(a) bisection ☐ *(b)* iteration ☐

(c) regula falsi ☐ *(d)* Newton-Raphson ☐

3. While constructing a backward difference table, if seven arguments are given, the backward difference table will contain term up to

(a) ^5y ☐ *(b)* ^6y ☐

(c) ^7y ☐ *(d)* ^8y ☐

4. If $f(x) \quad \frac{1}{x^3}$, then the divided difference of $\quad (1,\ 2)$ equals

(a) $\frac{1}{8}$ ☐ (b) $\frac{3}{8}$ ☐

(c) $\frac{5}{8}$ ☐ (d) $\frac{7}{8}$ ☐

5. Back substitution procedure of solving a simultaneous linear equation is given by

(a) Gauss elimination method ☐

(b) Crout's method ☐

(c) Gauss-Jordan method ☐

(d) Gauss-Seidel method ☐

6. In solving simultaneous linear equation by using Crout's method, we have $A \quad LU$, where A is the coefficient matrix, then

(a) L is diagonal triangular matrix ☐

(b) U is lower triangular matrix ☐

(c) L is upper triangular matrix ☐

(d) U is upper triangular matrix ☐

7. From the general quadrature formula, we can obtain a variate formula for calculating the numerical value of a definite integral by putting $n \quad 1,\ 2,\ 3,\ \ldots$. The best are found for

(a) $n \quad 1$ and $n \quad 2$ ☐

(b) $n \quad 2$ and $n \quad 6$ ☐

(c) $n \quad 2$ and $n \quad 4$ ☐

(d) $n \quad 4$ and $n \quad 6$ ☐

8. The Simpson's one-third rule in numerical integration needs at least

(a) a parabolic curve ☐

(b) a circular curve ☐

(c) four geometrical points ☐

(d) three geometrical points ☐

9. The *n*th approximation in Picard's iteration formula is given by

(a) $y_{n \quad 1} \quad y_n \quad hf(x_n, y_n)$ ☐

(b) $y_n \quad y_0 \quad {}_{x_0}^{x} f(x_n, y_n)\,dx$ ☐

(c) $y_n \quad y_0 \quad {}_{x_0}^{x} f(x, y_{n \quad 1})\,dx$ ☐

(d) $y_n \quad y_0 \quad {}_{x_0}^{x} f(x_{n \quad 1}, y_n)\,dx$ ☐

10. For solving ordinary differential equation numerically, the most reliable and the most accurate among the following is

(a) Taylor's method ☐

(b) Picard's method ☐

(c) Euler's method ☐

(d) Runge-Kutta method ☐

SECTION—B

(*Marks* : 15)

Answer the following questions : 3×5=15

1. Find $\ ^{n}u_x$, where $u_x \quad e^{ax \quad b}$ (here a and b are constants).

OR

Find the second difference of $y \quad 7x^4 \quad 12x^3 \quad 6x^2 \quad 5x \quad 3$, if $h \quad 2$.

2. Find the cubic polynomial interpolation which takes on the values

$$f(1) \quad 1, \; f(2) \quad 9, \; f(3) \quad 25, \; f(4) \quad 55$$

OR

If $f(x) \quad \frac{1}{x^3}$, find the divided difference of $\quad (a, b, c)$.

3. Define diagonally dominant matrix. Is the following system of equations diagonally dominant?

$$5x \quad 15y \quad 2z \quad 10, \quad x \quad y \quad 54z \quad 19, 27x \quad 6y \quad z \quad 3$$

OR

Solve the given equation by Crout's method :

$$x \quad y \quad 2; \; 2x \quad 3y \quad 5$$

4. Find the value of $_0^1 \frac{dx}{1 \quad x}$ using Simpson's one-third rule by taking $n \quad 2$.

OR

Derive the formula for trapezoidal rule for numerical integration.

5. Use Picard's method to solve $\frac{dy}{dx} \quad xy,\ y(0) \quad 1$.

OR

Apply Runge-Kutta formula of order 2, approximate the value of y for $x \quad 1 \ 1$, given $\frac{dy}{dx} \quad 3x \quad y^2,\ \ y(1) \quad 1 \ 2$.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* Express $3x^4 \quad 4x^3 \quad 6x^2 \quad 2x \quad 1$ in terms of factorial polynomial and find its fourth-order difference. 5

(b) Find the root of the equation using bisection method in four stages : 5

$$x^3 \quad 4x \quad 9 \quad 0$$

2. *(a)* Find a root of the equation $x^3 \quad x^2 \quad 3x \quad 3 \quad 0$ lying between 1 and 2 using regula falsi method. 5

(b) Write an algorithm for solving a given equation by using bisection method. 5

UNIT—II

3. *(a)* Obtain Newton's forward interpolation formula for interpolation with equal intervals of the argument. 5

(b) Find the missing values from the following table : 5

x	0	5	10	15	20	25
$f(x)$	6	10	–	17	–	31

4. *(a)* Obtain Lagrange's interpolation formula for interpolation with unequal intervals of the argument. 5

(b) Prepare a divided difference table for the following data :

x	1	2	4	7	12
y	22	30	82	106	216

Hence by using Newton's divided difference formula, find $f(5)$. 5

UNIT—III

5. *(a)* Solve the following system of equations by Gaussian elimination method : 5

$$3x \quad y \quad 2z \quad 3, \quad 2x \quad 3y \quad z \quad 3, \quad x \quad 2y \quad z \quad 4$$

(b) Solve the following system of equations by Crout's method : 5

$$4x \quad y \quad z \quad 13, \quad 3x \quad 5y \quad 2z \quad 21, \quad 2x \quad y \quad 6z \quad 14$$

6. *(a)* Solve the system of linear equations by Gauss-Jordan method : 5

$$x \quad y \quad z \quad 9, \quad 2x \quad 3y \quad 4z \quad 13, \quad 3x \quad 4y \quad 5z \quad 40$$

(b) Solve by using Gauss-Seidel method : 5

$$6x \quad y \quad z \quad 105, \quad 4x \quad 8y \quad 3z \quad 155, \quad 5x \quad 4y \quad 10z \quad 65$$

UNIT—IV

7. *(a)* Write an algorithm for Simpson's one-third rule. 5

(b) From the table below, evaluate f (14 9) : 5

x	14·1	14·3	14·5	14·7	14·9	15·1	15·3
y	7·25	8·17	9·04	10·42	11·99	14·11	17·08

8. *(a)* Obtain the general quadrature formula for equidistant points to find the approximate integration of any function for which numerical values are known. 5

(b) Evaluate $\,_{0}^{10} \dfrac{dx}{1 \quad x^2}$ by using trapezoidal rule. 5

UNIT—V

9. *(a)* Using Taylor's series method, obtain the solution of $\frac{dy}{dx} = 3x + y^2$ and $y = 1$ when $x = 0$. Find the value of y for $x = 0{\cdot}1$, correct to four places of decimals. 5

(b) Use Picard's method to solve

$$\frac{dy}{dx} = xy, \quad y(0) = 1$$

5

10. *(a)* Using Euler's method, find an approximate value of y corresponding to $x = 2$, given that $\frac{dy}{dx} = x + 2y$, $y(1) = 1$. 5

(b) Apply Runge-Kutta method (fourth order) to find an approximate value of y when $x = 0{\cdot}2$, given that $\frac{dy}{dx} = x + y^2$, $y(0) = 1$. 5

★ ★ ★