MATH/V/08 (b) **Student's Copy**

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(Pre-CBCS)

(5th Semester)

MATHEMATICS

EIGHTH (B) PAPER : MATH–354 (B)

(Probability Theory)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Tick ☑ the correct answer in the box provided : 1×10=10

1. If S is a sample space of a random experiment, then

(a) $P(S)$ 0 ☐

(b) $P(S)$ ☐

(c) $P(S)$ 1 ☐

(d) $P(S)$ $\frac{1}{2}$ ☐

2. When A and B are mutually exclusive events,

(a) $P(A \quad B) \quad 0$ ☐

(b) $P(A \quad B) \quad 1$ ☐

(c) $P(A \quad B) \quad P(B)$ ☐

(d) $P(A \quad B) \quad P(A)$ ☐

3. If

$$p(x) \qquad \begin{array}{ll} \dfrac{x}{15}, & x \quad 1, 2, 3, 4, 5 \\ 0, & \text{elsewhere} \end{array}$$

then $P(X \quad 1 \text{ or } 2)$ is

(a) $\frac{1}{5}$ ☐

(b) $\frac{2}{5}$ ☐

(c) $\frac{3}{5}$ ☐

(d) $\frac{4}{5}$ ☐

4. If X is a continuous random variable with probability density function $f(x)$, then the arithmetic mean is given by

(a) ${}_a^b f(x)dx$ ☐

(b) ${}_a^b x f(x)dx$ ☐

(c) ${}_a^b x^2 f(x)dx$ ☐

(d) ${}_a^b x^r f(x)dx$ ☐

5. If (X, Y) is a random variable, then the conditional probability mass function of X, given $Y \quad y$, is

(a) $P_{X|Y}(x|y) \quad \dfrac{P(X \quad x, Y \quad y)}{P(X \quad x)}$ ☐

(b) $P_{Y|X}(y|x) \quad \dfrac{P(X \quad x, Y \quad y)}{P(X \quad x)}$ ☐

(c) $P_{Y|X}(x|y) \quad \dfrac{P(X \quad x, Y \quad y)}{P(Y \quad y)}$ ☐

(d) $P_{X|Y}(x|y) \quad \dfrac{P(X \quad x, Y \quad y)}{P(Y \quad y)}$ ☐

6. The marginal density function of X is given by

(a) $f_X(x) \qquad f_{XY}(x, y)dy$ ☐

(b) $f_X(x) \qquad f_{XY}(x, y)dx$ ☐

(c) $f_Y(x) \qquad f_{XY}(x, y)dy$ ☐

(d) $f_Y(x) \qquad f_{XY}(x, y)dx$ ☐

7. For two random variables X and Y, the relation $E(XY) \quad E(X).E(Y)$ holds good

(a) if X and Y are independent ☐

(b) for all X and Y ☐

(c) if X and Y are identical ☐

(d) None of the above ☐

8. If X is a random variable and a and b are constants,

(a) $V(aX \quad b) \quad aV(X) \quad b$ ☐

(b) $V(aX \quad b) \quad aV(X) \quad V(b)$ ☐

(c) $V(aX \quad b) \quad a^2V(X)$ ☐

(d) $V(aX \quad b) \quad aV(X)$ ☐

9. The characteristic function of the Poisson distribution is

(a) $(q \quad pe^t)^n$ ☐

(b) $e^{\quad (e^{it} \quad 1)}$ ☐

(c) $(p \quad qe^t)^n$ ☐

(d) $\dfrac{q}{p^2}$ ☐

10. The relationship between mean and variance of geometric distribution is

(a) mean = variance ☐

(b) mean = 2 variance ☐

(c) mean > variance ☐

(d) mean < variance ☐

SECTION—B

(*Marks* : 15)

Answer the following questions : 3×5=15

1. If A and B are independent events, then show that $\overline{A}$ and B are independent.

2. The mean and variance of binomial distribution are 4 and $\dfrac{4}{3}$ respectively. Find $P(X \quad 1)$.

3. For two-dimensional random variable (X, Y) with joint distribution function $F_{XY}(x, y)$, prove that $f_{Y|X}(y|x) \quad \dfrac{f_{XY}(x, y)}{f_X(x)}$, where $f_X(x) \quad 0$.

4. If X and Y are random variables, prove that $E(aX \quad bY) \quad aE(X) \quad bE(Y)$, where a and b are constants.

5. Find the moment generating function of a gamma distribution.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, taking **one** from each Unit

UNIT—I

1. *(a)* State the axiomatic definition of probability. Prove that $P(\overline{A})\quad 1\quad P(A)$, where $\overline{A}$ is the complementary event of A. 5

(b) Two dice are tossed. Find the probability of getting 'an even number on the first die or a total of 8'. 5

2. State and prove Bayes' theorem. 10

UNIT—II

3. *(a)* Let X be a continuous random variable with probability density function

$$f(x) \quad \begin{matrix} ax, & 0 \quad x \quad 1 \\ a, & 1 \quad x \quad 2 \\ ax \quad 3a, & 2 \quad x \quad 3 \\ 0, & \text{elsewhere} \end{matrix}$$

(i) Determine the constant a.

(ii) Compute $P(X \quad 1\ 5)$. 5

(b) With the usual notations, find p for a binomial variate X, if $n \quad 6$ and $9P(X \quad 4) \quad P(X \quad 2)$. 5

4. *(a)* A continuous random variable X has a probability density function $f(x) \quad 3x^2$, $0 \quad x \quad 1$. Find a and b such that—

(i) $P(X \quad a) \quad P(X \quad a)$;

(ii) $P(X \quad b) \quad 0\ 05$. 5

(b) A random variable X has the following probability function :

x	:	0	1	2	3	4	5	6	7
$p(x)$	:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 \quad k$

Find—

(i) k ;

(ii) $P(X \quad 6)$. 5

UNIT—III

5. *(a)* For the bivariate probability distribution of X and Y given below :

Y / X	1	2	3	4	5	6
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$

Find—

(i) $P(X \quad 1, Y \quad 2)$;

(ii) $P(X \quad 1)$;

(iii) $P(X \quad 3)$. 5

(b) A two-dimensional random variable (X, Y) has a joint probability mass function $p(x, y) \quad \frac{1}{27}(2x \quad y)$, where x and y can assume 0, 1 and 2 only. Find the conditional distribution of Y for $X \quad x$. 5

6. *(a)* The joint probability density function of a two-dimensional random variable (X, Y) is given by

$$f(x, y) \quad \begin{matrix} 2, & 0 \quad x \quad 1, \quad 0 \quad y \quad x \\ 0, & \text{elsewhere} \end{matrix}$$

Find the—

(i) marginal density functions of X and Y;

(ii) conditional density function of Y given $X \quad x$;

(iii) conditional density function of X given $Y \quad y$. 5

(b) The joint probability density function of a two-dimensional random variable (X, Y) is given by

$$f(x, y) \quad \begin{matrix} 6x^2y, & 0 \quad x \quad 1, \quad 0 \quad y \quad 1 \\ 0, & \text{elsewhere} \end{matrix}$$

(i) Verify that ${}_0^1\,{}_0^1 f(x, y)dxdy \quad 1$.

(ii) Find $P(X \quad 1 | Y \quad 2)$. 5

UNIT—IV

7. Two random variables X and Y have the following joint probability density function :

$$f(x, y) \quad \begin{matrix} 2 \quad x \quad y, & 0 \quad x \quad 1, \quad 0 \quad y \quad 1 \\ 0, & \text{otherwise} \end{matrix}$$

Find the—

(a) marginal probability density functions of X and Y;

(b) conditional density functions;

(c) $\mathrm{Var}(X)$ and $\mathrm{Var}(Y)$;

(d) covariance between X and Y. 10

8. State and prove Chebyshev's inequality. 10

UNIT—V

9. *(a)* For a Poisson distribution, prove that $_{r\ 1} \quad r \quad _{r\ 1} \quad \frac{d\ \ _r}{d}$. 5

(b) Find the moment generating function of a normal distribution. 5

10. *(a)* Find the moment generating function, mean and variance of a geometric distribution. 5

(b) If X and Y are independent Poisson variates such that $P(X\quad 1)\quad P(X\quad 2)$ and $P(Y\quad 2)\quad P(Y\quad 3)$, find the variance of $X\quad 2Y$. 5

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