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(Pre-CBCS)

(5th Semester)

MATHEMATICS

SEVENTH PAPER

(MATH–353)

(Complex Analysis)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

Answer **all** questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. For z_1 i and z_2 $\sqrt{3}$ i, the arg$(z_1 \;\; z_2)$ is equal to

(a) $\frac{\ }{3}$ ☐

(b) $\frac{4}{3}$ ☐

(c) $\frac{5}{6}$ ☐

(d) $\frac{\ }{2}$ ☐

2. The value of ($\sqrt{3}$ $i)^3$ is

(a) i ☐

(b) $8i$ ☐

(c) $8i$ ☐

(d) $4\sqrt{2}i$ ☐

3. Which of the following is not harmonic function?

(a) $e^{x}(x\cos y \quad y\sin y)$ ☐

(b) $\frac{1}{2}\log(x^2 \quad y^2)$ ☐

(c) $\sin x \cosh y$ ☐

(d) $x^3 \quad 3xy^2$ ☐

4. The functions satisfying Laplace's equation are known as

(a) regular ☐

(b) homomorphic ☐

(c) harmonic ☐

(d) conjugate ☐

5. The series $\frac{z^n}{!n}$ is absolutely convergent for

(a) $z \quad 0$ ☐

(b) $z \quad 0$ ☐

(c) $|z| \quad 1$ ☐

(d) all values of z ☐

6. If R_1 and R_2 are the radii of convergence of the power series $a_n z^n$ and $b_n z^n$ respectively, then the radius of convergence of the series $a_n b_n z^n$ is

(a) $R_1 \quad R_2$ ☐

(b) $R_1 \; R_2$ ☐

(c) $\frac{R_1}{R_2}$ ☐

(d) $\frac{R_1 \; R_2}{2}$ ☐

7. The value of $\bar{z}\,dz$ along a semicircular arc $|z| \quad 1$ from $\quad 1$ to 1 is

(a) i ☐

(b) i ☐

(c) i ☐

(d) i ☐

8. A continuous arc without multiple point is called a

(a) Jordan curve ☐

(b) continuous arc ☐

(c) contour ☐

(d) rectifiable curve ☐

9. The function $f(z) \quad 3z^2 \quad 2z \quad 1$ has a singularity at infinity as a/an

(a) removable singularity ☐

(b) pole of order 2 ☐

(c) essential singularity ☐

(d) non-isolated singularity ☐

10. The function $f(z) \quad e^z$ has a singularity at infinity as a/an

(a) removable singularity ☐

(b) isolated essential singularity ☐

(c) essential singularity ☐

(d) non-isolated singularity ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

1. Show that $\arg(z) \quad \arg(\bar{z}) \quad 2n$.

2. Find the derivatives of $w \quad f(z)$ in polar form.

3. Find the radius of convergence of the series

$$1 \quad \frac{1}{n}^{n^2} \quad z^n$$

4. Evaluate $_L \bar{z}\, dz$ from $z \quad 0$ to $z \quad 4 \quad 2i$ along the curve L defined by the line from $z \quad 0$ to $z \quad 2i$ and then from $z \quad 2i$ to $z \quad 4 \quad 2i$.

5. Find the singularity of $\tan \frac{1}{z}$ at $z \quad 0$.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. *(a)* For any two complex numbers z_1 and z_2, show that $|z_1 \quad z_2|^2 \quad |z_1|^2 \quad |z_2|^2$ if and only if $z_1\bar{z}_2$ is purely imaginary. 5

(b) Prove that the sum and product of two complex numbers are real if and only if they are conjugate to each other. 5

2. *(a)* Find the equation in complex variable of all the circles which are orthogonal to $|z_1| \quad 1$ and $|z_1 \quad 1| \quad 4$. 5

(b) Find the region of the z-plane for which

$$\left|\frac{z \quad a}{z \quad \bar{a}}\right| \quad 1, \quad 1 \text{ or } \quad 1$$

where the real part of a is always positive. 5

UNIT—II

3. *(a)* State and prove Cauchy-Riemann equation for analyticity of a complex function. 5

(b) Determine the harmonic conjugate of $u \quad y^3 \quad 3x^2y$ and find the corresponding analytic function $f(z)$ in term of z. 5

4. *(a)* Show that $\frac{2}{x^2} \quad \frac{2}{y^2} \quad 4\frac{2}{z \; \bar{z}}$ for any complex function. 5

(b) Examine the nature of the function

$$f(z) \quad \frac{x^2y^5(x \quad iy)}{x^4 \quad y^{10}}, \; z \quad 0 \text{ and } f(0) \quad 0$$

in a region including origin. 5

UNIT—III

5. *(a)* Find the radius of convergence of the series

$$\frac{n\sqrt{2} \quad i}{1 \quad i \; 2n} z^n$$ 5

(b) Find the domain of convergence of the series

$$\frac{(\;\; 1)^{n \;\; 1}}{(2n \quad 1)!} z^{2n \;\; 1}$$ 5

6. *(a)* For what value of z, does the series $\;_{n \;\; 1} \frac{1}{(z^2 \quad 1)^n}$ converge? 5

(b) Find the domain of convergence of the power series

$$\frac{1 \;\; 3 \;\; 5 \cdots (2n \quad 1)}{!n} \; \frac{1 \quad z}{z}^{\; n}$$ 5

UNIT—IV

7. *(a)* Evaluate $\;_{L} \frac{z \quad 2}{z} dz$, where L is the semicircle $z \quad 2e^{it}$, $0 \quad t \quad$. 4

(b) Let $f(z)$ is an analytic function in a simple connected domain D bounded by a rectifiable curve C and is continuous in C. If a be any point of D, then show that

$$f(a) \quad \frac{1}{2 \;\; i} \;_{C} \frac{f(z)}{(z \quad a)} dz$$ 6

8. *(a)* Show that $\;_{C} \frac{e^{xz} \quad x^n}{n! z^{n \;\; 1}} dz \quad 2 \;\; i \;\; \frac{x^n}{!n}^{2}$. 5

(b) Using Cauchy integral formula, evaluate $\circ_{C} \frac{dz}{z(z^2 \quad 4)}$, where C is curve

$|z| \quad 1.$ 5

UNIT—V

9. *(a)* State and prove Taylor's theorem for complex function. 5

(b) Expand $\dfrac{z \quad 3}{z(z^2 \quad z \quad 2)}$ for the region $|z| \quad 2$. 5

10. *(a)* Show that $\exp \dfrac{c}{2}(z \quad z^{\quad 1})$ can be expanded in the form $\quad a_n z^n$ and find the value of a_n. 5

(b) Examine the natures of singularities of the following functions : 5

(i) $\sin \dfrac{1}{1 \quad z}$ at $z \quad 1$

(ii) $\dfrac{1}{\sin z \quad \cos z}$ at $z \quad \dfrac{}{4}$

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