MATH/V/06 **Student's Copy**

2 0 1 8

(Pre-CBCS)

(5th Semester)

MATHEMATICS

SIXTH PAPER (MATH–352)

(Real Analysis)

Full Marks : 75

Time : 3 hours

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Put a Tick ☑ mark against the correct answer in the box provided : 1×10=10

1. If every open cover of a set admits of a finite subcover, it is said to have the

(a) Cantor's intersection property ☐

(b) Heine-Borel property ☐

(c) Lindelöf covering property ☐

(d) None of the above ☐

2. A set is said to be compact, if it is both

(a) bounded and closed ☐

(b) bounded and open ☐

(c) open and closed ☐

(d) None of the above ☐

3. The range of a function continuous on a compact set is

(a) cover ☐

(b) subcover ☐

(c) compact ☐

(d) None of the above ☐

4. A function $f(x, y)$ is said to be continuous, if it is continuous at

(a) isolated point only ☐

(b) each point of its domain ☐

(c) some deleted neighbourhood of domain ☐

(d) None of the above ☐

5. If

$$\lim_{h \quad 0} \frac{f(x \quad h, y) \quad f(x, y)}{h}$$

exists, it is called the partial derivative of f with respect to

(a) x at (a, b) ☐

(b) y at (a, b) ☐

(c) x at (x, y) ☐

(d) y at (x, y) ☐

6. If $u \quad f(x, y)$ and $v \quad g(x, y)$ have continuous partial derivatives in a region R, a necessary and sufficient condition that they satisfy $F(u, v) \quad 0$ is that the Jacobian

(a) $\frac{(u, v)}{(x, y)} \quad F(u, v)$ ☐

(b) $\frac{(u, v)}{(x, y)} \quad 0$ ☐

(c) $\frac{(x, y)}{(u, v)} \quad 0$ ☐

(d) $\frac{(u, v)}{(x, y)} \quad 0$ ☐

7. In R^2, if (a, b) be a point in the domain of a function f such that f_x and f_y are both differentiable at (a, b), then

(a) $f_{xy}(a, b) \quad f_{yx}(a, b)$ ☐

(b) $f_{xy}(a, b) \quad f_{yx}(a, b)$ ☐

(c) f_{xy} and f_{yx} do not exist at (a, b) ☐

(d) None of the above ☐

8. When AC B^2 0, where A $f_{xx}(a, b)$, B $f_{xy}(a, b)$, C $f_{yy}(a, b)$, then f is

(a) minimum at (a, b) ☐

(b) maximum or minimum at (a, b) ☐

(c) maximum at (a, b) ☐

(d) neither maximum nor minimum ☐

9. Every compact subset of a metric space (x, d) is

(a) closed ☐

(b) open ☐

(c) bounded ☐

(d) both closed and bounded ☐

10. If every Cauchy sequence of X converges to a point of X, then a metric space (X, d) is

(a) compact ☐

(b) interior ☐

(c) complete ☐

(d) closure ☐

SECTION—B

(*Marks* : 15)

Write short answers to the following questions : 3×5=15

1. Show that the union of an arbitrary family of open sets is open.

2. Define convex set and uniform continuity.

3. If $u_1 \quad \frac{x_2 x_3}{x_1}$, $u_2 \quad \frac{x_1 x_3}{x_2}$, $u_3 \quad \frac{x_1 x_2}{x_3}$, then prove that $J(u_1, u_2, u_3) \quad 4$.

4. Find the stationary points of the function $f(x, y) \quad x^3 \quad y^3 \quad 3x \quad 12y \quad 20$.

5. Prove that closed subsets of compact sets are compact.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **one** question from each Unit

UNIT—I

1. State and prove Cantor's intersection theorem. 2+8=10

2. State and prove Bolzano-Weierstrass theorem. 2+8=10

UNIT—II

3. *(a)* Show that a function continuous on a compact domain is uniformly continuous. 6

(b) Show that the function

$$f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2 + y^2}} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$$

is continuous at the origin. 4

4. *(a)* Let f be a real-valued continuous function with domain $D \subseteq R^n$. Let the domain D be such that $x \in D, y \in D \Rightarrow tx + (1-t)y \in D, t \in [0, 1]$. Show that the function f assumes every value between $f(x)$ and $f(y)$. 6

(b) Examine the continuity of the function $f(x, y) = \sqrt{|xy|}$ at the origin. 4

UNIT—III

5. *(a)* Prove that if a function which is differentiable at a point is also continuous at the point. 5

(b) Given

$$f(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2} & , \text{ if } (x, y) \neq (0, 0) \\ 0 & , \text{ if } x = y = 0 \end{cases}$$

Show that both the partial derivatives exist at (0, 0) but the function is not continuous at (0, 0). 5

6. *(a)* If

$$u \quad \frac{x \quad y}{1 \quad xy}, \; v \quad \frac{(x \quad y)(1 \quad xy)}{(1 \quad x^2)(1 \quad y^2)}$$

then find $\frac{(u, v)}{(x, y)}$. Are they functionally related? If so, find the relationship. 6

(b) If $x \quad r\sin \quad \cos \;, y \quad r\sin \quad \sin \;, z \quad r\cos \;$, then show that

$$\frac{(x, y, z)}{(r, \;, \;)} \quad r^2 \sin$$ 4

UNIT—IV

7. State and prove Taylor's theorem. 2+8=10

8. *(a)* Let

$$f(x, y) \quad \begin{matrix} \frac{xy(x^2 \quad y^2)}{x^2 \quad y^2} & , (x, y) \quad (0, 0) \\ 0 & , (x, y) \quad (0, 0) \end{matrix}$$

Show that $f_{xy} \quad f_{yx}$ at the origin. 5

(b) If $u \quad \frac{x \quad y}{1 \quad xy}, \; v \quad \frac{(x \quad y)(1 \quad xy)}{(1 \quad x^2)(1 \quad y^2)}$, then find $\frac{(u, v)}{(x, y)}$. 5

UNIT—V

9. Define compact metric space. Prove that every compact subset of a metric space is closed. 3+7=10

10. Define complete metric space. Let X be the set of all continuous real-valued functions define on [0, 1] and also let $d(x, y) \quad |x(t) \quad y(t)|\, dt, \; x, y \quad X$.

Show that (X, d) is not complete. 2+8=10

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