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(Pre-CBCS)

(5th Semester)

MATHEMATICS

FIFTH PAPER (MATH–351)

(Computer-oriented Numerical Analysis)

Full Marks : 75

Time : 3 hours

Simple calculator can be used in this paper

(PART : A—OBJECTIVE)

(*Marks* : 25)

The figures in the margin indicate full marks for the questions

SECTION—A

(*Marks* : 10)

Each question carries 1 mark

Put a Tick ☑ mark against the correct answer in the box provided :

1. Which of the following relations is true?

(a) E 1 ☐

(b) 1 E ☐

(c) E ☐

(d) None of the above ☐

2. The value of $^{2}y_0$ is

(a) $y_2 \quad y_1 \quad y_0$ ☐

(b) $y_2 \quad 2y_1 \quad y_0$ ☐

(c) $y_2 \quad 2y_1 \quad y_0$ ☐

(d) $y_2 \quad y_1 \quad y_0$ ☐

3. If $f(0) \quad 5$, $f(1) \quad 1$, $f(2) \quad 9$, $f(3) \quad 25$ and $f(4) \quad 55$, then $f(5)$ is

(a) 105 ☐

(b) 115 ☐

(c) 125 ☐

(d) None of the above ☐

4. If $u_1 \quad 1$, $u_3 \quad 17$, $u_4 \quad 43$ and $u_5 \quad 89$, then $u_2 \quad$?

(a) 10 ☐

(b) 15 ☐

(c) 6 ☐

(d) 5 ☐

5. The system of equations $a_1x \quad a_2y \quad a_3z \quad d_1$; $b_1x \quad b_2y \quad b_3z \quad d_2$; $c_1x \quad c_2y \quad c_3z \quad d_3$ is a diagonal system, if

(a) $|a_1| \quad |a_2| \; |a_3|$, $|b_1| \quad |b_2| \; |b_3|$, $|c_1| \quad |c_2| \; |c_3|$ ☐

(b) $|a_1| \quad |a_2| \; |a_3|$, $|b_2| \quad |b_1| \; |b_3|$, $|c_3| \quad |c_2| \; |c_1|$ ☐

(c) $|a_1| \quad |a_2| \; |a_3|$, $|b_2| \quad |b_1| \; |b_3|$, $|c_3| \quad |c_2| \; |c_1|$ ☐

(d) None of the above ☐

6. In Gauss elimination method for solving system of equation $AX \quad B$, the matrix A is reduced to

(a) upper triangular matrix ☐

(b) lower triangular matrix ☐

(c) diagonal matrix ☐

(d) None of the above ☐

7. In the general quadrature formula, Simpson's one-third rule is obtained by putting

(a) $n \quad 1$ ☐

(b) $n \quad 2$ ☐

(c) $n \quad 3$ ☐

(d) $n \quad 4$ ☐

8. The value of $\,_{0}^{\overline{\;4}} \frac{dx}{1 \quad x^2}$ is

(a) 0 ☐

(b) 1 ☐

(c) 2 ☐

(d) None of the above ☐

9. Euler's method is the Runge-Kutta method of

(a) first order ☐

(b) second order ☐

(c) third order ☐

(d) None of the above ☐

10. For solving ordinary differential equation numerically, the most reliable and most accurate among the following is

(a) Taylor's method ☐

(b) Picard's method ☐

(c) Euler's method ☐

(d) Runge-Kutta method ☐

SECTION—B

(*Marks* : 15)

Each question carries 3 marks

Answer **all** questions

1. Prove that E $E^{\frac{1}{2}}$, where , , and E are forward, backward, central and shift operators.

2. Show that the divided differences are independent of the order of arguments, i.e., (x_0, x_1) (x_1, x_0).

3. Solve the given equations by Gauss elimination method x y 2; $2x$ $3y$ 5.

4. Compute the value of $_{1}^{2}\frac{dx}{x}$ using Simpson's $\frac{1}{3}$rd rule.

5. Using Runge-Kutta method of second order, find $y(1\ 0)$ for the differential equation $\frac{dy}{dx}$ xy y^2, $t(0)$ 1.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

The figures in the margin indicate full marks for the questions

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Express $y \quad 2x^3 \quad 3x^2 \quad 3x \quad 10$ in factorial notation and hence show that $^3y \quad 12$. 5

(b) *(i)* Construct a divided difference table for the following data :

X	:	1	2	4	7	12
$F(x)$	:	22	30	82	106	216

(ii) Prove that

$$\frac{1}{2}(E^{\frac{1}{2}} \quad E^{\;\frac{1}{2}})$$

where is the average operator and E is shift operator. 5

2. *(a)* Find a root of the equation $x^3 \quad 4x \quad 9 \quad 0$, using the bisection method correct up to 3 decimal places. 5

(b) Write an algorithm for finding the root of an equation by regula falsi method. 5

UNIT—II

3. *(a)* Derive Newton's forward interpolation. 5

(b) The table gives the distance in nautical miles of the visible horizon for the given height in feet above the Earth's surface :

X *height*	:	100	150	200	250	300	350	400
Y *distance*	:	10·63	13·03	15·04	16·81	18·42	19·90	21·27

Find the values of Y, when *(i)* $X \quad 218$ ft and *(ii)* $X \quad 410$ ft. 5

4. *(a)* State and prove Newton's divided difference formula. 5

(b) Using Lagrange's interpolation formula for unequal interval, find the values of $f(2)$ and $f(15)$ from the following data : 5

X	:	4	5	7	10	11	13
$f(x)$	:	48	100	294	900	1210	2028

UNIT—III

5. *(a)* Apply Gauss elimination method to solve the equations

$$x \quad 4y \quad z \quad\quad 5;\ x \quad y \quad 6z \quad\quad 12;\ 3x \quad y \quad z \quad 4$$ 5

(b) By Crout's method, solve the system $2x \quad 3y \quad z \quad\quad 1;\ 5x \quad y \quad z \quad 9;\ 3x \quad 2y \quad 4z \quad 11.$ 5

6. *(a)* Apply Gauss-Jordan method to solve the equations $x \quad y \quad z \quad 9;\ 2x \quad 3y \quad 4z \quad 13;\ 3x \quad 4y \quad 5z \quad 40.$ 5

(b) Write an algorithm for Gauss-Seidel interactions method. 5

UNIT—IV

7. *(a)* Derive the formula for finding first- and second-order derivatives using Newton's forward difference formula. 5

(b) Given that

X	:	1·0	1·1	1·2	1·3	1·4	1·5	1·6
Y	:	7·989	8·403	8·781	9·129	9·451	9·750	10·031

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at *(i)* $x \quad 1\ 1$ and *(ii)* $x \quad 1\ 6$. 5

8. *(a)* Derive Newton-Cotes quadrature formula. 5

(b) Use Simpson's $\frac{1}{3}$rd rule to find $^{0\ 6}_{0}\ e^{\ \ x^2}\, dx$ by taking seven ordinates. 5

UNIT—V

9. *(a)* Using Picard's process of successive approximations, obtain a solution up to the fifth approximation of the equation $\frac{dy}{dx} \quad y \quad x$ such that $y \quad 1$, when $x \quad 0$. Check your answer by finding the exact practical solution. 5

(b) Find the values of y at $x \quad 0\ 1$ and $x \quad 0\ 2$ to five decimal places from $\frac{dy}{dx} \quad x^2 y \quad 1$, $y(0) \quad 1$ by Taylor's series method. 5

10. *(a)* Apply Runge-Kutta method to find approximate value of y for $x \quad 0\ 2$, in steps of 0·1, if $\frac{dy}{dx} \quad x \quad y^2$, given that $y \quad 1$ where $x \quad 0$. 5

(b) Using Milne's predictor-corrector method, find $y(4\ 4)$ given $5xy \quad y^2 \quad 2 \quad 0$, given that $y(4) \quad 1$, $y(4\ 1) \quad 1\ 0049$, $y(4\ 2) \quad 1\ 0097$ and $y(4\ 3) \quad 1\ 0143$. 5

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