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(CBCS)

(6th Semester)

PHYSICS

FOURTEENTH PAPER

(Quantum Mechanics)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Put a Tick (✓) mark against the correct answer in the brackets provided : 1×10=10

1. Ritz combination principle describes

(a) the relationship between atomic spectral lines ( )

(b) the stability of an atom ( )

(c) quantization of energy ( )

(d) the lifetime of excited states of atom ( )

2. As temperature of a blackbody is increased, the peak in the blackbody spectrum

(a) remains the same for all temperatures ( )

(b) shifts to higher frequency ( )

(c) shifts to lower frequency ( )

(d) does not depend on temperature, but depends on the material of the body only ( )

3. Suppose an incident wave of energy E approaches a potential step of height V_0, then if $E > V_0$, then the transmission coefficient of the incident wave is

(a) $\frac{4k_1k_2}{(k_1+k_2)^2}$ ( )

(b) $\frac{4k_1k_2}{(k_1-k_2)^2}$ ( )

(c) $\frac{(k_1-k_2)^2}{(k_1+k_2)^2}$ ( )

(d) $\frac{(k_1+k_2)^2}{(k_1-k_2)^2}$ ( )

4. In coordinate space, the expression for momentum operator corresponding to classical momentum in the x-direction for a single particle is

(a) $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$ ( )

(b) $\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$ ( )

(c) $-i\hbar\frac{\partial}{\partial x}$ ( )

(d) $i\hbar\frac{\partial}{\partial x}$ ( )

5. An eigenfunction of the operator $\hat{A} \equiv \frac{d^2}{dx^2}$ is $\psi(x) = \sin 2x$. The corresponding eigenvalue is

(a) 2 ( )

(b) −2 ( )

(c) 4 ( )

(d) −4 ( )

6. The wave function for a hydrogen atom whose quantum numbers are $n = 2$, $l = 1$, $m = \pm 1$ is

(a) $\psi_{21, \pm 1} = R_{22} Y_1^{\pm 1}$ ( )

(b) $\psi_{21, \pm 1} = R_{22} Y_{\pm 1}^{1}$ ( )

(c) $\psi_{21, \pm 1} = R_{21} Y_1^{\pm 1}$ ( )

(d) $\psi_{21, \pm 1} = R_{21} Y_{\pm 1}^{1}$ ( )

7. How many components of $\vec{L}$, the angular momentum vector can be quantized (having definite values) in space such that the uncertainty principle is not violated?

(a) One ( )

(b) Two ( )

(c) Three ( )

(d) None of the above ( )

8. The commutation relationship between the raising operator and the L_z component of angular momentum is

(a) $[L_+, L_z] = +\hbar L_+$ ( )

(b) $[L_+, L_z] = -\hbar L_+$ ( )

(c) $[L_+, L_z] = +\hbar L_-$ ( )

(d) $[L_+, L_z] = -\hbar L_-$ ( )

9. Which of the following statements is false?

(a) The ket vector can be represented by column vector. ( )

(b) The bra vector can be represented by column vector. ( )

(c) Multiplication of ket vector by bra vector from left gives inner product. ( )

(d) The ket vector and bra vector are complex conjugates of each other. ( )

10. A set of n vectors $|1\rangle, |2\rangle, \ldots, |n\rangle$ is said to constitute an orthonormal set if

(a) the vectors are orthogonal to one another and if each one of them is normalized ( )

(b) the vectors are orthogonal to one another and not all of them is necessarily normalized ( )

(c) the vectors are each normalized but not necessarily orthogonal to every one of the other ( )

(d) the vectors need not necessarily be orthogonal to one another nor normalized ( )

(SECTION : B—SHORT ANSWER)

(*Marks* : 15)

Answer the following questions : 3×5=15

UNIT—I

1. State the laws of photoelectric emission.

OR

2. What is a wave function? What are the important conditions to be satisfied by a well-behaved wave function such that it represents a physically observable system?

UNIT—II

3. What is the momentum eigenvalue for a particle trapped in a cubical box of side a in the ground state (1, 1, 1)?

OR

4. Obtain the time-independent Schrödinger equation for a particle traveling along the x-direction. Write down the same expression in three dimensions.

UNIT—III

5. The wave function for a harmonic oscillator is

$$\psi_n(y) = \frac{1}{\sqrt{2^n n!}}\left(\frac{1}{\pi b^2}\right)^{1/4} H_n(y)e^{-y^2/2}$$

where $H_0(y) = 1$, $H_1(y) = 2y$ and $y = bx$, $b = (m\omega / \hbar)^{1/2}$. Find the expectation value $< x >$ for the first excited state of the harmonic oscillator.

OR

6. Show that the product of two Hermitian operators is Hermitian if and only if they commute.

UNIT—IV

7. Find the eigenvectors of $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

OR

8. Show that the L_z component of the angular momentum commutes with the kinetic energy operator.

UNIT—V

9. Show that the two basis vectors $v_1 = (1, 1)$ and $v_2 = (1, -1)$ are linearly independent.

OR

10. Check whether the three vectors $a = (1, 2, 3)$, $b = (3, 4, 4)$ and $c = (7, 10, 12)$ in R^3-space are linearly independent.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

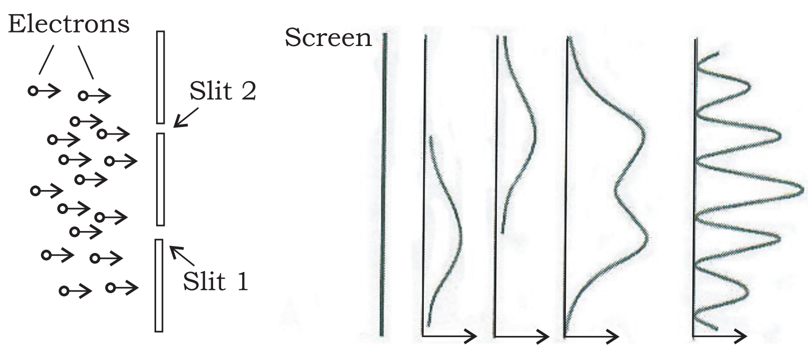
UNIT—I

1. Derive the expression for the change in wavelength for Compton effect. What is the Compton wavelength of an electron? Show that the maximum value of shift in wavelength is equal to twice the Compton wavelength of an electron. 8+1+1=10

OR

2. *(a)* Give the definitions for wave packet, group velocity and phase velocity. Show that the de Broglie wave group associated with a moving particle travels with the same velocity as the particle. 1+1+1+2=5

(b) An experimental arrangement is shown in the following figure for a two-slit experiment with electrons :



Looking at the above figure, obtain the probability densities corresponding to when only slit 1 is open (P_1), only slit 2 is open (P_2) and both the slits are open at the same time (P_{12}). Check if the probability density P_{12} is equal to the sum of P_1 and P_2. 5

UNIT—II

3. Consider a particle trapped inside a box of length L. Write down the expressions for the normalized wave function ψ_n and energy E_n of the particle. Also find the most probable locations of the particle corresponding to the states $n=2$ and $n=3$. 1+1+4+4=10

OR

4. *(a)* Find the expectation value $< p >$ of momentum for the particle in a box of length L. 2

(b) Find the probability that a particle trapped in a box L wide can be found between $0 \cdot 45L$ and $0 \cdot 55L$ for the ground $(n = 1)$, first $(n = 2)$ and second $(n = 3)$ excited states. 8

UNIT—III

5. For a particle in a cubical box of length L, obtain the wave function of the particle. Find the normalization constant and write down the normalized wave function. 7+3=10

OR

6. Write the expression for the time-independent Schrödinger equation for hydrogen atom. Separate this equation into three one-dimensional equations, i.e., the azimuthal (ϕ), radial (r) and polar (θ) equations and find the solution for the azimuthal equation. 1+6+3=10

UNIT—IV

7. *(a)* Obtain the commutation relation for angular momentum operator L^2 and any one of its components. 1

(b) Find the x, y and z-components of the angular momentum in spherical coordinates. 3+3+3=9

OR

8. *(a)* The operators S^2 and S_z of spin angular momentum commute and have simultaneous eigenstates which take the form $X_{s,\, m_s}$ where s is the spin quantum number and m_s is the spin magnetic quantum number. Find the eigenvalue equations for both S^2 and S_z for spin up and spin down. 3

(b) Suppose the spin eigenstates $X_{s,\, m_s}$ or $X_{\pm}$ are given by $X_{+} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $X_{-} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. Check whether these eigenfunctions *(i)* are orthogonal and *(ii)* can be normalized to unity. 2

(c) Prove that when the x-, y- and z-components of the Pauli spin matrices operate on the eigenfunction given in **8***(b)*, the eigenvalue equations are $\sigma_a X_{\pm} = \pm X_{\pm}$, where $a = x, y, z$. 5

UNIT—V

9. *(a)* State the theorem of Gram-Schmidt orthonormalization. 1

(b) Narrate the Gram-Schmidt procedures to get an orthonormal basis. 3

(c) Use the above procedures to transform the basis vectors $u_1 = (1, 1, 0)$, $u_2 = (1, -1, 0)$ and $u_3 = (0, 1, 2)$ into an orthonormal basis (v_1, v_2, v_3). 6

OR

10. *(a)* State the properties of inner product between two vectors $|\psi_m\rangle$ and $|\psi_n\rangle$. 2

(b) Prove that the above two vectors are said to be orthogonal if their inner product vanishes. 4

(c) If $|\psi_m\rangle = 3i|u_1\rangle + 4|u_2\rangle - 2|u_3\rangle$ and $|\psi_n\rangle = |u_1\rangle + 2i|u_2\rangle - 4|u_3\rangle$, find $|\psi_m\rangle^*$, $|\psi_n\rangle^*$, $\langle\psi_m|\psi_n\rangle$ and $\langle\psi_n|\psi_m\rangle$. 4

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