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(CBCS)

(6th Semester)

MATHEMATICS

TWELFTH (B) PAPER

(Elementary Number Theory)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. The greatest common divisor of 42823 and 6409 is

(*a*) 9 ( )

(*b*) 13 ( )

(*c*) 17 ( )

(*d*) 19 ( )

2. The least common multiple of 1687 and 482 is

(*a*) 1687 ( )

(*b*) 3374 ( )

(*c*) 2374 ( )

(*d*) 6748 ( )

3. A solution of the linear congruence $7x \quad 5 \pmod 8$ is

(a) 2 ( )

(b) 3 ( )

(c) 4 ( )

(d) 5 ( )

4. Which of the following is complete residue systems modulo 11?

(a) 0, 2, 12, 3, 15, 19, 21, 9, 7, 16, 17 ( )

(b) 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22 ( )

(c) 4, 3, 2, 1, 0, 1, 2, 3, 4, 5, 6 ( )

(d) 0, 1, 2, 3, 4, 5, 14, 15, 16, 17, 18 ( )

5. The remainder, if 8^{103} is divided by 103, is

(a) 0 ( )

(b) 1 ( )

(c) 103 ( )

(d) 8 ( )

6. Which of the following is true?

(a) For any integer $n \quad 2$, Euler's function (n) is even ( )

(b) For any integer $n \quad 2$, Euler's function (n) is odd ( )

(c) For any integer $n \quad 2$, Euler's function (n) is rational number ( )

(d) For any integer $n \quad 2$, Euler's function (n) is zero ( )

7. The congruence $x^2 \quad x \quad 4 \quad 0 \pmod 5$ has

(a) no solution ( )

(b) one solution ( )

(c) two solutions ( )

(d) four solutions ( )

8. The legendre symbol of $\left(\frac{29}{53}\right)$ is

(a) 1 ( )

(b) 1 ( )

(c) 0 ( )

(d) 2 ( )

9. The largest exponent of 5 which divides 764! is

(a) 140 ( )

(b) 155 ( )

(c) 176 ( )

(d) 189 ( )

10. The value of (20) is

(a) 480 ( )

(b) 546 ( )

(c) 594 ( )

(d) 600 ( )

(SECTION : B—SHORT ANSWER)

(*Marks* : 15)

Answer the following questions (any *three*) : 5×3=15

UNIT—I

1. Prove that if $a|b$ and $b|c$, then $a|c$.

OR

2. Show that $n(n \; 1)(2n \; 1)$ is a multiple of 6 for every natural number n.

UNIT—II

3. If $ca \quad cb$ (mod m) and $(c, m) \quad 1$, then prove that $a \quad b$ (mod m).

OR

4. Define complete residue system with example.

UNIT—III

5. If $n \quad 1$, then show that the sum of all positive integers which are less than n and prime to n is $\frac{1}{2}n \quad (n)$.

OR

6. Find the order of the integers 3 modulo 17.

UNIT—IV

7. Let p be an odd prime. Prove that

$$\frac{ab}{p} \qquad \frac{a}{p} \quad \frac{b}{p}$$

OR

8. If Q be positive and odd, then prove that

$$\frac{1}{Q} \qquad (\quad 1)^{\frac{Q \quad 1}{2}}$$

UNIT—V

9. If $n \quad 7056$, then find $\quad (n)$ and $\quad (n)$.

OR

10. Find the three different Pythagorean triples, not necessarily primitive of the form $16, y, z$.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer **one** question from each Unit

UNIT—I

1. *(a)* State and prove division algorithm. 1+5=6

(b) Find the number of distinct positive integral divisors and their sum for the integers 56700. 4

2. *(a)* Let a and b be integers, not both zero and let d be a positive integer. Prove that $d \quad \gcd(a, b)$ iff d satisfies 6

(i) $d|a$ and $d|b$;

(ii) if $c|a$ and $c|b$, then $c|d$.

(b) For positive integers a and b, prove that $\gcd(a, b)\ \mathrm{lcm}\ (a, b) \quad ab$. 4

UNIT—II

3. *(a)* Prove that the congruence $ax \quad b (\mathrm{mod}\ m)$ has a solution if and only if the greatest common divisor of a and m divides b. 6

(b) Prove that the number of positive primes is infinite. 4

4. *(a)* Let m be a fixed positive integer and $S \quad (0, 1, 2, 3, \cdots, m \quad 1)$. Prove that no two integers of S are congruent modulo m to each other and every $x \quad Z$ is congruent modulo to one of the integers of S. 6

(b) Solve the congruence :

$$13x \quad 9\ (\mathrm{mod}\ 25)$$ 4

UNIT—III

5. *(a)* State and prove Fermat's theorem. 1+5=6

(b) Find two primitive roots of 10. 4

6. *(a)* State and prove Wilson's theorem. 1+5=6

(b) Show that 16! + 86 is divisible by 323 by using Wilson's theorem. 4

UNIT—IV

7. *(a)* Let p be an odd prime and gcd(a, p) 1. Prove that a is a quadratic residue of p if and only if $a^{\frac{p\ \ 1}{2}}$ 1 (mod p). 6

(b) Solve $3x^2 \quad 9x \quad 7 \quad 0$ (mod 13) 4

8. *(a)* State and prove Chinese remainder theorem. 1+5=6

(b) Find all solutions of $f(x) \quad x^2 \quad x \quad 7 \quad 0$ (mod 3). 4

UNIT—V

9. *(a)* Let p be a prime and n, a positive integer. Prove that the exponent e such that

$$p^e | n! \text{ is } \quad _{k \ \ 1} \frac{n}{p^k}$$

6

(b) Prove that

$$_{d|n} \ (d) \quad \begin{matrix} 1 \text{ if } n \quad 1 \\ 0 \text{ if } n \quad 1 \end{matrix}$$

4

10. *(a)* Let a 0, b 0 and c be any three integers and d (a, b). Prove that the linear Diophantine equation $ax \quad by \quad c$ has a solution if and only if $d | c$. Further, if (x_0, y_0) is a particular solution of $ax \quad by \quad c$, then prove that any other solution of the equation is $x \quad x_0 \quad \frac{b}{d} t$, $y \quad y_0 \quad \frac{a}{d} t$, where t being any integer. 3+3=6

(b) If f is a multiplicative arithmetic function and F is defined by $F(n) \quad _{d|n} f(d)$. Prove that F is also multiplicative. 4

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