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(CBCS)

(6th Semester)

MATHEMATICS

ELEVENTH PAPER

(Mechanics)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. The least force *P* required to pull a body upon an inclined plane inclined at an angle to the horizontal is

(a) *P* *W* sin( ) ( )

(b) *P* *W* sin( ) ( )

(c) *P* *W* cos( ) ( )

(d) *P* *W* cos( ) ( )

2. If a body on a rough surface is in limiting equilibrium, then

(a) tan ( )

(b) tan ( )

(c) tan ( )

(d) tan ( )

3. The centre of gravity of a circular arc of radius 4 cm subtending at an angle 90° lies on the axis of symmetry at a distance of

(a) $\frac{8\sqrt{2}}{}$ from the centre ( )

(b) $\frac{4\sqrt{2}}{}$ from the centre ( )

(c) $\frac{2\sqrt{2}}{}$ from the centre ( )

(d) $\frac{6\sqrt{2}}{}$ from the centre ( )

4. The moment of inertia of a uniform solid sphere of radius a and mass m about a diameter is

(a) $\frac{1}{5}ma^2$ ( )

(b) $\frac{2}{5}ma^2$ ( )

(c) $\frac{2}{3}ma^2$ ( )

(d) $\frac{1}{24}ma^2$ ( )

5. If a particle moves along the x-axis under an attraction towards the origin O, varying inversely as the square of the distance from it, then the equation of motion is

(a) $\ddot{x} \quad x^2$ ( )

(b) $\ddot{x} \quad x^2$ ( )

(c) $\ddot{x} \quad \frac{}{x^2}$ ( )

(d) $\ddot{x} \quad \frac{}{x^2}$ ( )

6. The maximum displacement from the mean position of a particle executing simple harmonic motion is called

(a) frequency ( )

(b) trajectory ( )

(c) path ( )

(d) amplitude ( )

7. The path of a projectile is known as

(a) range ( )

(b) curvature ( )

(c) trajectory ( )

(d) course ( )

8. If a body is projected in a vertical plane with a velocity u and angle of projection , then the height of the directrix of the parabolic path above the plane of projection is

(a) $\frac{u}{2g}$ ( )

(b) $\frac{u^2}{2g}$ ( )

(c) $\frac{g}{2u}$ ( )

(d) $\frac{g^2}{2u}$ ( )

9. A smooth ball falling vertically from a height x impinges on a horizontal fixed plane. If e is the coefficient of restitution, then the ball rebounds to a height

(a) x ( )

(b) ex ( )

(c) e^2x ( )

(d) $\frac{e}{x^2}$ ( )

10. If e be the coefficient of restitution of collision of two perfectly elastic bodies, then

(a) e 1 ( )

(b) e $\frac{1}{2}$ ( )

(c) e 1 ( )

(d) e 0 ( )

(SECTION : B—SHORT ANSWER)

(*Marks* : 15)

Answer the following questions : 3×5=15

UNIT—I

1. Find the least force required to pull up a body of weight 100 kg on an inclined plane with inclination of 60° with the horizontal. Given that the angle of friction is 30°.

OR

2. Forces proportional to 1, 2, 3 and 4 act along the sides *AB*, *BC*, *AD* and *DC* respectively of a square *ABCD*, the length of each side is 2 feet. Find the magnitude and line of action of their resultant.

UNIT—II

3. Find the centre of gravity of the system of 3 particles of masses 1 kg, 3 kg and 4 kg placed at the points (0, 4, 4), (4, 0, 4) and (4, 4, 0) respectively.

OR

4. Prove that the centre of gravity of a triangular area coincides with that of three equal particles placed at the middle points of its sides.

UNIT—III

5. A particle moves in a plane with constant speed. Prove that its acceleration is perpendicular to its velocity.

OR

6. Find the total energy of a body of mass executing simple harmonic motion of period $\frac{2}{\ }$ and amplitude *a*.

UNIT—IV

7. Find the greatest height attained by the projectile thrown with a velocity *u* at an angle with the horizontal.

OR

8. Show that the least velocity with which the body can be projected to have a horizontal range is $\sqrt{gR}$ m/s.

UNIT—V

9. Prove that the rate of change of kinetic energy of a particle moving in a straight line is equal to its power.

OR

10. A sphere impinges directly on an equal sphere at rest. If the coefficient of restitution be e, then show that their velocities after impact are as (1 e) : (1 e).

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer **five** questions, selecting **one** from each Unit

UNIT—I

1. *(a)* Prove that a system of forces acting in one plane at different points of a rigid body can be reduced to a single force through a given point and a couple. 5

(b) A beam whose centre of gravity divides it into two portions a and b, is placed inside a smooth sphere. If be the inclination to the horizon in the position of equilibrium and 2 be the angle subtended by the beam at the centre of the sphere, then show that

$$\tan \quad \frac{b \quad a}{b \quad a} \tan$$ 5

2. *(a)* A uniform ladder rests in limiting equilibrium with its upper end against a smooth wall. If be the inclination of the ladder to the vertical, then prove that tan 2 , where is the coefficient of friction. 5

(b) Determine how high can a particle rest inside a rough hollow sphere of radius a, if the coefficient of friction is $\frac{1}{\sqrt{3}}$. 5

UNIT—II

3. *(a)* State and prove parallel axis theorem on moments of inertia. 5

(b) Find the centroid of the area formed by the curve $y \quad \sin x$ and $y \quad 0$, where $0 \quad x \quad$. 5

4. *(a)* Let AB and AC are two uniform rods of lengths $2a$ and $2b$ respectively. If $\quad BAC \quad$, then prove that the distance of the CG from A of the two rods is

$$\frac{(a^4 \quad 2a^2b^2 \cos \quad b^4)^{\frac{1}{2}}}{a \quad b}$$ 5

(b) A square hole is punched out of a circular lamina, the diagonal of the square being the radius of the circle. Show that the centre of gravity of the remainder is at a distance $\frac{a}{8 \quad 4}$ from the centre of the circle. 5

UNIT—III

5. *(a)* If the co-ordinate of a point moving with the constant acceleration be x_1, x_2, x_3 at instants t_1, t_2, t_3 respectively, then prove that the acceleration is

$$2 \frac{(x_2 \quad x_3)t_1 \quad (x_3 \quad x_1)t_2 \quad (x_1 \quad x_2)t_3}{(t_2 \quad t_3)(t_3 \quad t_1)(t_1 \quad t_2)}$$ 5

(b) Two cars start off to race with velocities u and v, and travel in a straight line with uniform accelerations and . If the two cars reached the finished line at the same time, then prove that the length of the course is

$$2 \frac{(u \quad v)(u \quad v \ )}{(a \quad)^2}$$ 5

6. *(a)* A particle is performing simple harmonic motion of period T about a centre O and it passes through a point P, where OP b, velocity u in the direction OP. Prove that the time which elapses before it returns to P is

$$\frac{T}{\ }\tan\ ^{1}\ \frac{uT}{2\ \ b}$$ 5

(b) A particle rests in equilibrium under the attraction of two centers of force which attracts directly as the distance, their intensities being , . The particle is displaced slightly towards one of them. Show that the time of small oscillation is $\frac{2}{\sqrt{\ \ }}$. 5

UNIT—IV

7. *(a)* For a given velocity of projection, the maximum range down an inclined plane is three times the range up the inclined plane. Show that the inclination of the plane to the horizontal is 30°. 5

(b) A particle is projected vertically upwards with a velocity v in a medium whose resistance is kv^2 per unit mass. Show that the greatest height attained by the particle is

$$\frac{1}{2k}\log\ 1\ \ \frac{kv^2}{g}$$ 5

8. *(a)* Particles are projected from the same point in a vertical plane with velocity $\sqrt{2gk}$. Prove that the locus of the vertices of their path is $x^2\ \ 4y(y\ \ k)\ \ 0$. 5

(b) A particle is projected from a point on the ground level and its height is h when it is at horizontal distance a and $2a$ from its point of projection. Prove that the velocity of projection u is given by

$$u^2\ \ \frac{g}{4}\ \frac{4a^2}{h}\ \ 9h$$ 5

Unit—V

9. *(a)* Deduce the work-energy equation. 5

(b) A bird of mass m is flying horizontally at a height h with velocity v when it is struck by a bullet of mass M moving vertically with a velocity V. If the bullet kills the bird and remains embedded in it, then prove that the bird will fall to the ground at a distance d from the point of projection of the bullet where

$$d \quad \frac{mv}{g(m \quad M)^2}[MV \quad \sqrt{M^2V^2 \quad 2gh(m \quad M)^2}$$

5

10. *(a)* A sphere impinges directly on an equal sphere which is at rest. Show that a fraction $\frac{1}{2}(1 \quad e^2)$ of the original kinetic energy is lost during the impact. 5

(b) Two spheres of masses m and M impinge directly when moving in opposite directions with velocities v and u respectively. If the sphere of mass m is brought to rest by collision, then show that $M(1 \quad e)u \quad v(m \quad eM)$. 5

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