Professional Course Examination, May 2023

(2nd Semester)

BACHELOR OF COMPUTER APPLICATIONS

(Discrete Mathematics)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(PART : A—OBJECTIVE)

(*Marks* : 25)

SECTION—I

(*Marks* : 15)

A. Tick (✓) the correct answer in the brackets provided : 1×10=10

1. Two sets A and B are said to be intersecting if

   (a) $A \cap B = \phi$ ( )

   (b) $A \cap B \neq \phi$ ( )

   (c) $A \cup B = \phi$ ( )

   (d) $A \cup B \neq \phi$ ( )

2. If A and B are two sets such that $n(A) = 24$, $n(B) = 22$ and $n(A \cap B) = 8$, then $n(B - A)$ is

(a) 10 ( )

(b) 12 ( )

(c) 14 ( )

(d) 16 ( )

3. In the conditional statement, $P \rightarrow Q$, the statement P is called

(a) hypothesis ( )

(b) conclusion ( )

(c) antecedent ( )

(d) consequent ( )

4. Which of the following is ***not*** a statement?

(a) Close the door ( )

(b) The earth is round ( )

(c) India is a country ( )

(d) $4 + 7 > 15$ ( )

5. $0! = ?$

(a) 0 ( )

(b) -1 ( )

(c) 1 ( )

(d) None of the above ( )

6. How many permutations of the letters of the word 'APPLE' are there?

(a) 30 ( )

(b) 40 ( )

(c) 50 ( )

(d) 60 ( )

7. The value of x satisfying $3x \equiv 2 \pmod 5$ is

(a) 2 ( )

(b) 1 ( )

(c) 4 ( )

(d) 3 ( )

8. The linear combination of $\gcd(252, 198) = 18$ is

(a) $252 * 4 - 198 * 5$ ( )

(b) $252 * 5 - 198 * 4$ ( )

(c) $252 * 5 - 198 * 2$ ( )

(d) $252 * 4 - 198 * 4$ ( )

9. A graph is a set of points, called

(a) lines ( )

(b) fields ( )

(c) edge ( )

(d) nodes ( )

10. Self loops are counted

(a) once ( )

(b) twice ( )

(c) thrice ( )

(d) multiple times ( )

B. Indicate whether the following statements are *True (T)* or *False (F)* by putting a Tick (✓) mark in the brackets provided : 1×5=5

1. In a Boolean algebra B, for all x, y in B, $x \cdot (x + y)$ is equal to x.

(T / F)

2. The proposition $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$ is a tautology.

(T / F)

3. If ${}^{n}P_{r} = 720$, ${}^{n}C_{r} = 120$, then r is equal to 3.

(T / F)

4. The inverse of 3 modulo 7 is –2.

(T / F)

5. A graph G is bipartite, then the chromatic number of G is 3.

(T / F)

SECTION—II

(*Marks* : 10)

C. Answer the following questions : 2×5=10

1. *(a)* A survey shows that 73% of the Indians like apples, whereas 65% like oranges. What percentage of Indians like both apples and oranges?

OR

(b) In a Boolean algebra B, show that $a + 1 = 1$.

2. *(a)* Write the truth table of $\sim (\sim p \wedge q)$.

OR

(b) Write a truth table of $p \rightarrow (q \wedge \sim p)$.

3. *(a)* Expand $(3x + 2y)^4$ by binomial theorem.

OR

(b) If ${}^{n}C_7 = {}^{n}C_5$, find n.

4. *(a)* Suppose a and c are relatively prime integers and b is an integer such that $b \mid c$. Prove that $\gcd(a, b) = 1$.

OR

(b) Solve : $6x \equiv 4 \pmod{10}$.

5. *(a)* What are multigraph and weighted graph?

OR

(b) Define a planar and non-planar graph.

(PART : B—DESCRIPTIVE)

(*Marks* : 50)

D. Answer the following questions : 10×5=50

1. *(a)* A class has 175 students. The following description gives the number of students studying one or more of the subjects in this class :

   Mathematics 100, Physics 70, Chemistry 46, Mathematics and Physics 30, Mathematics and Chemistry 28, Physics and Chemistry 23, Mathematics, Physics and Chemistry 18.

   Find *(i)* how many students are enrolled in Mathematics alone, Physics alone and Chemistry alone and *(ii)* the number of students who have not offered any of these subjects. 3+3=6

   (b) If $A = \{x : x \in N,\ x \leq 7\}$, $B = \{x : x$ is a prime, $x < 8\}$, and $C = \{x : x \in N,\ x$ is odd and $x < 10\}$, verify that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. 4

OR

2. *(a)* In a Boolean algebra $(B, +, \cdot, ')$, prove that

   $$(a + b)' = a' \cdot b' \text{ and } (a \cdot b)' = a' + b'$$ 5

   (b) Construct the switching table for the switching function f represented by the Boolean expression $xyz + x'(y + z)$. 5

3. *(a)* Construct a truth table for the statement formula

   $$(p \wedge \sim q) \vee (q \wedge (\sim p \vee r))$$ 5

   (b) Verify by truth table, $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$. 5

OR

4. *(a)* Prove that $p \rightarrow q,\ \sim p \rightarrow q \models q$. 5

   (b) Construct a truth table for the statement : 5

   $$(p \rightarrow (q \rightarrow r)) \rightarrow ((p \wedge q) \rightarrow r)$$

5. *(a)* Find the two middle terms in the expansion of

$$\left(3x - \frac{2}{x^2}\right)^{15}$$ 5

(b) Find the 10th term in the expansion of

$$\left(\frac{a}{b} - \frac{2b}{a^2}\right)^{12}$$ 5

OR

6. *(a)* Find the 5th term from the end in the expansion of

$$\left(x - \frac{1}{x}\right)^{12}$$ 5

(b) If ${}^{20}C_r = {}^{20}C_{r+6}$, find r. 5

7. *(a)* Find (26, 118) and express it in the form of $26x + 118y$, where $x, y \in Z$. 5

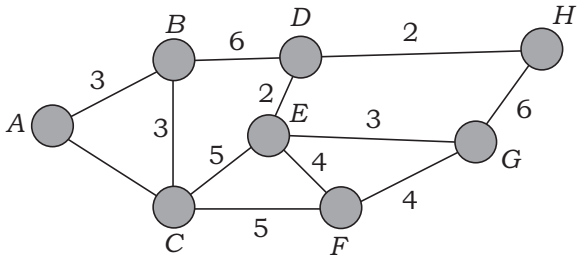
(b) Prove that the relation 'congruence modulo m' is an equivalence relation in the set of integers. 5

OR

8. *(a)* State and prove Fermat's theorem. 6

(b) Write the statement of Euler's Lemma. Compute gcd (803, 154). 4

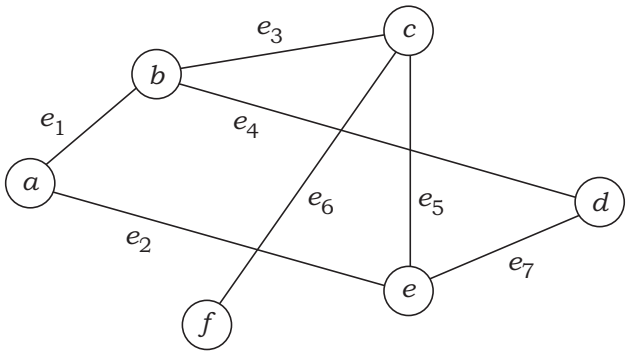
9. *(a)* Using Kruskal's algorithm, find the minimum spanning tree for the weighted graph of the figure : 6



(b) Prove that a tree with n vertices has $(n-1)$ edges. 4

OR

10. *(a)* Write the adjacency matrix and incidence matrix for the following graph : 6



(b) Define tree. Prove that a tree T is always separable. 4

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