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(CBCS)

(6th Semester)

PHYSICS

NINTH PAPER

(Quantum Mechanics)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. The de Broglie wavelength of a body of mass m and kinetic energy E is

(a) $\frac{2mh}{\sqrt{E}}$ ( )

(b) $\frac{h}{\sqrt{2mE}}$ ( )

(c) $\frac{h}{2mp}$ ( )

(d) $\frac{\sqrt{2mE}}{E}$ ( )

2. Quantum nature of light emerged in an attempt to explain

(a) radioactivity ( ) *(b)* interference of light ( )

(c) black-body radiation ( ) *(d)* pair production ( )

3. Which of the following wave functions is well-behaved?

(a) $(x) \quad Ae^{x}$ ( )

(b) $(x) \quad Ae^{\quad x}$ ( )

(c) $(x) \quad Ae^{x^2}$ ( )

(d) $(x) \quad Ae^{\quad x^2}$ ( )

where A is a constant and x .

4. For a free particle in step potential, let R and T be reflectance and transmittance, then

(a) $R \quad T \quad 1$ ( ) *(b)* $R \quad T$ ( )

(c) $R \quad T \quad 1$ ( ) *(d)* $RT \quad 1$ ( )

5. Eigenvalues of Hermitian operators

(a) are real only ( )

(b) are imaginary only ( )

(c) can be real or imaginary ( )

(d) are always complex ( )

6. The energy level of a one-dimensional harmonic oscillator according to Schrödinger equation is

(a) $n\hbar$ ( )

(b) $n \quad \frac{1}{2} \quad \hbar$ ( )

(c) $\frac{\hbar}{n \quad \frac{1}{2}}$ ( )

(d) $(n^2 \quad 1)\hbar$ ( )

7. Orbital magnetic moment of an electron is given (where L is angular momentum and m is mass of the electron) by

(a) $_L \quad \frac{eL}{2m}$ ( )

(b) $_L \quad \frac{ne\hbar}{2m}$ ( )

(c) $_L \quad \frac{neL}{2m}$ ( )

(d) Both *(a)* and *(b)* ( )

8. The trace of Pauli spin matrices is

(a) 1 each ( ) *(b)* i each ( )

(c) 1 each ( ) *(d)* 0 each ( )

9. If inner product between two vectors is zero, then the two vectors are

(a) orthogonal to each other ( )

(b) parallel to each other ( )

(c) Can be both *(a)* and *(b)* ( )

(d) opposite to each other ( )

10. Given the matrices

$$M_1 \quad \begin{matrix}1 & 0\\ 0 & 0\end{matrix}, \; M_2 \quad \begin{matrix}0 & 1\\ 0 & 0\end{matrix}, \; M_3 \quad \begin{matrix}0 & 0\\ 1 & 0\end{matrix}, \; M_4 \quad \begin{matrix}0 & 0\\ 0 & 1\end{matrix}$$

(a) The matrices are equivalent to 4 4 matrix ( )

(b) Any 2 2 real matrix can be written as the linear combination of these matrices ( )

(c) The eigenvalues of the matrices are 1 ( )

(d) The matrices are unit matrix ( )

(SECTION : B—SHORT ANSWER)

(*Marks* : 15)

Answer the following questions : 3×5=15

UNIT—I

1. Show that a material particle cannot be equivalent to a single wave starting from de Broglie relation.

OR

2. Show that when frequency of incident light is doubled, the kinetic energy of ejected electron from metal surface increases more than two times.

UNIT—II

3. Normalized wave function of a free particle in a box is given by

$$\sqrt{\frac{2}{L}}\sin\frac{n\ \ x}{L}$$

where $0 \quad x \quad L$. Obtain the probability of finding the particle within $0 \quad x \quad \frac{2}{L}$.

OR

4. Write a short note on quantum tunneling effect.

UNIT—III

5. Show that momentum operator is Hermitian.

OR

6. A wave function is given by $\quad e^{\ \ x}$. Obtain the eigenvalue w.r.t. the operator $\frac{d^2}{dx^2}$. What is the physical meaning of eigenvalue?

UNIT—IV

7. Show that spin gyromagnetic ratio is equal to two times that of orbital gyromagnetic ratio.

OR

8. Show that eigenvalue of L_z is $m\hbar$ where m is magnetic quantum number.

UNIT—V

9. Show that the spatial vectors $e_1 \quad (1, 0)$ and $e_2 \quad (0, 1)$ in 2-dimension are linearly independent and they form the basis set for real vectors in 2-dimension.

OR

10. Show that the vectors $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ and $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$ are orthogonal as well as normalized.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer the following : 10×5=50

UNIT—I

1. *(a)* What do you mean by duality of radiation and matter? Show that the de Broglie wavelength for a material particle of rest mass m_0 and charge q, accelerated from rest through a potential difference of V volt relativistically is given by

$$\frac{h}{\sqrt{2m_0qV \quad 1 \quad \frac{qV}{2m_0c^2}}}$$

2+5=7

(b) In an atom, an electron is moving with a speed of 600 m/s with accuracy of 0·005%. Calculate the certainty with which the position of the electron can be located. 3

OR

2. What do you mean by Compton effect? Which nature of light is necessary to explain the effect? Show that Compton shift is given by

$$\frac{h}{m_0 c}(1 \quad \cos \quad)$$

where m_0 is rest mass of electron and   is scattering angle. 10

UNIT—II

3. *(a)* Obtain Schrödinger time dependent equation. Give the physical meaning of the equation. 6

(b) A wave function is given by Ae^{ikx}. Show that the probability current density of the given wave function is given by $J \quad v|A|^2$ where v is velocity of the particle. 4

OR

4. A free particle of energy E is incident on a potential step given by $V \quad 0$, $x \quad 0$ and $V \quad V_0$, $x \quad 0$. Show that all the waves are reflected when $E \quad V_0$. 10

UNIT—III

5. *(a)* What do you mean by Hermitian operator? Show that two eigenfunctions of the same Hermitian operator belonging to two distinct eigenvalues are orthogonal. 1+3=4

(b) Obtain normalized wave function for particle in a 3-dimensional box. Hence discuss the degeneracy. 6

OR

6. Obtain the expression for energy eigenvalue of one-dimensional harmonic oscillator. What is zero-point energy? 10

UNIT—IV

7. *(a)* Write down Pauli spin matrices and show that $[\;^{2},\;\;_{x}]\;\;0$. 1+4=5

(b) What do you mean by orbital gyromagnetic ratio for an electron? Obtain the expression for it. 1+4=5

OR

8. *(a)* Show that the square of angular momentum commutes with any one of the components of angular momentum, i.e., $[L^2, L_x]\;\;0$. What is the physical meaning of the commutation? 4+1=5

(b) Let $_x$, $_y$, $_z$ be Pauli spin matrices. Let $\vec{A}$ and $\vec{B}$ be two vectors. Show that $(\vec{}\;\;\vec{A})(\vec{}\;\;\vec{B})\;\;\vec{A}\;\;\vec{B}\;\;\;i\vec{}\;\;(\vec{A}\;\;\;\vec{B})$. 5

UNIT—V

9. *(a)* Write down the addition and multiplication conditions to be satisfied by a vector space. 2

(b) Describe Gram-Schmidt orthogonalization process. Apply the process to find an orthonormal basis set for the subspace U of R^4 spanned by the following vectors : 3+5=8

$$v_1\quad(1,1,1,1),\; v_2\quad(1,2,4,5),\; v_3\quad(1,\;\;3,\;\;4,\;\;2)$$

OR

10. *(a)* Show that the vectors $u\;\;(1,2,3)$, $v\;\;(2,5,7)$, $w\;\;(1,3,5)$ are linearly independent. 4

(b) Let $|\;\;\;\;2|u_1\;\;3|u_2\;\;\;i|u_3$ and $|\;\;\;\;3|u_1\;\;2|u_2\;\;\;4|u_3$ and a constant $a\;\;2\;\;3i$. Compute the inner product $\;\;|\;\;$ and $|a\;\;$. 3+3=6

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