2023

(CBCS)

(6th Semester)

MATHEMATICS

ELEVENTH PAPER

(Mechanics)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. The equation of the resultant of any number of coplanar forces acting on a rigid body is given by

(a) $G \quad yR_y \quad xR_x \quad 0$ ( )

(b) $G \quad yR_x \quad xR_y \quad 0$ ( )

(c) $xR_y \quad yR_x \quad G$ ( )

(d) $xR_x \quad yR_y \quad G$ ( )

2. The least force P required to pull a body up on an inclined plane at an angle to the horizontal is

(a) $P \quad W \sin(\quad)$ ( )

(b) $P \quad W \sin(\quad)$ ( )

(c) $P \quad W \cos(\quad)$ ( )

(d) $P \quad W \cos(\quad)$ ( )

3. The moment of inertia of a circular disc of radius r about an axis through its centre, perpendicular to its plane is

(a) Mr^2 ( )

(b) $\frac{1}{3} Mr^2$ ( )

(c) $\frac{1}{2} Mr^3$ ( )

(d) $\frac{1}{2} Mr^2$ ( )

4. The centre of gravity of three uniform rods forming a triangle ABC is

(a) the incentre of the triangle formed by joining the mid-points of the sides of the triangle ABC ( )

(b) the orthocentre of the triangle ABC ( )

(c) the centroid of the triangle ABC ( )

(d) the incentre of the triangle ABC ( )

5. The velocity of a particle executing SHM is _____ at its mean position.

(a) zero ( ) *(b)* minimum ( )

(c) maximum ( ) *(d)* infinity ( )

6. If the position of a moving particle at time t referred to rectangular axes is given by $a \quad b \quad ct^2$, where a, b and c are constants, then its acceleration at time t is

(a) along the x-axis ( )

(b) $a \quad b \quad c$ along the x-axis ( )

(c) $\sqrt{a^2 \quad b^2}$ along the y-axis ( )

(d) $2c$ along the y-axis ( )

7. If a particle is projected with a velocity of $\sqrt{7}$ m/s at an angle 60° with the horizontal. The average velocity of the projectile between the instants of point of projection and reaching the highest point is

(a) 2 m/s ( ) *(b)* 5/4 m/s ( )

(c) 7/4 m/s ( ) *(d)* 3/4 m/s ( )

8. If a body is projected with a velocity 9·8 m/s making an angle 45° with the horizontal, then the range of the projectile is (take $g \quad 9{\cdot}8$ m/s^2)

(a) 39·2 m ( ) *(b)* 9·8 m ( )

(c) 4·9 m ( ) *(d)* 19·6 m ( )

9. A perfectly elastic sphere impinges directly with velocity u on another equal perfectly elastic sphere at rest. The velocity of the first sphere after collision is

(a) u ( )
(b) $-u$ ( )
(c) 0 ( )
(d) None of the above ( )

10. A smooth sphere impinges directly with velocity u on another smooth sphere of equal mass at rest. If the spheres are perfectly elastic, the velocity of the second sphere after collision will be

(a) 0 ( )
(b) u ( )
(c) $\frac{u}{2}$ ( )
(d) None of the above ( )

(SECTION : B—SHORT ANSWER)

(*Marks* : 15)

Answer the following : 3×5=15

UNIT—I

1. Forces 13, 10, 5 lbs weight act along the sides *BC*, *CA* and *AB* of an equilateral triangle *ABC*. Find the direction and magnitude of their resultant and the point where the line of action meets *BC*.

OR

2. A uniform ladder rests in limiting equilibrium with its upper end against a smooth wall. If be the inclination of the ladder to the vertical, prove that tan 2 , where is the coefficient of friction.

UNIT—II

3. Prove that the centre of gravity of a triangular area coincides with that of three equal particles placed at its angular points.

OR

4. Find the centre of gravity of a uniform sector of a circle.

UNIT—III

5. If the angular velocity of a point moving in a plane curve be constant about a fixed origin, show that its transverse acceleration varies as its radial velocity.

OR

6. A particle rests in equilibrium under the attraction of two centre of forces which attract directly as the distance, their intensities being and . Show that the time of small oscillation is $\frac{2}{\quad}$.

UNIT—IV

7. If h and h be the greatest heights in the two paths of a projectile with a given velocity for a given range R, then show that $R \quad 4\sqrt{hh}$.

OR

8. If a particle is projected with velocity u from the ground at an angle with the horizontal, find the time to reach the greatest height.

UNIT—V

9. The earth's attraction on a particle varies inversely as the square of its distance from the earth's centre. A particle whose weight on the surface of the earth is W, falls to the surface of the earth from a height $5a$ above it. Show that the work done by the earth's attraction is $\frac{5aW}{6}$, where a is the radius of the earth.

OR

10. A smooth sphere impinges directly with another smooth sphere of equal mass. If the two spheres are perfectly elastic, then show that the velocity of the second sphere is same as that of the first sphere.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer the following : 10×5=50

UNIT—I

1. *(a)* If a system of coplanar forces reduces to a single couple whose moment is *G*, such that when each force is turned round its point of application through a right angle, it reduces to a couple *H*. Prove that when each force is turned through an angle , the system is equivalent to a couple whose moment is *G*cos *H* sin . 5

(b) A uniform ladder rests in limiting equilibrium with its upper end against a smooth vertical wall. If be the inclination of the ladder to the vertical, prove that tan 2 , where is the coefficient of friction. 5

OR

2. *(a)* A beam whose centre of gravity divides it into two portions *a* and *b*, is placed inside a smooth sphere. If be the inclination to the horizon in the position of equilibrium and 2 be the angle subtended by the beam at the centre of the sphere, then show that tan $\frac{b \quad a}{b \quad a}$ tan . 5

(b) How high can a particle rest inside a hollow sphere of radius *a*, if the coefficient of friction be $\frac{1}{\sqrt{3}}$? 5

UNIT—II

3. *(a)* Find the CG of the homogeneous area bounded by the parabola y^2 4*ax*, the *x*-axis and the ordinate *x* *h*. 5

(b) A square hole is punched out of a circular lamina, the diagonal of the square being the radius of the circle, *a*. Show that the centre of gravity of the remainder is at a distance $\frac{a}{8 \quad 4}$ from the centre of the circle. 5

OR

4. *(a)* State and prove the perpendicular axis theorem on moments of inertia. 5

(b) Three heavy particles are placed at angles A, B, C of a triangle, their weights being in the ratio $a : b : c$. Show that the distance of CG of a particle from the vertex A is $\frac{2bc\cos\frac{A}{2}}{a \quad b \quad c}$. 5

UNIT—III

5. *(a)* For a particle moving in a plane curve, show that the radial and transverse acceleration components are given by $f_r \quad \ddot{r} \quad \dot{r}^2$ and $f \quad r \quad 2\dot{r}$. 5

(b) A particle moves with SHM in a straight line. In the 1st second after starting from rest, it travels a distance a and in the next second, it travels a distance b in the same direction. Prove that the amplitude of the motion is $\frac{2a^2}{3a \quad b}$ and its period is

$$\frac{2}{\cos^{1}\frac{b \quad a}{2a}}$$

5

OR

6. *(a)* Prove that if the tangential and normal components of acceleration of a particle describing a plane curve be constant throughout the motion, the angle through the direction of motion turns in time t is given by

$$A\log(1 \quad Bt)$$

where A and B are constants. 5

(b) The greatest possible acceleration of a train is 1 m/s^2 and the greatest retardation is $\frac{4}{3}$ m/s^2. Find the least time taken to run between two stations 12 km apart, if the maximum speed is 22 m/s. 5

UNIT—IV

7. *(a)* Two bodies are projected from the same point in directions making angles $_1$, $_2$ with the horizontal and strike at the same point in the horizontal plane through the point of projection. If t_1, t_2 be their times of flight, prove that

$$\frac{t_1^2 \quad t_2^2}{t_1^2 \quad t_2^2} \quad \frac{\sin(\ _1 \quad \ _2)}{\sin(\ _1 \quad \ _2)}$$

5

(b) Particles are projected from the same point in a vertical plane with velocity $\sqrt{2gk}$. Prove that the locus of the vertices of their paths is the ellipse $x^2 \quad 4y(y \quad k) \quad 0$.

5

OR

8. *(a)* A particle is projected vertically upward with a velocity v in a medium whose resistance is kv^2 per unit mass. Show that the greatest height attained by the particle is

$$\frac{1}{2k}\log\ 1 \quad \frac{kv^2}{g}$$

5

(b) A cannon ball has a range R on the horizontal plane. If h and h are the greatest heights in the two paths for which this is possible, prove that $R \quad 4\sqrt{hh}$.

5

UNIT—V

9. *(a)* A shell of mass $(m_1 \quad m_2)$ moving with velocity v, breaks up into masses m_1 and m_2 which move in the same direction with relative velocity V. Show that the energy of the explosion is given by

$$E \quad \frac{1}{2}\frac{m_1 m_2}{m_1 \quad m_2}V^2$$

5

(b) A uniform elastic string has the length a_1 when the tension is T_1 and the length a_2 when the tension is T_2. Show that its natural length is

$$\frac{a_2T_1 \quad a_1T_2}{T_1 \quad T_2}$$

and the amount of work done in stretching it from its natural length to a length $(a_1 \quad a_2)$ is

$$\frac{1}{2}\frac{(a_1T_1 \quad a_2T_2)^2}{(T_1 \quad T_2)(a_1 \quad a_2)}$$ 5

OR

10. *(a)* A shot of mass m is projected from a gun of mass M by an explosion which generates a kinetic energy E. Show that the gun recoils with a velocity $\sqrt{\frac{2mE}{M(M \quad m)}}$ and the initial velocity of the shot is $\sqrt{\frac{2ME}{M(M \quad m)}}$. 5

(b) A smooth sphere impinges on an equal sphere at rest. Before impact, the first sphere was moving in a direction making an angle with the line of centres at the moment of impact. If the direction of motion of the first sphere is turned through an angle by the impact, show that

$$\tan \quad \frac{(1 \quad e)\tan}{1 \quad e \quad 2\tan^2}$$ 5

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