**PHY/II/EC/03** **Student's Copy**

**2 0 2 3**

( CBCS )

( 2nd Semester )

# PHYSICS

SECOND PAPER

**( Thermodynamics and Mathematical Physics—I )**

*Full Marks* : 75

*Time* : 3 hours

*The figures in the margin indicate full marks for the questions*

## ( Section : A—objective )

( *Marks* : 10 )

Tick (✓) the correct answer in the brackets provided : 1×10=10

**1.** The density of hydrogen at NTP is 0·000089 g/cc. Then r.m.s. velocity of hydrogen is

(*a*) 1 8 $10^5$ cm/s ( ) (*b*) 1 8 $10^3$ cm/s ( )

(*c*) 3 2 $10^5$ cm/s ( ) (*d*) 3 2 $10^3$ cm/s ( )

**2.** In an Ingen-Hauz experiment, wax melted over 10 cm of Cu rod and over 4 cm of Fe rod. If the thermal conductivity of Cu is 0·90, then the thermal conductivity of Fe is

(*a*) 0·36 ( ) (*b*) 2·25 ( )

(*c*) 5·625 ( ) (*d*) 0·144 ( )

**3.** Let $V_i$ and $V_f$ be the initial and final volume of one mole of a perfect gas. Then work done $W$ at constant temperature $T$ is

*(a)* $W \quad P(V_f \quad V_i)$ ( )

*(b)* $W \quad P(V_i \quad V_f)$ ( )

*(c)* $W \quad RT \log_e \frac{V_f}{V_i}$ ( )

*(d)* $W \quad RT \log_e \frac{V_i}{V_f}$ ( )

**4.** In a reversible isobaric-adiabatic process, which of the following remains constant?

*(a)* Gibbs function ( )

*(b)* Enthalpy ( )

*(c)* Helmholtz function ( )

*(d)* Internal energy ( )

**5.** For any vector $\vec{a}$, the value of $\hat{i} \quad (\vec{a} \quad \hat{i}) \quad \hat{j} \quad (\vec{a} \quad \hat{j}) \quad \hat{k} \quad (\vec{a} \quad \hat{k})$ is

*(a)* $\hat{a}$ ( )

*(b)* $\vec{a}$ ( )

*(c)* $2\vec{a}$ ( )

*(d)* 0 ( )

**6.** If $\vec{r} \quad x\hat{i} \quad y\hat{j} \quad z\hat{k}$, then $\vec{\ }(r^2)$ is equal to

*(a)* $\vec{r}$ ( )

*(b)* $2\vec{r}$ ( )

*(c)* $r^2$ ( )

*(d)* $2r^2$ ( )

**7.** Let $A \quad [a_{ij}]$ be an $m \quad n$ matrix. Then which of the following are true?

*(i)* $(A^\dagger)^\dagger \quad \overline{\{(\overline{A})^t\}^t}$

*(ii)* $(A^\dagger)^\dagger \quad \overline{(\overline{A})}$

*(iii)* $(A^\dagger)^\dagger \quad A$

*(a)* *(i)* and *(ii)* ( )

*(b)* *(ii)* and *(iii)* ( )

*(c)* *(i)* and *(iii)* ( )

*(d)* *(i)*, *(ii)* and *(iii)* ( )

**8.** Let $A$ be an orthogonal matrix, a characteristic root of $A$ and $X$ the corresponding characteristic vector. Then

*(a)* 1 ( ) *(b)* 1 ( )

*(c)* 1 ( ) *(d)* 0 ( )

**9.** The value of (2 $m$) (1 $m$) is

*(a)* $\frac{}{\sin m}$ ( ) *(b)* $\frac{m}{\sin m}$ ( )

*(c)* $\frac{(m \quad 1)}{\sin m}$ ( ) *(d)* $\frac{m\,(m \quad 1)}{\sin m}$ ( )

**10.** The integral $_0\; e^{\;x^2}\,dx$ is equal to

*(a)* $\sqrt{\;}$ ( ) *(b)* $\frac{\sqrt{\;}}{2}$ ( )

*(c)* ( ) *(d)* $\frac{}{2}$ ( )

## ( SECTION : B—SHORT ANSWER )

( *Marks* : 15 )

Answer the following in brief : 3×5=15

### UNIT—I

**1.** Define Boyle's temperature ($T_B$). Show that $T_B \;\; \frac{a}{Rb}$, where $a$, $b$ are van der Waals' constants.

**OR**

**2.** Define degrees of freedom. Show that for a gas possessing $f$ degrees of freedom, the ratio of specific heats ( ) is equal to $1 \;\; \frac{2}{f}$.

## UNIT—II

**3.** Define adiabatic process. Explain why the temperature of a gas drops during an adiabatic expansion.

**OR**

**4.** State Carnot's theorem. "Efficiency of a Carnot's engine is always less than one." Explain.

## UNIT—III

**5.** Show that $\vec{\ } \ \ (\ \ \vec{A}) \quad (\vec{\ } \ \ ) \ \vec{A} \qquad (\vec{\ } \ \ \vec{A})$, where   is a scalar and $\vec{A}$ is a vector field.

**OR**

**6.** Show that the vector $\vec{A} \quad z\sin \ \ \hat{e} \quad z\cos \ \ \hat{e} \qquad \cos \ \ \hat{z}$ is solenoidal.

## UNIT—IV

**7.** Show that the matrix $B^{\dagger}AB$ is Hermitian or skew-Hermitian according as $A$ is Hermitian or skew-Hermitian.

**OR**

**8.** Show that the necessary and sufficient condition for a square matrix to be invertible is that it is non-singular.

## UNIT—V

**9.** Show that $\ _0^1 \frac{x^{m\ \ 1}(1\quad x)^{n\ \ 1}}{(a\quad x)^{m\ \ n}}dx \quad \frac{(m,\ n)}{a^n(1\quad a)^m}$, where the symbols have their usual meanings.

**OR**

**10.** Show that $\ _0^1 x^{m\ \ 1}(1\quad x^a)^n\,dx \quad \frac{1}{a}\,\frac{\ \frac{m}{a}\quad n!}{\ \frac{m}{a}\quad n\quad 1}$, where the symbols have their usual meanings.

## ( SECTION : C—DESCRIPTIVE )

( *Marks* : 50 )

Answer the following questions : 10×5=50

### UNIT—I

**1.** *(a)* Derive the general equation representing the rectilinear flow of heat along a bar of uniform cross-sectional area. Hence, obtain an expression for excess temperature for a bar of infinite length at a point distant $x$ from the hot end after the steady state is reached. 4+3=7

*(b)* An icebox is built of wood of thickness 1·75 cm, lined inside with cork of thickness 3 cm. If the temperature of the inner surface of the cork is 0 °C and that of the outer surface of wood is 12 °C, calculate the temperature of the interface. (Given : Thermal conductivities of wood and cork are 0·0006 and 0·00012 cgs units) 3

**OR**

**2.** *(a)* Discuss kinetic interpretation of temperature. Hence show that the root-mean-square velocity of a gas is directly proportional to the root of its absolute temperature. 3+1=4

*(b)* The number of molecules per cc. of a gas is $2\ 7\ \ 10^{19}$ at NTP. Calculate the number of molecules per cc. of the gas at 0 °C and $10^{\ 6}$ mm pressure of Hg. 2

*(c)* The critical temperature and critical pressure of $CO_2$ are 31 °C and 73 atm respectively. Assuming that $CO_2$ obeys van der Waals' equation, compute the critical volume of $CO_2$ and estimate the diameter of $CO_2$ molecule. 4

### UNIT—II

**3.** *(a)* State the zeroth law of thermodynamics. Hence derive the equation of state. 1+3=4

*(b)* Using the first law of thermodynamics, derive the adiabatic equation of state for a perfect gas. 3

(c) A Carnot's engine whose low-temperature reservoir is at 7 °C has an efficiency of 50%. By how much °C, the temperature of the high-temperature reservoir should be raised to increase the efficiency to 70%? 3

**OR**

**4.** (a) Discuss the thermodynamic or work scale of temperature. Hence define the absolute zero temperature. 4+1=5

(b) Show that Helmholtz free energy remains constant during an isothermal isochoric process. 2

(c) Explain the principle of increase of entropy. 3

UNIT—III

**5.** (a) Find the directional derivative of $\quad x^2yz \quad 4xz^2$ at (1, $\quad$ 2, $\quad$ 1) in the direction of $2\hat{i} \quad \hat{j} \quad 2\hat{k}$. 3

(b) Transform the vector $\vec{A} \quad x\hat{i} \quad 2z\hat{j} \quad y\hat{k}$ to cylindrical coordinate system. 3

(c) Define symmetric and skew-symmetric tensors. Show that the components of a contravariant tensor of rank 2 can be expressed as the sum of a symmetric contravariant tensor and a skew-symmetric contravariant tensor, both of rank 2. 1+1+2=4

**OR**

**6.** (a) Show that the vector $\vec{A} \quad \frac{\vec{r}}{r}$, where $\vec{r} \quad x\hat{i} \quad y\hat{j} \quad z\hat{k}$ and $r \quad |\vec{r}|$ is irrotational. 3

(b) Represent the vector $\vec{F} \quad y\hat{i} \quad z\hat{j} \quad x\hat{k}$ in spherical polar coordinate system. 3

(c) Define contravariant tensor. Show that the velocity of the fluid at any point is component of a contravariant vector. 2+2=4

## UNIT—IV

**7.** *(a)* Define symmetric and skew-symmetric matrix. Let $A$ and $B$ be two symmetric matrices. Then show that $AB$ is symmetric if and only if $A$ and $B$ commute. 1+1+2=4

*(b)* Show that any two eigenvectors corresponding to two distinct eigenvalues of Hermitian matrix are orthogonal. 3

*(c)* Using matrix method, solve the system of equations

$$x \quad y \quad z \quad 6, \; x \quad y \quad z \quad 2 \text{ and } 2x \quad y \quad z \quad 1$$ 3

**OR**

**8.** *(a)* Define Hermitian and skew-Hermitian matrix. Show that every square matrix $A$ can be uniquely expressed as the sum of a Hermitian and a skew-Hermitian matrix. 1+1+2=4

*(b)* Find the value of  for which the matrix

$$A \quad \begin{matrix} \cos & \sin & 0 \\ \sin & \cos & 0 \\ 0 & 0 & \end{matrix}$$

is orthogonal. 3

*(c)* If $A$ and $B$ are two non-singular square matrices of order $n$, then show that

$$(AB)^{\;1} \quad B^{\;1}A^{\;1}$$ 3

## UNIT—V

**9.** *(a)* Using the expression for $(n)$, show that

$$\frac{1}{2} \quad \sqrt{\phantom{x}}$$ 4

*(b)* Show that

$$(m,\, n) \quad \frac{(m \quad 1)!(n \quad 1)!}{(m \quad n \quad 1)!} \quad \frac{(m) \quad (n)}{(m \quad n)}$$ 3

*(c)* Show that

$$\int_0^\infty \frac{y^{m-1}}{(1+y)^{m+n}}\,dy = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)} \qquad 3$$

**OR**

**10.** *(a)* Show that

$$\int_0^\infty \frac{y^{m-1}}{(1+y)^{m+n}}\,dy = \int_0^1 \frac{y^{m-1}+y^{n-1}}{(1+y)^{m+n}}\,dy = \beta(m,\ n) \qquad 4$$

*(b)* Prove that

$$\int_0^1 \frac{35x^3}{32\sqrt{(1-x)}}\,dx = 1 \qquad 3$$

*(c)* Show that

$$\int_0^{\pi/2} \sin^p\theta\,\cos^q\theta\,d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\,\Gamma\left(\frac{p+q+2}{2}\right)} \qquad 3$$

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