2025

(NEP—2020)

(4th Semester)

PHYSICS (MAJOR)

(Mathematical Physics—II)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. Which of the following is the differential equation for which the solution is $y = c_1 e^x + c_2 e^{-x} + 3$?

   (a) $\frac{d^2 y}{dx^2} - y - 3 = 0$ ( )

   (b) $\frac{d^2 y}{dx^2} - y + 3 = 0$ ( )

   (c) $\frac{d^2 y}{dx^2} + y - 3 = 0$ ( )

   (d) $\frac{d^2 y}{dx^2} + y - 6 = 0$ ( )

2. Which of the following is ***not*** a differential equation?

(a) $\frac{dy}{dx} + \frac{x}{y} = 0$ ( )

(b) $(1 + x^2)y'' + 2xy' = 0$ ( )

(c) $\frac{\partial^2 y}{\partial x^2} = \sqrt{1 + \frac{\partial^2 y}{\partial t^2}}$ ( )

(d) $\cos\left(\frac{dy}{dx}\right) + y = 0$ ( )

3. For the differential equation $x^2 y'' + y' + y = 0$

(a) $x = 0$ is an ordinary point ( )

(b) $x = 0$ is a regular singular point ( )

(c) $x = 0$ is an irregular singular point ( )

(d) None of the above ( )

4. The solution of the differential equation $\frac{dy}{dx} + y^2 = 0$ is

(a) $y = \frac{1}{x + c}$ ( )

(b) $y = \frac{1}{x^2 + c}$ ( )

(c) $y = \frac{x}{x + c}$ ( )

(d) $y = -\frac{1}{x + c}$ ( )

5. Frobenius method of power series solution of a differential equation is possible if $x = x_0$ is

(a) an ordinary point ( )

(b) a regular singular point ( )

(c) an irregular singular point ( )

(d) Either (a) or (b) ( )

6. A differential equation $M(x, y)dx + N(x, y)dy = 0$ is said to be exact differential equation, if it satisfies

(a) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ ( )

(b) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ ( )

(c) $\frac{\partial y}{\partial x} = \frac{\partial N}{\partial M}$ ( )

(d) $\frac{\partial M}{\partial y} = -\frac{\partial N}{\partial x}$ ( )

7. The value of $\frac{2}{3}P_2(x) + \frac{1}{3}P_0(x)$ is

(a) x ( )

(b) x^2 ( )

(c) x^3 ( )

(d) $x^2 + \frac{2}{3}$ ( )

8. The value of $H_2(x)$ is

(a) $4x^3 + 2$ ( )

(b) $8x^2 - 12x$ ( )

(c) $4x^2 - 2$ ( )

(d) -1 ( )

9. If J_0 and J_1 are Bessel's function, then $J_1'(x)$ is given by

(a) $J_0(x) - \frac{1}{x} J_1(x)$ ( )

(b) $-J_0(x)$ ( )

(c) $J_0(x) + \frac{1}{x} J_1(x)$ ( )

(d) $J_0(x) - \frac{1}{x^2} J_1(x)$ ( )

10. Which of the following is not equal to 1?

(a) P_0 ( )

(b) $J_0(x)$ ( )

(c) $H_0(x)$ ( )

(d) $L_0(x)$ ( )

(SECTION : B—SHORT ANSWERS)

(*Marks* : 25)

Answer *five* questions, taking at least *one* from each Unit : 5×5=25

UNIT—I

1. Find the general solution of the equation

$$xy\frac{dy}{dx} = 1 + x + y + xy$$

2. Solve the equation $y'' - 3y' + 2y = 0$, given that $y(0) = 1$ and $y'(0) = 0$.

UNIT—II

3. Show that $x = 0$ is an irregular singular point and $x = 2$ is a regular singular point of the following differential equation :

$$x^3(x-2)\frac{d^2y}{dx^2} - (x-2)\frac{dy}{dx} + 3xy = 0$$

4. Show that the general solution of the two-dimensional Laplace's equation in Cartesian coordinates $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is $u = Ce^{k_1 x + k_2 y}$, where C, k_1 and k_2 are arbitrary constants.

UNIT—III

5. Show that $P_{2n}(x) = (-1)^n \frac{(2n)!}{2^{2n}(n!)^2}$.

6. Show that $\int_{-1}^{1} P_n^m(x) P_l^m(x)\, dx = 0$ if $n \neq l$.

UNIT—IV

7. Show that $J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin x$.

8. Show that $L_3^1(x) = -\frac{1}{2}x^2 + 3x - 3$.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 40)

Answer *four* questions, taking at least *one* from each Unit : 10×4=40

UNIT—I

1. (a) Solve the differential equation $(x+y)(dx-dy)=dx+dy$. 4

   (b) Find the general solution of the differential equation :

$$\frac{dy}{dx}=\frac{y+x-2}{y-x-4}$$

6

2. (a) Solve the differential equation $(x^3+y^3)dx-xy^2dy=0$, given that $y(1)=1$. 5

   (b) Find the general solution of the differential equation

$$x^2\frac{d^2y}{dx^2}-x(x+2)\frac{dy}{dx}+(x+2)y=0$$

5

UNIT—II

3. (a) Solve the differential equation $\frac{d^2y}{dx^2}+y=0$ by power series method near $x=0$. 6

   (b) For the differential equation $x(1-x)\frac{d^2y}{dx^2}-3x\frac{dy}{dx}-y=0$, find the roots of its indicial equation. 4

4. (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2}+\frac{1}{2x}\frac{dy}{dx}+\frac{1}{4x}y=0$$

by Frobenius method about $x=0$. 6

(b) Show that the general solution of one-dimensional heat diffusion equation $\frac{\partial \theta}{\partial t} = h^2 \frac{\partial^2 \theta}{\partial x^2}$ is given by $\theta = (A\cos nx + B\sin nx)Ce^{-n^2 h^2 t}$, where A, B, C and n are arbitrary constants. 4

UNIT—III

5. (a) Prove that

$$P_n(x) = \sum_{n=0}^{\infty}\left[\sum_{k=0}^{N} \frac{(-1)^n (2n-2k)!}{2^n k!(n-k)!(n-2k)!} x^{n-2k}\right]$$

is the coefficient of t^n in the expansion of $(1-2xt+t^2)^{-\frac{1}{2}}$ and hence show that $P_n(x) = (-1)^n P_n(-x)$. 5+2=7

(b) Show that $H_4(x) = 16x^4 - 48x^2 + 12$. 3

6. (a) Show that

$$\int_{-\infty}^{+\infty} e^{-x^2} H_n(x) H_m(x)\, dx = 2^n n! \sqrt{\pi}\, \delta_{nm}$$ 6

(b) Show that

$$P_n^{m+1}(x) - \frac{2mx}{(1-x^2)^{1/2}} P_n^m(x) + \{n(n+1) - m(m+1)\} P_n^{m-1}(x) = 0$$ 4

UNIT—IV

7. (a) Prove the following recurrence relations for $J_n(x)$: 3+3=6

(i) $xJ_n'(x) = nJ_n(x) - xJ_{n+1}(x)$

(ii) $2J_n'(x) = J_{n-1}(x) - J_{n+1}(x)$

(b) Evaluate the values of L_0, L_1, L_2. 4

8. (a) Show that $\int_0^{\infty} e^{-x} L_m(x) L_n(x)\, dx = \delta_{mn}$, where $\delta_{mn} = \begin{vmatrix} 0 & \text{for } m \neq n \\ 1 & \text{for } m = n \end{vmatrix}$. 6

(b) Prove that $J_0(x) = \frac{1}{\pi}\int_0^{\pi} \cos(x\sin\theta)\, d\theta$. 4

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