2025

(CBCS)

(2nd Semester)

PHYSICS

SECOND PAPER

(Thermodynamics and Mathematical Physics)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. Which of the following statements is true according to the kinetic theory of gases?

(a) Kinetic energy depends on gas type ( )

(b) Kinetic energy depends on temperature ( )

(c) Kinetic energy is same for all molecules ( )

(d) Temperature is proportional to volume ( )

2. The van der Waals' equation considers

(a) volume and intermolecular forces ( )

(b) temperature dependence ( )

(c) pressure-volume change in ideal gas ( )

(d) effect of external pressure ( )

3. Which of the following is a consequence of the third law of thermodynamics?

(a) It is impossible to reach absolute zero ( )

(b) The entropy of a system decreases as temperature increases ( )

(c) The heat capacity of all substances is zero at absolute zero ( )

(d) The entropy of a system is always positive ( )

4. Which of the following is the correct equation according to the Clausius-Clapeyron relation?

(a) $\frac{\partial P}{\partial T} = \frac{T\Delta V}{L}$ ( )

(b) $\frac{\partial V}{\partial T} = \frac{L}{P\Delta T}$ ( )

(c) $\frac{\partial V}{\partial T} = \frac{P}{L\Delta T}$ ( )

(d) $\frac{\partial P}{\partial T} = \frac{L}{T\Delta V}$ ( )

5. The divergence of curl of a vector function $\vec{F}$ is

(a) unity ( )

(b) infinite ( )

(c) zero ( )

(d) undefined ( )

6. What is the significance of Kronecker delta δ_{ij} in tensor analysis?

(a) To convert a covariant tensor into a contravariant tensor ( )

(b) To compute the determinant of a tensor ( )

(c) To express the identity matrix and to relate the components of a tensor in different coordinate systems ( )

(d) To represent the inverse of a tensor ( )

7. Which of the following matrices is always equal to its conjugate transpose?

(a) A real matrix ( )

(b) A Hermitian matrix ( )

(c) A skew-symmetric matrix ( )

(d) An orthogonal matrix ( )

8. Which of the following matrices is singular?

(a) A matrix with determinant greater than 1 ( )

(b) A matrix with all elements equal to 1 ( )

(c) A matrix with determinant equal to zero ( )

(d) A matrix with all diagonal elements non-zero ( )

9. What is the value of $\Gamma(1)$?

(a) 0 ( )

(b) ∞ ( )

(c) 1 ( )

(d) Undefined ( )

10. Which of the following properties is true for the beta function $\beta(x, y)$?

(a) $\beta(x, y) = \frac{\beta(y, x)}{x}$ ( )

(b) $\beta(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$ ( )

(c) $\beta(x, y) = \frac{\Gamma(x+y)}{\Gamma(x)\Gamma(y)}$ ( )

(d) $\beta(x, y) = \Gamma(x) + \Gamma(y)$ ( )

(SECTION : B—SHORT ANSWERS)

(*Marks* : 15)

Answer the following : 3×5=15

UNIT—I

1. Calculate the temperature at which the r.m.s. velocity of oxygen molecules will be equal to that of hydrogen molecules at 0 °C, given that pressure is the same in both cases.

OR

2. What is meant by the critical temperature of a substance and why is it important in determining the state of the substance?

UNIT—II

3. What is Carnot cycle, and how does it relate to the efficiency of a heat engine?

OR

4. Explain the concept of a reversible process in thermodynamics.

UNIT—III

5. Show that the vector $\vec{V} = (x + 3y)\hat{i} + (y - 3z)\hat{j} + (x - 2z)\hat{k}$ is solenoidal.

OR

6. If A_{ij} is a skew-symmetric tensor, then show that $(\delta^i_j\delta^k_l + \delta^i_l\delta^k_j)A_{ik} = 0$.

UNIT—IV

7. If A and B are symmetric matrices, then show that AB is symmetric if and only if A and B commute.

OR

8. Prove that $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

UNIT—V

9. Show that $2^n \Gamma\left(n+\frac{1}{2}\right) = 1.3.5...(2n-1)\sqrt{\pi}$.

OR

10. Evaluate $\int_0^\infty \sqrt[4]{x} e^{-\sqrt{x}} dx$.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer the following : 10×5=50

UNIT—I

1. *(a)* Describe kinetic model of a gas with basic postulates. Derive the expression for the r.m.s. speed of a gas molecule. 1+6=7

(b) At what temperature, pressure remaining constant, will the r.m.s. velocity of a gas be half its value at 0 °C? 3

OR

2. Set up the differential equation for rectilinear flow of heat in a rod heated at one end and solve it for a rod of *(a)* infinite length and *(b)* finite length. 10

UNIT—II

3. *(a)* Derive Maxwell's four general relations for the thermodynamic system. 6

(b) Write the statement of third law of thermodynamics. Explain the impossibility of attaining the absolute zero temperature. 1+3=4

OR

4. *(a)* What is a Carnot engine? Show that no engine between two given temperatures can be more efficient than a reversible Carnot's engine working between the same two temperatures. 1+6=7

(b) Show that the net change in entropy is zero in reversible cycle. 3

UNIT—III

5. *(a)* Show that the vector $\frac{\vec{r}}{r^3}$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $r = |\vec{r}|$, is both solenoidal and irrotational. 6

(b) Differentiate between symmetric and skew-symmetric tensors. 2

(c) Show that the velocity of a fluid at any point is component of a contravariant vector. 2

OR

6. *(a)* Deduce the expressions for the divergence and curl of a vector field in cylindrical coordinates. 6

(b) Show that the gradient of a scalar function and velocity are respectively a covariant and a contravariant tensor both of rank 1. 2+2=4

UNIT—IV

7. *(a)* Express the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & -1 & -2 \\ 4 & 2 & 0 \end{bmatrix}$ as the sum of a symmetric and skew-symmetric matrix. 5

(b) Find the inverse of the matrix $A = \begin{bmatrix} 3 & -10 & -1 \\ -2 & 8 & 2 \\ 2 & -4 & -2 \end{bmatrix}$. 5

OR

8. *(a)* Determine the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$. 5

(b) Use matrix method to solve the following system of linear equations : 5

$$3x + 4y + 2z = 8$$
$$2y - 3z = 3$$
$$x - 2y + 6z = -2$$

UNIT—V

9. *(a)* Show that $\Gamma(1+m)\Gamma(m) = \dfrac{m\pi}{\sin m\pi}$. 5

(b) Evaluate $\int_{-1}^{+1}(1+x)^{p-1}(1-x)^{q-1}dx$. 5

OR

10. *(a)* Show that $\Gamma(m)\Gamma\left(m+\dfrac{1}{2}\right) = \dfrac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$. 6

(b) Show that $\int_0^{\infty}\dfrac{x^8(1-x^6)}{(1+x)^{24}} = 0$. 4

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