2025

(CBCS)

(6th Semester)

MATHEMATICS

ELEVENTH PAPER

(Mechanics)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. The equation of the resultant of any number of coplanar forces acting on a rigid body is given by

   (a) $G - yR_y + xR_x = 0$ ( )

   (b) $G - yR_x + xR_y = 0$ ( )

   (c) $xR_y - yR_x = G$ ( )

   (d) $xR_x - yR_y = G$ ( )

2. If a body rests in limiting equilibrium on a rough inclined plane for which the coefficient of friction is $\frac{1}{4}$, then the angle of friction is

(a) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$ ( )

(b) $\tan^{-1}\left(\frac{1}{4}\right)$ ( )

(c) $\pi + \tan^{-1}\left(\frac{1}{4}\right)$ ( )

(d) None of the above ( )

3. The moment of inertia of a circular disc of radius r about an axis through its centre perpendicular to its plane is

(a) Mr^2 ( )

(b) $\frac{1}{3} Mr^2$ ( )

(c) $\frac{1}{2} Mr^3$ ( )

(d) $\frac{1}{2} Mr^2$ ( )

4. The centre of gravity of a circular arc of radius 4 cm subtending at an angle 90° lies on the axis of symmetry at a distance of

(a) $\frac{2\sqrt{2}}{\pi}$ from the centre ( )

(b) $\frac{4\sqrt{2}}{\pi}$ from the centre ( )

(c) $\frac{6\sqrt{2}}{\pi}$ from the centre ( )

(d) $\frac{8\sqrt{2}}{\pi}$ from the centre ( )

5. The maximum velocity of a body moving with simple harmonic motion is 2 cm/sec and its period is $\frac{1}{5}$ sec. Then its amplitude is

(a) $\frac{1}{3\pi}$ cm ( )

(b) $\frac{1}{4\pi}$ cm ( )

(c) $\frac{1}{5\pi}$ cm ( )

(d) $\frac{1}{6\pi}$ cm ( )

6. If time t be regarded as a function of velocity v, then the rate of decrease of acceleration f, is given by

(a) $f^4 \frac{d^2t}{dv^2}$ ( )

(b) $f^3 \frac{d^2t}{dv^2}$ ( )

(c) $f^2 \frac{d^2t}{dv^2}$ ( )

(d) $f \frac{d^2t}{dv^2}$ ( )

7. If a particle moves along the x-axis under an attraction towards the origin O, varying inversely as the square of the distance from it, then the equation of motion is

(a) $\ddot{x} = \frac{\mu}{x^2}$ ( )

(b) $\ddot{x} = \frac{-\mu}{x^2}$ ( )

(c) $\ddot{x} = \mu x^2$ ( )

(d) $\ddot{x} = -\mu x^2$ ( )

8. If a particle is projected with a velocity 9·8 m/s from the ground at an angle 45° with the horizontal, then the range of the projectile is (take $g = 9 \cdot 8 \text{ m/s}^2$)

(a) 9·8 m ( )

(b) 4·9 m ( )

(c) 19·6 m ( )

(d) 39·2 m ( )

9. The SI unit of work done is

(a) watt ( )

(b) m/s^2 ( )

(c) joule ( )

(d) newton ( )

10. The loss in kinetic energy is zero only for

(a) a perfectly elastic impact ( )

(b) all elastic impacts ( )

(c) a perfectly inelastic impact ( )

(d) all inelastic impacts ( )

(SECTION : B—SHORT ANSWERS)

(*Marks* : 15)

Answer the following questions : 3×5=15

UNIT—I

1. Forces equal to $3P$, $7P$ and $5P$ act respectively along the sides AB, BC and CA of an equilateral triangle ABC. Find the magnitude, direction and line of action of the resultant.

OR

2. A uniform ladder rests in limiting equilibrium with its upper end against a smooth wall. If θ be the inclination of the ladder to the vertical, prove that $\tan\theta = 2\mu$, where μ is the coefficient of friction.

UNIT—II

3. Find the CG of the homogeneous area bounded by the parabola $y^2 = 4ax$, the x-axis and the ordinate $x = h$.

OR

4. Find the center of gravity of a uniform sector of a circle.

UNIT—III

5. If the angular velocity of a point moving in a plane curve be constant about a fixed origin, show that its transverse acceleration varies as its radial velocity.

OR

6. If v_1 and v_2 are the velocities of a particle moving in simple harmonic motion at distances x_1 and x_2 from the center, show that

$$4\pi\eta^2 (x_1^2 - x_2^2) = v_2^2 - v_1^2$$

where η is the frequency.

UNIT—IV

7. If h and h' be the greatest heights in the two paths of a projectile with a given velocity for a given range R, then show that $R = 4\sqrt{hh'}$.

OR

8. For a given velocity of projection, the maximum range down an inclined plane is three times the range up the inclined plane. Show that the inclination of the plane to the horizontal is 30°.

UNIT—V

9. The earth's attraction on a particle varies inversely as the square of its distance from the earth's centre. A particle whose weight on the surface of the earth is W, falls to the surface of the earth from a height $5a$ above it. Show that the work done by the earth's attraction is $\frac{5aW}{6}$, where a is the radius of the earth.

OR

10. Two balls impinge directly and the impact interchanges their velocities. Then prove that the two balls must be of equal mass and perfectly elastic.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer the following questions : 10×5=50

UNIT—I

1. *(a)* If a system of coplanar forces reduces to a single couple whose moment is G, such that when each force is turned round its point of application through a right angle, it reduces to a couple H. Prove that when each force is turned through an angle α, the system is equivalent to a couple whose moment is $G\cos\alpha + H\sin\alpha$. 5

(b) The altitude of a cone is h and the radius of its base is r. A string is fastened to the vertex and to a point on the circumference of the circular base, and is put over a smooth peg. Show that if the cone rests with its axis horizontal, the length of the string must be $\sqrt{h^2 + 4r^2}$. 5

OR

2. (a) A uniform rod rests on a fixed smooth sphere with its lower end pressing against a smooth vertical wall which touches the sphere. If θ is the angle which the rod makes with the vertical, when in equilibrium, prove that

$$a = 2l \sin\left(\frac{\theta}{2}\right) \cos^3\left(\frac{\theta}{2}\right)$$

where l is the length of the rod and a is the radius of the sphere. 5

(b) How high can a particle rest inside a hollow sphere of radius a, if the coefficient of friction be $\frac{1}{\sqrt{3}}$? 5

UNIT—II

3. (a) State and prove the perpendicular axis theorem on moments of inertia. 5

(b) A square hole is punched out of a circular lamina, the diagonal of the square being the radius of the circle. Show that the centre of gravity of the remainder is at a distance $\frac{a}{8\pi - 4}$ from the centre of the circle, where a is its diameter. 5

OR

4. (a) A thin uniform wire is bent into the form of a triangle ABC and heavy particles of weight P, Q, R are placed at the angular points. If the centre of mass of the particles coincides with that of the wire, prove that

$$\frac{P}{b+c} = \frac{Q}{c+a} = \frac{R}{a+b}$$ 5

(b) Calculate the moment of inertia of a rectangular lamina about a line through its centre and parallel to one of its edges. 5

UNIT—III

5. *(a)* For a particle moving in a plane curve, show that the radial and transverse acceleration components are given by $f_r = \ddot{r} - r\dot{\theta}^2$ and $f_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$. 7

(b) A particle rests in equilibrium under the attraction of two centre of forces which attract directly as the distance, their intensities being η and η'. Show that the time of small oscillation is $\frac{2\pi}{\eta + \eta'}$. 3

OR

6. *(a)* A train travels a distance s in t sec. It starts from rest and ends at rest. In the first part of the journey it moves with constant acceleration f and in the second part with constant retardation f'. Show that if s is the distance between two stations, then

$$t = \sqrt{2s\left(\frac{1}{f} + \frac{1}{f'}\right)}$$ 5

(b) A particle moves with SHM in a straight line. In the first second after starting from rest, it travels a distance a and in the next second, it travels a distance b in the same direction. Prove that the amplitude of the motion is $\frac{2a^2}{3a - b}$ and its period is

$$\frac{2\pi}{\cos^{-1}\left(\frac{b-a}{2a}\right)}$$ 5

UNIT—IV

7. *(a)* If R be the maximum range up an inclined plane of inclination β for a given velocity of projection, prove that $R = \frac{1}{2}gT^2$, where T is the corresponding time of flight. 5

(b) Two bodies are projected from the same point in directions making angles α_1, α_2 with the horizontal and strike at the same point in the horizontal plane through the point of projection. If t_1, t_2 be their times of flight, prove that

$$\frac{t_1^2 - t_2^2}{t_1^2 + t_2^2} = \frac{\sin(\alpha_1 - \alpha_2)}{\sin(\alpha_1 + \alpha_2)}$$ 5

OR

8. (a) Particles are projected from the same point in a vertical plane with velocity $\sqrt{2gk}$. Prove that the locus of the vertices of their paths is the ellipse $x^2 + 4y(y - k) = 0$. 5

(b) A particle is projected vertically upwards with a velocity v in a medium whose resistance is kv^2 per unit mass. Show that the greatest height attained by the particle is

$$\frac{1}{2k}\log\left[1 + \frac{kv^2}{g}\right]$$ 5

UNIT—V

9. (a) A uniform elastic string has the length a_1 when the tension is T_1 and the length a_2 when the tension is T_2. Show that its natural length is $\frac{a_2T_1 - a_1T_2}{T_1 - T_2}$ and the amount of work done in stretching it from its natural length to a length of $(a_1 + a_2)$ is

$$\frac{1}{2}\frac{(a_1T_1 - a_2T_2)^2}{(T_1 - T_2)(a_1 - a_2)}$$ 5

(b) A gun of mass M fires a shell of mass m horizontally and the energy of explosion is such as would be sufficient to project the shell vertically to a height h. Prove that the velocity of the recoil is

$$\left\{\frac{2m^2gh}{M(M + m)}\right\}^{\frac{1}{2}}$$ 5

OR

10. *(a)* A sphere impinges directly on an equal sphere which is at rest. Show that a fraction $\frac{1}{2}(1-e^2)$ of the original KE is lost during the impact. 5

(b) A smooth sphere impinges on an equal sphere at rest. Before impact, the first sphere was moving in a direction making an angle α with the line of centres at the moment of impact. If the direction of motion of the first sphere is turned through an angle ϕ by the impact, show that

$$\tan\phi = \frac{(1+e)\tan\alpha}{1-e+2\tan^2\alpha}$$ 5

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