MATH201 (MAJOR/MINOR)

Student's Copy

2024

(NEP-2020)

(3rd Semester)

MATHEMATICS (MAJOR/MINOR)

(Differential Equations)

Full Marks: 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A-OBJECTIVE)

(Marks: 10)

Tick (\checkmark) the correct answer in the brackets provided :

- 1. The differential equation of the family of curves $y = a \cos(mx + b)$, a and b being arbitrary constants, is
 - (a) $\frac{d^2y}{dx^2} + m^2y = 0$ ()
 - (b) $\frac{d^2y}{dx^2} m^2y = 0$ ()
 - (c) $\frac{d^2y}{dx^2} + my = 0$ ()

(d)
$$\frac{d^2 y}{dx^2} - my = 0$$
 ()

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| Contd.

1×10=10

The solution to the differential equation $\frac{dy}{dx} = e^{x-y}$ is ci.

(a)
$$e^x = e^y$$
 (b) $e^x = e^{-y}$ (c) $e^y = e^{-x}$ (c) $e^y = e^{-x}$ (c) (d) None of the above

The integrating factor of the equation xdy - ydx = 0 is ë.

a)
$$\frac{1}{y^2}$$
 () (b) $\frac{1}{x^2}$ ()

$$(c) - \frac{1}{xy}$$
 () (d) None of the above

+4y = 0 is $\frac{d^2y}{dx^2}$ The general solution of the differential equation 4.

(a)
$$y = c_1 \cos 2x + c_2 \sin 2x$$
 (
(b) $y = c_1 \cos 2x - c_2 \sin 2x$

(c)
$$y = c_1 \cos x + c_2 \sin x$$

$$(a) \quad y = c_1 \cos x - c_2 \sin x \qquad ($$

equation differential the , is ъ Ğ ъ (Id) $(D^2 + 5 - 2D)y = 10 \sin x$, where D =integral particular The ю.

a)
$$\sin x + \cos 2x$$
 (

$$b$$
 2 sin $x + \cos 2x$

(c)
$$2\sin x + \cos x$$
 (

(d)
$$\sin x + 2\cos x$$
 (

 $\frac{d^2y}{dx^2} + P \frac{dy}{dx} + Qy = 0,$ 11 where P and Q are functions of x only or constant, if 1 - P + Q = 0, then y 6. In the linear differential equation of second-order

a)
$$e^{-x}$$
 () (b) x^2 ()
c) x () (d) e^{-x} ()

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Cont

7. The differential equation Pdx + Qdy + Rdz = 0 is integrable, if (OP de) (de (ar (NG (ao

$$(a) \quad P\left(\frac{\partial \nabla}{\partial z} - \frac{\partial X}{\partial y}\right) + Q\left(\frac{\partial X}{\partial x} - \frac{\partial Z}{\partial z}\right) + R\left(\frac{\partial Y}{\partial y} - \frac{\partial Y}{\partial x}\right) = 0 \quad (1)$$

$$(b) \quad P\left(\frac{\partial Q}{\partial x} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial y} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial z} - \frac{\partial Q}{\partial x}\right) = 0 \quad (1)$$

$$(c) \quad P\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) = 0 \quad (1)$$

- equation differential partial 0= $(d) \quad P\left(\frac{\partial Q}{\partial y} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial z} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial x} - \frac{\partial Q}{\partial x}\right)$ the of degree 12. $\frac{\partial z}{\partial x}$ and $+\frac{\partial^3 z}{\partial y^3} = 2x(-)$ order The az az ø.
 - $(a) \quad \partial y^{2} \quad (ax)$ (a) order = 1, degree = 3 (b) order = 3, degree = 2 $(c) \quad (c) \quad ($
 - (b) order = 3, degree = 2
 (c) order = 3, degree = 1
 - (d) order = 2, degree = 3 (
- The PDE obtained from z = (x + a)(y + b) by removing a and b is 6
- b+d=z(q) (q) bd = z(a)
 - bd = zhxb-d=z(c)
 - **10.** The general solution of the PDE $p^2 + q^2 = x + y$ is

(a)
$$z = \frac{2}{3}(x+a)^{1/2} + \frac{2}{3}(y-a)^{1/2} + b$$
 ()
(b) $z = \frac{2}{3}(x+a)^{3/2} + \frac{2}{3}(y-a)^{3/2} + b$ ()
(c) $z = \frac{2}{3}(x-a)^{3/2} + \frac{2}{3}(y+a)^{3/2} + b$ ()

(c)
$$z = \frac{2}{3}(x-a)^{3/2} + \frac{2}{3}(y+a)^{3/2} + b$$
 ()
(d) $z = \frac{2}{3}(x-a)^{1/2} + \frac{2}{3}(y+a)^{1/2} + b$ ()

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(SECTION : B-SHORT ANSWERS)

(Marks : 15)

Answer five questions, taking at least one from each Unit :

 $3 \times 5 = 15$

UNIT-I

- 1. Find $y = e^{x} (A \cos x + B \sin x)$, where A and B are arbitrary constants. the differential equation ç, the family g, curves
- Ņ Solve :

$$x^2 - x^2 y$$
)dy + ($xy^2 + y^2$)dx = 0

UNIT-╘

ω Solve :

$$\frac{d^2y}{dx^2} - a^2y = e^{ax}$$

4. Solve p(p + x) = y(x + y), where $p = \frac{dy}{dx}$.

UNIT-III

- ģ $\frac{dy}{dt} + x = 1.$ Solve the simultaneous linear differential equation di a -y = tand
- 6 Solve :

(yz + 2x)dx + (zx - 2z)dy + (xy - 2y)dz = 0

UNIT-IV

- 7 Show that the differential equations $p = x^2 - ay$, $q = y^2 - ax$ are compatible
- °. Find the surface satisfying the

- - and find their common solution.
- $(x-y)y^2p + (y-x)x^2q = (x^2 + y^2)z$ and passing through $xz = a^3$ partial equation

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y = 0.

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SECTION : C-DESCRIPTIVE)

(Marks : 50)

Answer five questions, taking at least one from each Unit :

 $10 \times 5 = 50$

(a) Solve
$$\frac{1}{2x} \frac{dy}{dx} + \frac{(x+y)}{(x^2+y^2)} = 0.$$
 5

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(b) Solve
$$\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2}$$
 given that $y = 0$ when $x = 1$. 5

2 a) Reduce equation and solve it. the equation $\frac{dy}{dx} + \frac{y}{x} \log y = \frac{y}{x^2} (\log y)^2$ to a linear differential S

(b) Solve
$$(1 + 3e^{x/y})dx + 3e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0.$$
 5

- ω (a) Solve the differential equation $\frac{d^2y}{dx^2} + 4y = x \cos x$.
- (b) Solve $(D^3 + 3D^2 + 2D)y = x^2$ where $D = \frac{d}{dx}$.

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- (a) Reduce the equation $xyp^2 - p(x^2 + y^2 - 1) + xy = 0$ where đ
- 4
- $p = \frac{dy}{dx}$

- the equation represents a family of conics touching the four sides of a Clairut's form by the substitutions $x^2 = u$ and $y^2 = v$. Hence show that

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6

Find

the 2a

orthogonal trajectories of the family of curves

given by

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 $1 + \cos \theta$

square.

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ŝ (a) Solve the homogeneous differential equation

$$(x+a)^2 \frac{d^2y}{dx^2} - 4(x+a)\frac{dy}{dx} + 6y = x$$

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ট Apply the method of variation of parameters to solve

$$(x-1)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = (x-1)^2$$

<u>م</u> <u>a</u> Solve :

$$x^{2} \frac{d^{2}y}{dx^{2}} - (x^{2} + 2x)\frac{dy}{dx} + (x + 2)y = x^{3}e^{x}$$

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9 Show that the equation $x \frac{d^3y}{dx^3} + (x^2 + x + 3) \frac{d^2y}{dx^2} + (4x + 2) \frac{dy}{dx} + 2y = 0$ is

exact and solve it.

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E V

7 <u>a</u> Find the general solution to the following PDE by

$$x(x^2 + 3u^2)_0 - u(3x^2 + u^2)_0 - 00x^2 = 0$$
 using the second second

6

- Θ Find the surface which intersects the AL D $-y(3x^{-}+y^{-})q = 22(y^{-}-x^{-})$ surfaces
- z(x + y) = c(1 + 3z) orthogonally and which passes through the circle
- œ (a) Let $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$. Find the integral surface of the given $f(xyz, x^2 + y^2 - 2z) = 0$ be the general solution PDE 101

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- Ð Using Charpit's method, find the complete integral of the following PDE which contains the straight line, x + y = 0, z = 1. G

$$pxy + pq + qy - yz = 0$$

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G25-280

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MATHEMATICS (MAJOR/MINOR)

(Differential Equations)

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(SECTION : A-OBJECTIVE)

(Marks : 10)

Tick (\mathbf{v}) the correct answer in the brackets provided :

- 1×10=10
- .-The differential equation of the family of curves $y = a \cos(mx + b)$, a and b being arbitrary constants, is

(a)
$$\frac{d^2 y}{dx^2} + m^2 y = 0$$
 ()
(b) $\frac{d^2 y}{dx^2} - m^2 y = 0$ ()
(c) $\frac{d^2 y}{dx^2} + my = 0$ ()

(d)
$$\frac{d^2 y}{dx^2} - my = 0$$
 (

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Contd.

2. The solution to the differential equation $\frac{dy}{dx} = e^{x-y}$ is $\sqrt{570}$

(a)
$$e^x = e^y$$
 () (b) $e^x = e^{-y}$ (

0 ey 11 :e-x (d) None of the above

ω The integrating factor of the equation xdy - ydx = 0 is

(a)
$$\frac{1}{y^2}$$
 () (b) $\frac{1}{x^2}$ ()

(d) None of the above

ğ

4 The general solution of the differential equation Q $\frac{d^2y}{dx^2}$ +4y = 0 is

(b)
$$y = c_1 \cos 2x + c_2 \sin 2x$$
 ()
(c) $y = c_1 \cos 2x - c_2 \sin 2x$ ()
(c) $y = c_1 \cos x + c_2 \sin x$ ()
(d) $u = c_1 \cos x$

(a)
$$y = c_1 \cos x - c_2 \sin x$$
 ()
5. The particular integral (p)

 $(D^2 + 5 - 2D)y = 10 \sin x$, where D =dx, 2 01 S the differential equation

Θ (a) $\sin x + \cos 2x$

0 $2\sin x + \cos 2x$

(d) $2\sin x + \cos x$

 $\sin x + 2\cos x$

9 h

a) where P and Q are functions of x only or constant, if 1 - P + Q = 0, then y = 0the linear differential equation of second-order (b) *2 $r \frac{d^2y}{dx^2} + P \frac{dy}{dx} + Qy = 0,$

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e-x

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(d)

 $z = \frac{2}{3}(x-a)^{1/2} + \frac{2}{3}(y+a)^{1/2} + b$

7 10. œ <u>ب</u> The differential equation Pdx + Qdy + Rdz = 0 is integrable, if The Ð a) (d) <u>(</u>) The general solution of the PDE $p^2 + q^2 = x + y$ is The PDE obtained from z = (x + a)(y + b) by removing a and b zb (d 0 6 a) 0 <u>a</u> (a) order = 1, degree = 36 0 $P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right)$ Ъ Ъ Ъ b - d = z $z = \frac{2}{3}(x+a)^{1/2} + \frac{2}{3}(y-a)^{1/2} + b$ order order = 3, degree = 2z = pqorder = 3, degree $\left(\frac{\partial^{2} z}{\partial y^{3}} = 2x \left(\frac{\partial z}{\partial x}\right)$ is $z = \frac{2}{3}(x+a)^{3/2} + \frac{2}{3}(y-a)^{3/2} + b$ $z = \frac{2}{3}(x-a)^{3/2} + \frac{2}{3}(y+a)^{3/2} + b$ $\left(\frac{\partial Q}{\partial y} - \frac{\partial R}{\partial y}\right)$ <u>de</u> de de order and $-\frac{\partial R}{\partial y} + Q \left(\frac{\partial R}{\partial y} - \frac{\partial P}{\partial z} \right) + R \left(\frac{\partial P}{\partial z} - \frac{\partial Q}{\partial x} \right)$ = 2, degree <u>s</u> Ø $\left(+ Q \left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z} \right) + R \left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x} \right) \right)$ $\Big) + Q\Big(\frac{\partial R}{\partial z} - \frac{\partial P}{\partial z}\Big) + R\Big(\frac{\partial P}{\partial x} - \frac{\partial Q}{\partial x}\Big) = 0$ $+Q\left(\frac{\partial R}{\partial x}-\frac{\partial P}{\partial z}\right)+R\left(\frac{\partial Q}{\partial z}\right)$ degree ။ ယ = of the - are (d) (b) partial = 0 =0 = 0 z = p + qxyz = pqdifferential S equation

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(Marks: 15)

Answer five questions, taking at least one from each Unit :

UNIT-I

- curves $y = e^x (A \cos x + B \sin x)$, where A and B are arbitrary constants. of family the of equation differential the 1. Find
 - 2. Solve :

$$(x^{2} - x^{2}y)dy + (xy^{2} + y^{2})dx = 0$$

UNIT-II

3. Solve :

$$\frac{d^2y}{dx^2} - a^2y = e^{ax}$$

4. Solve p(p + x) = y(x + y), where $p = \frac{dy}{dx}$.

UNIT-III

- and $\frac{dx}{dt} - y = t$ equation simultaneous linear differential Solve the $\frac{dy}{dt} + x = 1.$ ġ ທ່
- 6. Solve :

$$(yz + 2x)dx + (zx - 2z)dy + (xy - 2y)dz =$$

0

UNIT-IV

- Show that the differential equations $p = x^2 ay$, $q = y^2 ax$ are compatible 1
- equation y = 0 $(x - y)y^2p + (y - x)x^2q = (x^2 + y^2)z$ and passing through $xz = a^3$ the satisfying surface 8. Find the

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Contd.

3×5=15

4

(SECTION : C-DESCRIPTIVE

(Marks: 50)

Answer *five* questions, taking at least *one* from each Unit :

 $10 \times 5 = 50$

UNIT-I

(a) Solve
$$\frac{1}{2x} \frac{dy}{dx} + \frac{(x+y)}{(x^2+y^2)} = 0.$$
 5
du $2xy$ 1 that ... 0 when $x = 1.$ 5

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(b) Solve
$$\frac{dy}{dx} + \frac{2xy}{1+x^2} = \frac{1}{(1+x^2)^2}$$
 given that $y = 0$ when $x = 1$.

S Reduce the equation $\frac{dy}{dx} + \frac{y}{x} \log y = \frac{y}{x^2} (\log y)^2$ to a linear differential equation and solve it. **2**. (a)

(b) Solve
$$(1 + 3e^{x/y})dx + 3e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0.$$

UNIT--II

 $+4y = x\cos x$. Solve the differential equation $\frac{d^2y}{dx^2}$ 3. (a)

S

(b) Solve
$$(D^3 + 3D^2 + 2D)y = x^2$$
 where $D = \frac{d}{dx}$.

S 5 Clairut's form by the substitutions $x^2 = u$ and $y^2 = v$. Hence show that the equation represents a family of conics touching the four sides of a Reduce the equation $xyp^2 - p(x^2 + y^2 - 1) + xy = 0$ where $p = \frac{dy}{dx^2}$ square. **4**. (a)

Find the orthogonal trajectories of the family of curves given by $1 + \cos \theta$ 2а 1 (q)

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Contd.

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UNIT-III

Solve the homogeneous differential equation 5. (a)

$$(x+a)^2 \frac{d^2y}{dx^2} - 4(x+a)\frac{dy}{dx} + 6y = x$$

4

Apply the method of variation of parameters to solve (q)

$$(x-1)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = (x-1)^2$$

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6. (a) Solve :

$$c^{2} \frac{d^{2}y}{dx^{2}} - (x^{2} + 2x)\frac{dy}{dx} + (x + 2)y = x^{3}e^{x}$$

Show that the equation $x \frac{d^3y}{dx^3} + (x^2 + x + 3) \frac{d^2y}{dx^2} + (4x + 2) \frac{dy}{dx} + 2y = 0$ is exact and solve it. (q)

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UNIT-IV

Find the general solution to the following PDE by Lagrange's method 7. (a)

$$x(x^2 + 3y^2)p - y(3x^2 + y^2)q = 22(y^2 - x^2)$$

circle system z(x + y) = c(1 + 3z) orthogonally and which passes through the of the surface which intersects the surfaces $x^2 + y^2 = 1, \ z = 1.$ Find the (a

4

- PDE $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$. Find the integral surface of the given Let $f(xyz, x^2 + y^2 - 2z) = 0$ be the general solution of the PDE which contains the straight line, x + y = 0, z = 1. 8. (a)
- S ŝ Using Charpit's method, find the complete integral of the following partial differential equation : (q)

$$0 = zh - hb + bd + hxd$$

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