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(NEP–2020)

(3rd Semester)

MATHEMATICS (MAJOR/MINOR)

(Differential Equations)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. The differential equation of the family of curves $y = a\cos(mx + b)$, a and b being arbitrary constants, is

   (a) $\dfrac{d^2y}{dx^2} + m^2y = 0$ ( )

   (b) $\dfrac{d^2y}{dx^2} - m^2y = 0$ ( )

   (c) $\dfrac{d^2y}{dx^2} + my = 0$ ( )

   (d) $\dfrac{d^2y}{dx^2} - my = 0$ ( )

2. The solution to the differential equation $\frac{dy}{dx} = e^{x-y}$ is

(a) $e^x = e^y$ ( )
(b) $e^x = e^{-y}$ ( )
(c) $e^y = e^{-x}$ ( )
(d) None of the above ( )

3. The integrating factor of the equation $xdy - ydx = 0$ is

(a) $\frac{1}{y^2}$ ( )
(b) $\frac{1}{x^2}$ ( )
(c) $\frac{1}{xy}$ ( )
(d) None of the above ( )

4. The general solution of the differential equation $\frac{d^2y}{dx^2} + 4y = 0$ is

(a) $y = c_1 \cos 2x + c_2 \sin 2x$ ( )
(b) $y = c_1 \cos 2x - c_2 \sin 2x$ ( )
(c) $y = c_1 \cos x + c_2 \sin x$ ( )
(d) $y = c_1 \cos x - c_2 \sin x$ ( )

5. The particular integral (PI) of the differential equation $(D^2 + 5 - 2D)y = 10 \sin x$, where $D = \frac{d}{dx}$, is

(a) $\sin x + \cos 2x$ ( )
(b) $2 \sin x + \cos 2x$ ( )
(c) $2 \sin x + \cos x$ ( )
(d) $\sin x + 2 \cos x$ ( )

6. In the linear differential equation of second-order $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = 0$, where P and Q are functions of x only or constant, if $1 - P + Q = 0$, then $y =$

(a) e^x ( )
(b) x^2 ( )
(c) x ( )
(d) e^{-x} ( )

7. The differential equation $Pdx + Qdy + Rdz = 0$ is integrable, if

(a) $P\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) = 0$ ( )

(b) $P\left(\frac{\partial Q}{\partial x} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial y} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial z} - \frac{\partial Q}{\partial x}\right) = 0$ ( )

(c) $P\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right) + Q\left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial Q}{\partial z} - \frac{\partial R}{\partial y}\right) = 0$ ( )

(d) $P\left(\frac{\partial Q}{\partial y} - \frac{\partial R}{\partial y}\right) + Q\left(\frac{\partial R}{\partial z} - \frac{\partial P}{\partial z}\right) + R\left(\frac{\partial P}{\partial x} - \frac{\partial Q}{\partial x}\right) = 0$ ( )

8. The order and degree of the partial differential equation $\left(\frac{\partial z}{\partial x}\right)^2 + \frac{\partial^3 z}{\partial y^3} = 2x\left(\frac{\partial z}{\partial x}\right)$ is

(a) order = 1, degree = 3 ( )

(b) order = 3, degree = 2 ( )

(c) order = 3, degree = 1 ( )

(d) order = 2, degree = 3 ( )

9. The PDE obtained from $z = (x+a)(y+b)$ by removing a and b is

(a) $z = pq$ ( ) (b) $z = p + q$ ( )

(c) $z = p - q$ ( ) (d) $xyz = pq$ ( )

10. The general solution of the PDE $p^2 + q^2 = x + y$ is

(a) $z = \frac{2}{3}(x+a)^{1/2} + \frac{2}{3}(y-a)^{1/2} + b$ ( )

(b) $z = \frac{2}{3}(x+a)^{3/2} + \frac{2}{3}(y-a)^{3/2} + b$ ( )

(c) $z = \frac{2}{3}(x-a)^{3/2} + \frac{2}{3}(y+a)^{3/2} + b$ ( )

(d) $z = \frac{2}{3}(x-a)^{1/2} + \frac{2}{3}(y+a)^{1/2} + b$ ( )

(SECTION : B—SHORT ANSWERS)

(*Marks* : 15)

Answer *five* questions, taking at least *one* from each Unit : 3×5=15

UNIT—I

1. Find the differential equation of the family of curves $y = e^x(A\cos x + B\sin x)$, where A and B are arbitrary constants.

2. Solve :

$$(x^2 - x^2y)dy + (xy^2 + y^2)dx = 0$$

UNIT—II

3. Solve :

$$\frac{d^2y}{dx^2} - a^2y = e^{ax}$$

4. Solve $p(p + x) = y(x + y)$, where $p = \frac{dy}{dx}$.

UNIT—III

5. Solve the simultaneous linear differential equation $\frac{dx}{dt} - y = t$ and $\frac{dy}{dt} + x = 1$.

6. Solve :

$$(yz + 2x)dx + (zx - 2z)dy + (xy - 2y)dz = 0$$

UNIT—IV

7. Show that the differential equations $p = x^2 - ay$, $q = y^2 - ax$ are compatible and find their common solution.

8. Find the surface satisfying the partial differential equation $(x - y)y^2p + (y - x)x^2q = (x^2 + y^2)z$ and passing through $xz = a^3$, $y = 0$.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer *five* questions, taking at least *one* from each Unit : 10×5=50

UNIT—I

1. (a) Solve $\frac{1}{2x}\frac{dy}{dx}+\frac{(x+y)}{(x^2+y^2)}=0.$ 5

   (b) Solve $\frac{dy}{dx}+\frac{2xy}{1+x^2}=\frac{1}{(1+x^2)^2}$ given that $y=0$ when $x=1$. 5

2. (a) Reduce the equation $\frac{dy}{dx}+\frac{y}{x}\log y=\frac{y}{x^2}(\log y)^2$ to a linear differential equation and solve it. 5

   (b) Solve $(1+3e^{x/y})dx+3e^{x/y}\left(1-\frac{x}{y}\right)dy=0.$ 5

UNIT—II

3. (a) Solve the differential equation $\frac{d^2y}{dx^2}+4y=x\cos x.$ 5

   (b) Solve $(D^3+3D^2+2D)y=x^2$ where $D=\frac{d}{dx}$. 5

4. (a) Reduce the equation $xyp^2-p(x^2+y^2-1)+xy=0$ where $p=\frac{dy}{dx}$ to Clairut's form by the substitutions $x^2=u$ and $y^2=v$. Hence show that the equation represents a family of conics touching the four sides of a square. 5

   (b) Find the orthogonal trajectories of the family of curves given by $r=\frac{2a}{1+\cos\theta}$. 5

UNIT—III

5. (a) Solve the homogeneous differential equation

$$(x+a)^2 \frac{d^2y}{dx^2} - 4(x+a)\frac{dy}{dx} + 6y = x$$

4

(b) Apply the method of variation of parameters to solve

$$(x-1)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = (x-1)^2$$

6

6. (a) Solve : 5

$$x^2 \frac{d^2y}{dx^2} - (x^2+2x)\frac{dy}{dx} + (x+2)y = x^3 e^x$$

(b) Show that the equation $x\frac{d^3y}{dx^3} + (x^2+x+3)\frac{d^2y}{dx^2} + (4x+2)\frac{dy}{dx} + 2y = 0$ is exact and solve it.

5

UNIT—IV

7. (a) Find the general solution to the following PDE by Lagrange's method : 6

$$x(x^2+3y^2)p - y(3x^2+y^2)q = 22(y^2-x^2)$$

(b) Find the surface which intersects the surfaces of the system $z(x+y) = c(1+3z)$ orthogonally and which passes through the circle $x^2+y^2=1$, $z=1$.

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8. (a) Let $f(xyz, x^2+y^2-2z)=0$ be the general solution of the PDE $x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$. Find the integral surface of the given PDE which contains the straight line, $x+y=0$, $z=1$.

5

(b) Using Charpit's method, find the complete integral of the following partial differential equation :

$$pxy + pq + qy - yz = 0$$

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