MATH200 (MAJOR)

Student's Copy

2024

(NEP-2020)

(3rd Semester)

MATHEMATICS (MAJOR)

(Modern Algebra—I)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A-OBJECTIVE)

(Marks: 10)

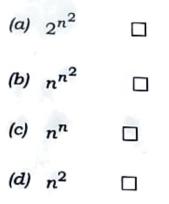
Tick \square the correct answer in the boxes provided :

- 1. The inverse of the element a in the group of all positive rational numbers under the composition $a * b = \frac{ab}{2}$ is
 - (a) 2 🗌
 - (b) a / 4
 - (c) 4/a
 - (d) 4a 🗆

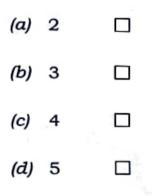
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1×10=10

2. The number of binary compositions on a finite set A having n elements is



3. In (Z_6, \oplus_6) , where $Z_6 = \{0, 1, 2, 3, 4, 5\}$ and \oplus_6 is the addition modulo 6, the order of 4 is



- 4. If a and b are any two elements of a group G and H is any subgroup of G, then Ha = Hb if and only if
 - (a) $ab \in H$
 - (b) $a^{-1}b \in H$ \Box
 - (c) $ab^{-1} \in H$
 - (d) $(ab)^{-1} \in H$

- 5. Let $T = \{\pm 1, \pm i, \pm j, \pm k\}$ be a quaternion group. Then
 - (a) T is non-Abelian with order 4 \Box
 - (b) T is non-Abelian with order 8 \Box
 - (c) T is Abelian with order 4 \cdot
 - (d) T is Abelian with order 8 \Box
- 6. Which of the following is incorrect?
 - (a) A group of order 1 is Abelian.
 - (b) Every commutative group is Abelian.
 - (c) In a group (G, \times), where \times denotes ordinary multiplication, a^{-1} and b are commute if a and b are commute $\forall a, b \in G$.
 - (d) If every element of a group G is its own inverse, then G is non-Abelian.
- 7. The order of a finite Abelian group (G, \oplus_4), where $G = \{0, 1, 2, 3\}$ and \oplus_4 as addition modulo 4, is
 - (a) 6 🛛
 - (b) 5 🗆
 - (c) 4 🛛
 - (d) 3

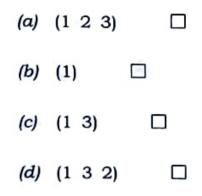
- **8.** Let a be any element of an Abelian group G such that $ax = xa \forall x \in G$. Then
 - (a) N(a) > G
 - (b) N(a) < G
 - (c) N(a) = G
 - $(d) \quad N(a) = \{ \} \qquad \Box$

where N(a) is normalizer of an element of G.

- 9. The generator of the cyclic group (ω , ω^2 , 1) is
 - (a) ω and ω^2

 - (c) ω^2 only \Box (d) ω , ω^2 and ω^3 \Box

10. The inverse of the product of (1 2) and (2 3) of the symmetric group S_3 is



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(SECTION : B-SHORT ANSWERS)

(Marks: 15)

Answer five questions, taking at least one from each Unit : 3×5=15

Unit—I

- 1. In the set of real numbers, show that the composition $x * y = x^y$ for all $x, y \in \mathbb{R}$ is not associative.
- 2. Prove that the inverse of the product of two elements of a group G is the product of the inverse taken in the reverse order.

Unit—II

- 3. Prove that the intersection of two subgroups of a group G is a subgroup of G.
- Prove that any two right cosets of a subgroup are either disjoint or identical.

UNIT—III

- 5. Prove that a group G is an Abelian if $a^2 = e \ \forall a \in G$, where e is the identity element of G.
- 6. Show that the symmetric group S_n is Abelian for n = 2 and non-Abelian other than n > 2.

Unit—IV

- 7. Find the remainder when 45^{31} is divided by 33.
- 8. If a finite group of order n contains an element of order n, then prove that the group must be cyclic.

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(SECTION : C-DESCRIPTIVE)

(Marks : 50)

Answer five questions, taking at least one from each Unit :

UNIT-I

- 1. (a) Let A be the set of fourth root of unity, i.e., $A = \{1, -1, i, -i\}$. Then show that the algebraic structure (A, \times) is a group, where \times is an ordinary multiplication.
 - (b) Prove that the set $G = \{1, 2, 3, 4, 5, 6\}$ is a group with respect to multiplication modulo 7, using group table.
- 2. (a) Prove that the identity element in a group is unique. Also prove that the inverse of each element of a group is unique.
 - (b) If a and b are any two elements of a group G, then show that the equations ax = b and ya = b have unique solutions in G.

Unit—II

- **3.** (a) Prove that the necessary and sufficient condition for a non-empty subset K of a group G to be a subgroup is that $r \in K$, $s \in K \Rightarrow rs^{-1} \in K$; where s^{-1} is the inverse of s in G.
 - (b) If H and K are two subgroups of a group G, then show that HK is a subgroup of G if and only if HK = KH.
- 5

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10×5=50

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- **4.** (a) Prove that the order of every element of a finite group is finite and is less than or equal to the order of the group.
 - (b) State and prove Lagrange's theorem on the order of a group.

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Unit—III

5. Prove that-

- (a) the set of all nth of unity forms a finite Abelian group of order n with respect to multiplication;
- (b) G is Abelian if it is a group with (ab)^m = a^mb^m for three consecutive integers m ∀a, b ∈ G.
 7+3=10

6. (a) Show that if G is an Abelian group, then for any integer n such that

$$(ab)^n = a^n b^n \,\forall a, b \in G$$

(b) Prove that a group G is Abelian if and only if

$$(pq)^{-1} = p^{-1}q^{-1} \forall p, q \in G$$
 5

UNIT-IV

7. (a) Show that every finite group is isomorphic to a permutation group. 7

(b) Find the order of permutation of (23)(134)(13) in the symmetric group S₄.

Prove that—

- (a) for any set S, A(S) is a group with respect to composition mapping where A(S) denotes the set of all one to one mapping of S onto itself;
- (b) the normalizer of a group G is a subgroup of G. 5+5=10

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(Modern Algebra-I)

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(SECTION : A-OBJECTIVE)

(Marks: 10)

Tick ☑ the correct answer in the boxes provided :

1×10=10

- 1. The inverse of the element a in the group of all positive rational numbers under the composition $a * b = \frac{ab}{2}$ is
 - (a) 2 □ (b) a/4 □ (c) 4/a □
 - (d) 4a 🛛

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	(a)	(q)	(c)	(d)	3. In th	(a)	(q)	(c)	(q)	4. If a the	(a)	(q)	(c)	(p)	/258

- **5.** Let $T = \{\pm 1, \pm i, \pm j, \pm k\}$ be a quaternion group. Then
- 4 T is non-Abelian with order (a)

T is non-Abelian with order 8 (q)

T is Abelian with order 4 <u>ی</u>

T is Abelian with order 8 (q)

- 6. Which of the following is **incorrect**?
- A group of order 1 is Abelian. (a)

Every commutative group is Abelian. (q

- In a group (G, x), where x denotes ordinary multiplication, a^{-1} and bare commute if a and b are commute $\forall a, b \in G$. <u></u>
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- **7**. The order of a finite Abelian group (G, \oplus_4), where G = {0, 1, 2, 3} and \oplus_4 as addition modulo 4, is

(a) 6	(b) 5	(c) 4	(q) 3

e (q)

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8. Let *a* be any element of an Abelian group *G* such that $ax = xa \forall x \in G$. Then

- (a) N(a) > G
- $(b) \quad N(a) < G$

 $(c) \quad N(a) = G$

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where $N(\alpha)$ is normalizer of an element of G.

- 9. The generator of the cyclic group ($\omega,\,\omega^2,\,l)$ is
- (a) ω and ω^2
- (b) @ only
- (c) w² only

- (d) ω , ω^2 and ω^3
- 10. The inverse of the product of (1 2) and (2 3) of the symmetric group S_3 is
- (a) (1 2 3)
- (p) (1)
- (c) (1 3)
- (d) (1 3 2)

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Contd.

(SECTION : B-SHORT ANSWERS)

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(SECTION : C-DESCRIPTIVE)	(<i>Marks</i> : 50) Answer <i>five</i> questions, taking at least <i>one</i> from each Unit : 10×5=50	UNITI	a) Let A be the set of fourth root of unity, i.e., $A = \{1, -1, i, -i\}$. Then show that the algebraic structure (A, \times) is a group, where \times is an ordinary multiplication.	(b) Prove that the set $G = \{1, 2, 3, 4, 5, 6\}$ is a group with respect to multiplication modulo 7, using group table.	(a) Prove that the identity element in a group is unique. Also prove that the inverse of each element of a group is unique.	(b) If a and b are any two elements of a group G, then show that the equations $ax = b$ and $ya = b$ have unique solutions in G.	UNITII	Prove that the necessary and sufficient condition for a non-empty subset K of a group G to be a subgroup is that $r \in K$, $s \in K \Rightarrow rs^{-1} \in K$; where s^{-1} is the inverse of s in G.	(b) If H and K are two subgroups of a group G, then show that HK is a subgroup of G if and only if $HK = KH$.	(a) Prove that the order of every element of a finite group is finite and is less than or equal to the order of the group.	(b) State and prove Lagrange's theorem on the order of a group. 5	6 [Contd.
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UNIT-III

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- Show that if G is an Abelian group, then for any integer n such that (a) ٠ ف

$$(ab)^n = a^n b^n \forall a, \ b \in G$$

Prove that a group G is Abelian if and only if (q)

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UNIT-IV

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- 8. Prove that-
- for any set S, A(S) is a group with respect to composition mapping where A(S) denotes the set of all one to one mapping of S onto itself; (a)
- the normalizer of a group G is a subgroup of G. (q)

5+5=10

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