2024

(NEP–2020)

(2nd Semester)

MATHEMATICS (MAJOR/MINOR)

(Algebra)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : $1\times10=10$

1. If $f(x)$ and $g(x)$ are polynomials of degrees m and n respectively, then degree of $f(x)\cdot g(x)$ is

(a) $m\cdot n$ ( )

(b) m/n ( )

(c) $m+n$ ( )

(d) $m-n$ ( )

2. The value of k for which a polynomial $x^4 - x^3 + kx + 5$ is divisible by $(x+1)$ is

(a) 4 ( )

(b) 5 ( )

(c) 6 ( )

(d) 7 ( )

3. The polynomial $x^2 - x + 1$ is

(a) reducible over R ( )

(b) irreducible over Q ( )

(c) irreducible over C ( )

(d) None of the above ( )

4. The equation

$$x^5 + 2x^4 + 2x^3 + 4x^2 + x + 2 = 0$$

has

(a) no multiple root ( )

(b) one multiple root ( )

(c) two multiple roots ( )

(d) three multiple roots ( )

5. Every algebraic equation of odd degree has

(a) at least one real root ( )

(b) at least one complex root ( )

(c) at least one surd root ( )

(d) None of the above ( )

6. If the signs of the terms of an equation are all positive, then

(a) it has at least one positive root ( )

(b) it cannot have a positive root ( )

(c) it cannot have a negative root ( )

(d) it cannot have any real root ( )

7. The roots of the equation $x^3 - 5x^2 - 4x + 20 = 0$, given that two of its roots are equal in magnitude but are of opposite signs, are

(a) 4, 5 and –5 ( )

(b) –4, 20 and –20 ( )

(c) 5, 4 and –4 ( )

(d) 5, 2 and –2 ( )

8. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then $\alpha^2 + \beta^2 + \gamma^2$ equals

(a) $-p^2 + 2q$ ( )

(b) $p^2 + 2q$ ( )

(c) $p^2 - 2q$ ( )

(d) $-p^2 - 2q$ ( )

9. If $n = 5$ and $\theta = \frac{\pi}{10}$, then the value of $(\cos\theta + i\sin\theta)^n$ is

(a) i ( )

(b) $-i$ ( )

(c) 0 ( )

(d) 1 ( )

10. The expansion of $\sin n\theta$ in powers of $\cos\theta$ and $\sin\theta$, n being a positive integer, is

(a) $n\cos^{n-1}\theta\sin\theta - \frac{n(n-1)(n-2)}{3!}\cos^{n-3}\theta\sin^3\theta + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!}\cos^{n-5}\theta\sin^5\theta - \cdots$ ( )

(b) $n\cos^{n-1}\theta\sin\theta + \frac{n(n-1)(n-2)}{3!}\cos^{n-3}\theta\sin^3\theta + \frac{n(n-1)(n-2)(n-3)(n-4)}{5!}\cos^{n-5}\theta\sin^5\theta + \cdots$ ( )

(c) $n\cos^{n-1}\theta\sin\theta - \frac{n(n-1)(n-2)}{3!}\cos^{n-3}\theta\sin^3\theta - \frac{n(n-1)(n-2)(n-3)(n-4)}{5!}\cos^{n-5}\theta\sin^5\theta - \cdots$ ( )

(d) $n\cos^{n-1}\theta\sin\theta + \frac{n(n-1)(n-2)}{3!}\cos^{n-3}\theta\sin^3\theta - \frac{n(n-1)(n-2)(n-3)(n-4)}{5!}\cos^{n-5}\theta\sin^5\theta + \cdots$ ( )

(SECTION : B—SHORT ANSWERS)

(*Marks* : 15)

Answer *five* questions, taking at least *one* from each Unit : 3×5=15

UNIT—I

1. If a polynomial $f(x)$ of degree $n > 2$ is divided by $(x-\alpha)^2$, prove that the remainder is $(x-\alpha)f'(\alpha) + f(\alpha)$.

2. Show that $g(x) = 3x^4 + 15x^2 + 10$ is irreducible over the field of rational numbers.

Student's

UNIT—II

3. Show that 2 is a multiple root of $x^3 + x^2 - 16x + 20 = 0$ of multiplicity 2.

4. Solve the equation $x^4 - 10x^3 + 29x^2 - 22x + 4 = 0$, given that one root is $(2 + \sqrt{3})$.

UNIT—III

5. Prove that the equation $x^{10} - 4x^6 + x^4 - 2x - 3 = 0$ has at least four imaginary roots.

6. If the sum of two roots of the equation $x^3 + a_1x^2 + a_2x + a_3 = 0$ be zero, show that $a_1a_2 - a_3 = 0$.

UNIT—IV

7. Find the values of the expression $(1 + i)^{1/7}$ using De Moivre's theorem.

8. Expand $\cos 5\theta$ in powers of $\cos\theta$.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer *five* questions, taking at least *one* from each Unit : 10×5=50

UNIT—I

1. (a) State and prove the division algorithm for polynomials. 1+7=8

   (b) Find the quotient polynomial and the remainder when $x^3 + 3x^2 - 2x + 5$ is divided by $(x - 2)$. 2

2. (a) State and prove the remainder theorem. 1+2=3

   (b) If a polynomial $f(x)$ is divided by $(x - \alpha)(x - \beta)$, $\alpha \neq \beta$, prove that the remainder is

$$\frac{(x - \beta)f(\alpha) - (x - \alpha)f(\beta)}{(\alpha - \beta)}$$

   Using this result, find the remainder when $x^5 - 3x^4 + 4x^2 + x + 4$ is divided by $(x + 1)(x - 2)$. 5+2=7

UNIT—II

3. State the fundamental theorem of algebra. Using it, prove that every algebraic equation of n-th degree has n roots, real or imaginary, and no more. 1+9=10

4. *(a)* Show that if an equation $f(x)=0$, whose coefficients are all rational quantities, has a surd root of the form $(\alpha+\sqrt{\beta})$, then the conjugate surd $(\alpha-\sqrt{\beta})$ is also a root of the same equation. 5

   (b) Show that if an equation $f(x)=0$ whose coefficients are all real quantities, has a complex number of the form $(\alpha+i\beta)$ as one of its roots, then the conjugate complex number $(\alpha-i\beta)$ is also a root of the same equation. 5

UNIT—III

5. *(a)* Prove that the equation $x^3+x^2-5x-1=0$ has one positive root lying in $(1, 2)$ and two negative roots lying in $(-1, 0)$ and $(-3, -2)$. 4

   (b) Show that if the equation $x^3-ax^2+bx-c=0$ has a pair of roots of the form $\alpha(1 \pm i)$, where α is real and $i=\sqrt{-1}$, then $(a^2-2b)(b^2-2ac)=c^2$. 6

6. *(a)* If $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4+px^3+qx^2+rx+s=0$, show that $(1+\alpha^2)(1+\beta^2)(1+\gamma^2)(1+\delta^2)=(1-q+s)^2+(p-r)^2$. 5

   (b) If α, β, γ are the roots of the equation $2x^3+x^2+x+1=0$, find the equation whose roots are $\frac{1}{\beta^3}+\frac{1}{\gamma^3}-\frac{1}{\alpha^3}, \frac{1}{\gamma^3}+\frac{1}{\alpha^3}-\frac{1}{\beta^3}, \frac{1}{\alpha^3}+\frac{1}{\beta^3}-\frac{1}{\gamma^3}$. 5

UNIT—IV

7. State and prove De Moivre's theorem. 1+9=10

8. *(a)* Solve $x^3-30x+133=0$ by Cardan's method. 5

   (b) Solve the cubic equation $x^3-6x-9=0$ by Cardan's method. 5

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