Student's Copy

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(NEP–2020)

(1st Semester)

MATHEMATICS (MAJOR/MINOR)

(Calculus)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Put a Tick (✓) mark against the correct answer in the brackets provided : $1\times10=10$

1. The graph of the function $x(y-2)=1$

(a) crosses the line $y=2$ ( )

(b) crosses the Y-axis ( )

(c) crosses the X-axis at $-1/2$ ( )

(d) crosses the line $y-2=-x$ ( )

2. The gradient of the curve $y = 3x^2 - 2x + 5$ at $x = 0$ is

(a) 0 ( )

(b) 1 ( )

(c) −1 ( )

(d) −2 ( )

3. If $f(x) = [x]$ (the greatest integer function), then

(a) $\lim_{x \to 1} f(x)$ exists ( )

(b) $f(x)$ is continuous at $x = 2$ ( )

(c) $f(x)$ is continuous at $x = 2 \cdot 5$ ( )

(d) $f(x)$ is derivable at $x = 0$ ( )

4. Geometrical interpretation of Rolle's theorem says that there exists a point such that the slope of the tangent is 0 at that point which means the tangent to the curve at that point is

(a) parallel to x-axis ( )

(b) perpendicular to x-axis ( )

(c) Neither parallel nor perpendicular to x-axis ( )

(d) None of the above ( )

5. If a and b are two roots of the equation $f(x)=0$, then $f'(x)=0$ will have at least one root between a and b, provided

(a) $f(x)$ is derivable in $[a, b]$ ( )

(b) $f(x)$ is continuous in $[a, b]$ ( )

(c) $f'(x)$ does not exist in (a, b) ( )

(d) None of the above ( )

6. If $f(x)=\int_{x}^{x^2} e^t dt$, then $\frac{df(x)}{dx}$ is equal to

(a) $2x^2e^{x^2}-e^{x^2}$ ( )

(b) $2xe^{x^2}-e^x$ ( )

(c) $2x^2e^{x^2}+e^{x^2}$ ( )

(d) None of the above ( )

7. If $f(2a-x)=-f(x)$, then the value of $\int_0^{2a} f(x)dx$ is equal to

(a) $2\int_0^a f(x)dx$ ( )

(b) $\int_0^a f(x)dx$ ( )

(c) 2 ( )

(d) 0 ( )

8. If $u = f(y-z, z-x, x-y)$, then the value of $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ is

(a) 0 ( )

(b) 3 ( )

(c) $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}$ ( )

(d) None of the above ( )

9. Let $f(x, y) = \frac{x-y}{x^2+y^2}$. Then $\lim f(x, y)$ as $(x, y) \to (1, 2)$ along the line $y = 2$ is

(a) $-\frac{1}{5}$ ( )

(b) $\frac{3}{5}$ ( )

(c) $\frac{1}{5}$ ( )

(d) Does not exist ( )

10. The sequence $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if

(a) $p < 1$ ( )

(b) $p = 1$ ( )

(c) $p > 1$ ( )

(d) For any value of p ( )

(SECTION : B—SHORT ANSWERS)

(*Marks* : 15)

Answer *five* questions, taking at least *one* from each Unit :

3×5=15

UNIT—I

1. Evaluate the limit $\lim_{x \to \infty} (x + e^x)^{2/x}$.

2. Show that a function which is derivable at a point is necessarily continuous at that point.

UNIT—II

3. Expand e^x in the power of $(x+3)$.

4. Verify Rolle's theorem for the function $f(x) = x\sqrt{a^2 - x^2}$ in $[0, a]$.

UNIT—III

5. Evaluate the integral $\int e^x \frac{x-1}{(x+1)^3} dx$.

6. Using the reduction formula for $\int_0^{\pi/2} \cos^n x \, dx$, evaluate $\int_0^{\pi/2} \cos^4 x \sin^2 x \, dx$.

UNIT—IV

7. Show that the sequence $\left\{ \left(\frac{(3n)!}{(n!)^3} \right)^{1/n} \right\}$ converges and find its limit.

8. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Test the continuity of f at the origin.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer *five* questions, taking at least *one* from each Unit : $10\times5=50$

UNIT—I

1. *(a)* Draw the graph of the function f defined by

$$f(x)=\begin{cases}\dfrac{|x|-x}{2}, & -2\le x<0\\ x\ \ , & 0\le x<1\\ 2x\ \ , & 1\le x<2\end{cases}$$

From the graph, can you conclude that $f(x)$ is continuous in the interval $[-2, 2]$? $5+1=6$

(b) The relation between volume (V) and presssure (P) of a gas is given by $V=\dfrac{200}{P}$. Find the average rate of change of volume with respect to pressure when P increases from 30 to 35. Also find the instantaneous rate of change of volume at $P=30$. 4

2. *(a)* Use ε-δ definition of continuity to prove that $y=\sin x$ is continuous at every value of x. 4

(b) If $y=(x+\sqrt{1+x^2})^m$, then prove that

$$(1+x^2)y_{n+2}+(2n+1)xy_{n+1}+(n^2-m^2)y_n=0$$

Also find $y_n(0)$. $4+2=6$

UNIT—II

3. *(a)* State and prove Cauchy's mean value theorem. 5

(b) Expand $\cos x$ in a finite series in power of x, with remainder in Cauchy's form. 5

4. *(a)* State and prove Lagrange's mean value theorem. 5

(b) Expand $(1+x)^m$ in a finite series with remainder in Cauchy's form. 5

UNIT—III

5. *(a)* Evaluate $\int_1^2 \frac{1}{x^2}\,dx$ using definite integral as limit of a sum. 5

(b) Obtain a reduction formula for $\int \sec^n x\,dx$ and hence find the value of

$$\int \sec^6 x\,dx$$

5

6. *(a)* Obtain the reduction formula

$$\int_0^{\pi/2} \sin^n x dx = \begin{cases} \frac{(n-1)(n-3)(n-5)\ldots 3\cdot 1}{n(n-2)(n-4)\ldots 4\cdot 2}\cdot\frac{\pi}{2}, & n \text{ is even} \\ \frac{(n-1)(n-3)(n-5)\ldots 4\cdot 2}{n(n-2)(n-4)\ldots 3\cdot 1}\times 1, & n \text{ is odd} \end{cases}$$

6

(b) Show that $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\,dx = \frac{\pi^2}{4}$. 4

UNIT—IV

7. *(a)* State and prove Cauchy's general principle of convergence for a sequence. 5

(b) If $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$, $x^2+y^2+z^2 \neq 0$, then prove that $u(x, y, z)$ is a harmonic function. 5

8. *(a)* State and prove Euler's theorem on a homogeneous function for three variables. 5

(b) Test the convergence or divergence of the series

$$\sum_{n=0}^{\infty} (\sqrt[3]{n^3+1} - n)$$

5

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2024

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