MATH100 (MAJOR) **Student's Copy**

2024

(NEP–2020)

(1st Semester)

MATHEMATICS (MAJOR)

(Vector Analysis)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. For any two vectors $\vec{u}$ and $\vec{v}$, if $\vec{v} = \frac{d\vec{u}}{dt}$, then $\frac{d}{dt}\left(\vec{u} \times \frac{d\vec{u}}{dt}\right)$ is equal to

   (a) 0 ( )

   (b) $\frac{d\vec{u}}{dt} \times \frac{d\vec{u}}{dt}$ ( )

   (c) $\vec{u} \times \frac{d\vec{u}}{dt}$ ( )

   (d) $\vec{u} \times \frac{d^2\vec{u}}{dt^2}$ ( )

2. The derivative of any constant vector c is

(a) 2 ( )

(b) c ( )

(c) 0 ( )

(d) 1 ( )

3. If $\vec{r} = \sin t\hat{i} + \cos t\,\hat{j} + t\hat{k}$, then the value of the acceleration is

(a) $\cos t\hat{i} - \sin t\,\hat{j} + \hat{k}$ ( )

(b) $-\sin t\hat{i} - \cos t\,\hat{j} + \hat{k}$ ( )

(c) $-\sin t\hat{i} - \cos t\,\hat{j}$ ( )

(d) $\sin t\hat{i} + \cos t\,\hat{j}$ ( )

4. If $\vec{a}$ is a constant vector, then grad $(\vec{a} \cdot \vec{r})$ is equal to

(a) $\vec{a} \cdot \vec{r}$ ( )

(b) $\vec{r}$ ( )

(c) $\vec{a}$ ( )

(d) 0 ( )

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(b) $\vec{r}$ ( )

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(d) 0 ( )

5. If $\phi(x, y, z) = x^2 + y^2 + z^2$, then the value of grad ϕ at the point (1, 2, 3) is

(a) $2\hat{i} + 3\hat{j} + 6\hat{k}$ ( )

(b) $2\hat{i} + 4\hat{j} + 6\hat{k}$ ( )

(c) $3\hat{i} + 4\hat{j} + 5\hat{k}$ ( )

(d) $3\hat{i} + 2\hat{j} + 5\hat{k}$ ( )

6. If $\vec{F} = (5xy - 6x^2)\hat{i} + (2y - 4x)\hat{j}$, then the value of $\int_C \vec{F}\, d\vec{r}$ along the curve C in the xy-plane, $y = x^3$ from the point (1, 1) to (2, 8), is

(a) 25 ( )

(b) 15 ( )

(c) 45 ( )

(d) 35 ( )

7. The circulation of $\vec{F}$ around the curve C, where $\vec{F} = y\hat{i} + z\hat{j} + x\hat{k}$ and C is the circle $x^2 + y^2 = 1$, $z = 0$, is

(a) π ( )

(b) $-\pi$ ( )

(c) 2π ( )

(d) -2π ( )

8. If V is the closed region bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $2x + 2y + z = 4$, then the value of $\iiint_V \nabla \cdot \vec{F}\, d\vec{V}$ is

(a) $\frac{8}{3}$ ( )

(b) $\frac{3}{8}$ ( )

(c) $-\frac{8}{3}$ ( )

(d) $\frac{5}{3}$ ( )

9. If $\vec{F}$ is a continuously differentiable vector point function in a region V and S is a closed surface enclosing V, then $\int \vec{F} \cdot \hat{n}\, d\vec{S}$ is equal to

(a) $\int \text{div} \times \vec{F}\, d\vec{V}$ ( )

(b) $\int \text{div} \cdot \vec{F}\, d\vec{V}$ ( )

(c) $\int \text{curl} \times \vec{F}\, d\vec{V}$ ( )

(d) $\int \text{curl} \cdot \vec{F}\, d\vec{V}$ ( )

10. If C is a simple closed curve in the xy-plane not enclosing the origin, then the value of $\int_C \vec{F}\, d\vec{r}$, where $\vec{F} = \dfrac{-y\hat{i} + x\hat{j}}{x^2 + y^2}$, is

(a) $x^2 + y^2$ ( )

(b) $-y\hat{i} + x\hat{j}$ ( )

(c) 0 ( )

(d) $\dfrac{-y\hat{i} + x\hat{j}}{x^2 + y^2}$ ( )

(SECTION : B—SHORT ANSWERS)

(*Marks* : 15)

Answer *five* questions, taking at least *one* from each Unit : $3 \times 5 = 15$

UNIT—I

1. Prove that a necessary and sufficient condition that a proper vector $\vec{u}$ has a constant length is $\vec{u} \cdot \frac{d\vec{u}}{dt} = 0$.

2. A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = -t - 5$, where t is the time. Find the components of its velocity and acceleration at time $t = 1$ in the direction of $\hat{i} - 2\hat{j} + 2\hat{k}$.

UNIT—II

3. Show that $\nabla^2 \left(\frac{1}{r} \right) = 0$.

4. Find the directional derivative of $\phi(x, y, z) = 4xz^3 - 3x^2y^2z$ at $(2, -1, 2)$ in the direction of $2\hat{i} - 3\hat{j} + 6\hat{k}$.

UNIT—III

5. If $\vec{A} = (2y + 3)\hat{i} + xy\hat{j} + (yz - x)\hat{k}$, evaluate $\int_C \vec{A} d\vec{r}$, along the paths C, $x = 2t^2$, $y = t$, $z = t^3$ from $t = 0$ to $t = 1$.

6. Evaluate $\iint_S \vec{A} \cdot \vec{n} \, d\vec{S}$, where $\vec{A} = z\hat{i} + x\hat{j} - 3y^2z\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$.

UNIT—IV

7. Evaluate by Stokes' theorem

$$\int_C (e^x dx + 2y\, dy - dz)$$

where C is the curve $x^2 + y^2 = 4,\ z = 2$.

8. If S is any closed surface enclosing a volume V and $\vec{F} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$, show that $\int_S \vec{F} \cdot \vec{n}\, d\vec{S} = 6V$.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer *five* questions, taking at least *one* from each Unit : 10×5=50

UNIT—I

1. *(a)* If $\vec{w}$ is a constant vector, $\vec{r}$ and $\vec{s}$ are vector functions of a scalar variable t and if $\frac{d\vec{r}}{dt} = \vec{w} \times \vec{r}$, $\frac{d\vec{s}}{dt} = \vec{w} \times \vec{s}$, then show that

$$\frac{d}{dt}(\vec{r} \times \vec{s}) = \vec{w} \times (\vec{r} \times \vec{s})$$ 4

(b) Find $\vec{T}$, $\vec{N}$, k for the plane curve $\vec{r}(t) = t\hat{i} + (\log \cos t)\,\hat{j}$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$. 3

(c) Find the unit tangent vector and arc length of the curve

$$\vec{r}(t) = 2\cos t\,\hat{i} + 2\sin t\,\hat{j} + \sqrt{5}\, t\,\hat{k},\ 0 \le t \le \pi$$ 3

2. *(a)* If $\vec{r}(t) = a\cos t\,\hat{i} + a\sin t\,\hat{j} + at\tan\alpha\,\hat{k}$, then find the values of

$$\left|\frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2}\right| \text{ and } \frac{d\vec{r}}{dt} \cdot \left(\frac{d^2\vec{r}}{dt^2} \times \frac{d^3\vec{r}}{dt^3}\right)$$ 5

(b) Suppose $\phi(x, y, z) = xy^2z$ and $\vec{A} = xz\hat{i} - xy^2\hat{j} + yz^2\hat{k}$. Find $\frac{\partial^3}{\partial x^2 \partial z}(\phi\vec{A})$ at the point $(2, -1, 1)$. 5

UNIT—II

3. (a) Show that $\nabla^2 r^n = n(n+1)r^{n-2}$, where n is constant. 6

(b) Let $\vec{A} = (6xy + z^3, 3x^2 - z, 3xz^2 - y)$. Show that $\vec{A}$ is irrotational and that $\vec{A} = \text{grad}\,\phi$, for some scalar point function ϕ. Find ϕ. 4

4. (a) Find $(\vec{A} \times \nabla) \times \vec{B}$ at the point $(1, -1, 2)$, if $\vec{A} = xz^2\hat{i} + 2y\hat{j} - 3xz\hat{k}$ and $\vec{B} = 3xz\hat{i} + 2yz\hat{j} - z^2\hat{k}$. 4

(b) If $\vec{\omega}$ is a constant vector and $\vec{v} = \vec{\omega} \times \vec{r}$, then prove that $\vec{\omega} = \frac{1}{2}\text{curl}\,\vec{v}$. 3

(c) Prove that $\text{div}(r^n\vec{r}) = (n+3)r^n$. 3

UNIT—III

5. (a) Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = xy\,\hat{i} + (x^2 + y^2)\hat{j}$ and C is the x-axis from $x = 2$ to $x = 4$ and the line $x = 4$ from $y = 0$ to $y = 12$. 5

(b) Evaluate $\iiint_V \vec{F} \cdot dV$, where $\vec{F} = 2xz\hat{i} - x\hat{j} + y^2\hat{k}$ and V is the region bounded by the surfaces $x = 0$, $y = 0$, $z = x^2$ and $z = 4$. 5

6. (a) Evaluate $\int_C \{yz\,dx + (xz+1)\,dy + xy\,dz\}$, where C is any path from $(1, 0, 0)$ to $(2, 1, 4)$. 5

(b) Evaluate $\iint_S \vec{F} \cdot \vec{n} \cdot dS$, where $\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ and S is that part of the surface of the sphere $x^2 + y^2 + z^2 = 1$ which lies in the first octant. 5

UNIT—IV

7. *(a)* Verify Stokes' theorem for $\vec{A} = (y - z + 2)\hat{i} + (yz + 4)\hat{j} - xz\hat{k}$, where S is the surface of the cube $x = 0$, $y = 0$, $z = 0$ and $x = 2$, $y = 2$, $z = 2$ above the xy-plane. 5

(b) Show that

$$\int_S (ax\hat{i} + by\hat{j} + cz\hat{k}) \cdot \vec{n}\, dS = \frac{4}{3}\pi (a + b + c)$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$. 5

8. *(a)* State Stokes' theorem and hence apply for $\vec{F} = y\hat{i} + z\hat{j} + x\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. 5

(b) Evaluate by Green's theorem

$$\int_C (x^2 - \cosh y)\, dx + (y + \sin x)\, dy$$

where C is the rectangle with vertices $(0, 0)$, $(\pi, 0)$, $(\pi, 1)$, $(0, 1)$. 5

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