Student's Copy

2 0 2 4

(CBCS)

(6th Semester)

PHYSICS

NINTH PAPER

(Quantum Mechanics)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. In the interaction of light with matter, light behaves

(*a*) as a wave ( )

(*b*) as a wave and as a particle ( )

(*c*) as a particle ( )

(*d*) neither as wave nor as particle ( )

2. According to Wien's displacement law, the intensity maximum of blackbody radiation shifts towards _____ with increase in temperature of the blackbody.

(a) shorter wavelength ( )

(b) longer wavelength ( )

(c) lower frequency ( )

(d) Intensity is independent of temperature ( )

3. A stationary state is that for which the probability of finding the particle at a point in space is

(a) a function of time (t) ( )

(b) a function of position (x) ( )

(c) a function of both x and t ( )

(d) independent of both x and t ( )

4. The wave function $\psi(\vec{r}, t)$ is said to be normalized, if

(a) $\int |\psi(\vec{r}, t)| \, d\tau = 1$ ( )

(b) $\int |\psi(\vec{r}, t)|^2 \, d\tau = 1$ ( )

(c) $\int |\psi(\vec{r}, t)|^2 \, d\tau = 1/N^2$ ( )

(d) $\int |\psi^*(\vec{r}, t)| \, d\tau = 1$ ( )

5. If $\tilde{A}$ is a linear operator and C is a constant, then

(a) $\tilde{A}C = C\tilde{A}$ ( )

(b) $\tilde{A}C = \tilde{A}$ ( )

(c) $\tilde{A} = \tilde{A} + C$ ( )

(d) $\tilde{A}^2 = C\tilde{A}$ ( )

6. The value of $\left[\hat{x}, \frac{\hat{\partial}}{\partial x}\right]$ is

(a) 0 ( )

(b) 1 ( )

(c) -1 ( )

(d) ∞ ( )

7. Which of the following is correct?

(a) $[\tilde{\sigma}^2, \tilde{\sigma}_x] = 2i\sigma_z$ ( )

(b) $[\tilde{\sigma}^2, \tilde{\sigma}_x] \neq 0$ ( )

(c) $[\tilde{\sigma}^2, \tilde{\sigma}_x] = 0$ ( )

(d) $[\tilde{\sigma}^2, \tilde{\sigma}_x] = \sigma_z$ ( )

8. The eigenvalue of L^2 is

(a) $l^2\hbar^2$ ( )

(b) $l(l+1)\hbar^2$ ( )

(c) $(l+1)\hbar^2$ ( )

(d) $(l+1)^2\hbar^2$ ( )

9. If S be a linear vector space, which of the following is not true?

(a) If $|A\rangle \in S$, then $c|A\rangle \in S$, where c is a complex number ( )

(b) If $|A\rangle$ and $|B\rangle$ belong to S, then $(|A\rangle + |B\rangle) \in S$ ( )

(c) For any $|A\rangle \in S$, there exists another element $|A'\rangle \in S$, such that $|A\rangle + |A'\rangle \neq |0\rangle$ ( )

(d) If $|A\rangle \in S$, $|A\rangle + |0\rangle = |A\rangle$, where $|0\rangle$ is a null element ( )

10. $|A\rangle$ is said to be normalized, if

(a) $|\langle A|A\rangle|^{1/2} = 1$ ( )

(b) $|\langle A|A\rangle|^{1/2} = 0$ ( )

(c) $\langle A|A\rangle = 1$ ( )

(d) $\langle A|A\rangle = 0$ ( )

(SECTION : B—SHORT ANSWERS)

(*Marks* : 15)

Answer the following : 3×5=15

UNIT—I

1. Show that the maximum change in the wavelength of incident X-ray photon occurs when photon gets scattered in the direction of incidence.

OR

2. Show that the group velocity v_g is equal to the particle velocity v.

UNIT—II

3. If $\psi_1(x, t)$ and $\psi_2(x, t)$ are two solutions of Schrödinger wave equation for a given potential $V(x, t)$, then show that $\psi = a_1\psi_1 + a_2\psi_2$ is also a solution of the Schrödinger wave equation.

OR

4. For the wave function $\psi(x) = \sqrt{a}\, e^{-ax}$, calculate the probability of finding the particle within $x = 1/a$ and $x = 2/a$.

UNIT—III

5. For the step potential

$$V(x) = \begin{cases} 0, & \text{for } x < 0 \\ V_0, & \text{for } x > 0 \end{cases}$$

show that wave function vanishes in the region $x > 0$ if $V_0 \to \infty$.

OR

6. Calculate the transmission probability if electrons of energy 2 eV are incident on a barrier of 3 eV height and 0·4 nm width.

UNIT—IV

7. The ground-state of a harmonic oscillator is given by $\psi(x) = Ae^{-\alpha^2 x^2}$, where $\alpha^2 = \frac{k}{2\hbar\omega}$ and k is the force constant. Show that the mean potential energy $\langle V \rangle = \frac{1}{4}\hbar\omega$.

OR

8. Calculate the radial probability function for the ground-state of hydrogen atom.

UNIT—V

9. Write down the addition and multiplication conditions to be satisfied by a vector space.

OR

10. Describe the Gram-Schmidt orthogonalization process.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer the following : 10×5=50

UNIT—I

1. *(a)* Explain photoelectric effect. Derive the Einstein's photoelectric equation. Explain the characteristics of photoelectric phenomena based on Einstein's equation. 2+2+3=7

   (b) Explain why Compton effect is not observed with visible light. 3

OR

2. *(a)* Describe Davisson and Germer's experiment and interpret its results. 7

   (b) On the basis of uncertainty principle, explain why the wave and particle aspect of a physical entity are not observed simultaneously. 3

UNIT—II

3. (a) Derive the one-dimensional Schrödinger's time-dependent wave equation for a particle of mass m moving under the action of a conservative force field derivable from $V(x)$, hence deduce Schrödinger's time-independent wave equation. 4+2=6

(b) Discuss the physical interpretation of the wave function ψ. Explain Born's interpretation of the wave function. 4

OR

4. (a) A particle travelling with energy E is incident on a potential barrier of height V_0 and width a. Show that there is a finite probability of transmission even if $E < V_0$. 6

(b) Define probability current density. Derive the equation of continuity for probability current density. 4

UNIT—III

5. (a) What are Hermitian operators? Show that $\hat{p}_x$ is Hermitian. 1+2=3

(b) The ground-state wave function for the hydrogen atom is

$$\psi = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

Show that the average distance of the electron from the nucleus is $1 \cdot 5 a_0$. 4

(c) Show that $\langle p^2 \rangle = \left(\frac{\pi\hbar}{L}\right)^2$ for the wave function

$$\psi(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin\frac{\pi x}{L}, & 0 < x < L \\ 0, & |x| > L \end{cases}$$

3

OR

6. (a) Derive the expressions for normalized wave function and total linear momentum for a free particle of mass m moving in a rectangular potential box with sides a, b and c parallel to x, y and z axes respectively. 5+2=7

(b) Show that $[x^n, \hat{p}_x] = i\hbar n x^{n-1}$, where n is a positive integer. 3

UNIT—IV

7. *(a)* Derive the commutation relation for the components of angular momentum L_x, L_y, L_z and show that all the three components commute with $L^2 = L_x^2 + L_y^2 + L_z^2$ 4+4=8

(b) Find the eigenvalues of Pauli spin matrix $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. 2

OR

8. *(a)* Describe Stern-Gerlach experiment with a suitable schematic diagram. Discuss how the experiment demonstrated the existence of two spin states of the electron. 4+2=6

(b) Show that the magnetic dipole moment of an electron due to its orbital motion is quantized. Hence define Bohr magneton. 3+1=4

UNIT—V

9. *(a)* Apply Gram-Schmidt process to derive orthogonal basis vectors (v_1, v_2, v_3) from the basis vectors $u_1 = (1, 1, 1)$, $u_2 = (-1, 1, 0)$ and $u_3 = (1, 2, 1)$. 4

(b) For the states $|\psi\rangle = 3i|v_1\rangle - 7i|v_2\rangle$ and $|\phi\rangle = -|v_1\rangle + 2i|v_2\rangle$ where $|v_1\rangle$ and $|v_2\rangle$ are orthonormal bases.

(i) Evaluate $|\psi + \phi\rangle$ and $\langle \psi + \phi|$. 2+2=4

(ii) Show that $\langle \psi|\phi\rangle = \langle \phi|\psi\rangle^*$. 2

OR

10. *(a)* Write down the procedure to construct orthonormal basis set from non-orthonormal vectors $(v_1, v_2, v_3, \cdots, v_n)$. 4

(b) Construct orthonormal basis set from the given vectors $u_1 = (1, 2, 3)$, $u_2 = (3, -1, 1)$ and $u_3 = (1, 1, -2)$. 6

***

PHY/VI/CC/16 **Student's Copy**

2024

(CBCS)

(6th Semester)

PHYSICS

NINTH PAPER

(Quantum Mechanics)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : $1\times10=10$

1. In the interaction of light with matter, light behaves

(*a*) as a wave ( )

(*b*) as a wave and as a particle ( )

(*c*) as a particle ( )

(*d*) neither as wave nor as particle ( )

2. According to Wien's displacement law, the intensity maximum of blackbody radiation shifts towards ____ with increase in temperature of the blackbody.

(a) shorter wavelength ( )

(b) longer wavelength ( )

(c) lower frequency ( )

(d) Intensity is independent of temperature ( )

3. A stationary state is that for which the probability of finding the particle at a point in space is

(a) a function of time (t) ( )

(b) a function of position (x) ( )

(c) a function of both x and t ( )

(d) independent of both x and t ( )

4. The wave function $\psi(\vec{r}, t)$ is said to be normalized, if

(a) $\int |\psi(\vec{r}, t)| \, d\tau = 1$ ( )

(b) $\int |\psi(\vec{r}, t)|^2 \, d\tau = 1$ ( )

(c) $\int |\psi(\vec{r}, t)|^2 \, d\tau = 1 / N^2$ ( )

(d) $\int |\psi^*(\vec{r}, t)| \, d\tau = 1$ ( )

5. If $\tilde{A}$ is a linear operator and C is a constant, then

(a) $\tilde{A}C = C\tilde{A}$ ( )

(b) $\tilde{A}C = \tilde{A}$ ( )

(c) $\tilde{A} = \tilde{A} + C$ ( )

(d) $\tilde{A}^2 = C\tilde{A}$ ( )

6. The value of $\left[\hat{x}, \frac{\hat{\partial}}{\partial x}\right]$ is

(a) 0 ( )

(b) 1 ( )

(c) −1 ( )

(d) ∞ ( )

7. Which of the following is correct?

(a) $[\tilde{\sigma}^2, \tilde{\sigma}_x] = 2i\sigma_z$ ( )

(b) $[\tilde{\sigma}^2, \tilde{\sigma}_x] \neq 0$ ( )

(c) $[\tilde{\sigma}^2, \tilde{\sigma}_x] = 0$ ( )

(d) $[\tilde{\sigma}^2, \tilde{\sigma}_x] = \sigma_z$ ( )

8. The eigenvalue of L^2 is

(a) $l^2\hbar^2$ ( )

(b) $l(l+1)\hbar^2$ ( )

(c) $(l+1)\hbar^2$ ( )

(d) $(l+1)^2\hbar^2$ ( )

9. If S be a linear vector space, which of the following is not true?

(a) If $|A\rangle \in S$, then $c|A\rangle \in S$, where c is a complex number ( )

(b) If $|A\rangle$ and $|B\rangle$ belong to S, then $(|A\rangle + |B\rangle) \in S$ ( )

(c) For any $|A\rangle \in S$, there exists another element $|A'\rangle \in S$, such that $|A\rangle + |A'\rangle \neq |0\rangle$ ( )

(d) If $|A\rangle \in S$, $|A\rangle + |0\rangle = |A\rangle$, where $|0\rangle$ is a null element ( )

10. $|A\rangle$ is said to be normalized, if

(a) $|\langle A|A\rangle|^{1/2} = 1$ ( )

(b) $|\langle A|A\rangle|^{1/2} = 0$ ( )

(c) $\langle A|A\rangle = 1$ ( )

(d) $\langle A|A\rangle = 0$ ( )

(SECTION : B—SHORT ANSWERS)

(*Marks* : 15)

Answer the following : $3 \times 5 = 15$

UNIT—I

1. Show that the maximum change in the wavelength of incident X-ray photon occurs when photon gets scattered in the direction of incidence.

OR

2. Show that the group velocity v_g is equal to the particle velocity v.

UNIT—II

3. If $\psi_1(x, t)$ and $\psi_2(x, t)$ are two solutions of Schrödinger wave equation for a given potential $V(x, t)$, then show that $\psi = a_1\psi_1 + a_2\psi_2$ is also a solution of the Schrödinger wave equation.

OR

4. For the wave function $\psi(x) = \sqrt{a}\, e^{-ax}$, calculate the probability of finding the particle within $x = 1/a$ and $x = 2/a$.

UNIT—III

5. For the step potential

$$V(x) = \begin{cases} 0, & \text{for} \quad x < 0 \\ V_0, & \text{for} \quad x > 0 \end{cases}$$

show that wave function vanishes in the region $x > 0$ if $V_0 \to \infty$.

OR

6. Calculate the transmission probability if electrons of energy 2 eV are incident on a barrier of 3 eV height and 0·4 nm width.

UNIT—IV

7. The ground-state of a harmonic oscillator is given by $\psi(x) = Ae^{-\alpha^2 x^2}$, where $\alpha^2 = \frac{k}{2\hbar\omega}$ and k is the force constant. Show that the mean potential energy $\langle V \rangle = \frac{1}{4}\hbar\omega$.

OR

8. Calculate the radial probability function for the ground-state of hydrogen atom.

UNIT—V

9. Write down the addition and multiplication conditions to be satisfied by a vector space.

OR

10. Describe the Gram-Schmidt orthogonalization process.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer the following : 10×5=50

UNIT—I

1. *(a)* Explain photoelectric effect. Derive the Einstein's photoelectric equation. Explain the characteristics of photoelectric phenomena based on Einstein's equation. 2+2+3=7

(b) Explain why Compton effect is not observed with visible light. 3

OR

2. *(a)* Describe Davisson and Germer's experiment and interpret its results. 7

(b) On the basis of uncertainty principle, explain why the wave and particle aspect of a physical entity are not observed simultaneously. 3

UNIT—II

3. *(a)* Derive the one-dimensional Schrödinger's time-dependent wave equation for a particle of mass m moving under the action of a conservative force field derivable from $V(x)$, hence deduce Schrödinger's time-independent wave equation. 4+2=6

(b) Discuss the physical interpretation of the wave function ψ. Explain Born's interpretation of the wave function. 4

OR

4. *(a)* A particle travelling with energy E is incident on a potential barrier of height V_0 and width a. Show that there is a finite probability of transmission even if $E < V_0$. 6

(b) Define probability current density. Derive the equation of continuity for probability current density. 4

UNIT—III

5. *(a)* What are Hermitian operators? Show that $\hat{p}_x$ is Hermitian. 1+2=3

(b) The ground-state wave function for the hydrogen atom is

$$\psi = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

Show that the average distance of the electron from the nucleus is $1 \cdot 5 a_0$. 4

(c) Show that $\langle p^2 \rangle = \left(\frac{\pi\hbar}{L}\right)^2$ for the wave function

$$\psi(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin\frac{\pi x}{L}, & 0 < x < L \\ 0, & |x| > L \end{cases}$$

3

OR

6. *(a)* Derive the expressions for normalized wave function and total linear momentum for a free particle of mass m moving in a rectangular potential box with sides a, b and c parallel to x, y and z axes respectively. 5+2=7

(b) Show that $[x^n, \hat{p}_x] = i\hbar n x^{n-1}$, where n is a positive integer. 3

UNIT—IV

7. (a) Derive the commutation relation for the components of angular momentum L_x, L_y, L_z and show that all the three components commute with $L^2 = L_x^2 + L_y^2 + L_z^2$ 4+4=8

(b) Find the eigenvalues of Pauli spin matrix $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. 2

OR

8. (a) Describe Stern-Gerlach experiment with a suitable schematic diagram. Discuss how the experiment demonstrated the existence of two spin states of the electron. 4+2=6

(b) Show that the magnetic dipole moment of an electron due to its orbital motion is quantized. Hence define Bohr magneton. 3+1=4

UNIT—V

9. (a) Apply Gram-Schmidt process to derive orthogonal basis vectors (v_1, v_2, v_3) from the basis vectors $u_1 = (1, 1, 1)$, $u_2 = (-1, 1, 0)$ and $u_3 = (1, 2, 1)$. 4

(b) For the states $|\psi\rangle = 3i|v_1\rangle - 7i|v_2\rangle$ and $|\phi\rangle = -|v_1\rangle + 2i|v_2\rangle$ where $|v_1\rangle$ and $|v_2\rangle$ are orthonormal bases.

(i) Evaluate $|\psi + \phi\rangle$ and $\langle\psi + \phi|$. 2+2=4

(ii) Show that $\langle\psi|\phi\rangle = \langle\phi|\psi\rangle^*$. 2

OR

10. (a) Write down the procedure to construct orthonormal basis set from non-orthonormal vectors $(v_1, v_2, v_3, \cdots, v_n)$. 4

(b) Construct orthonormal basis set from the given vectors $u_1 = (1, 2, 3)$, $u_2 = (3, -1, 1)$ and $u_3 = (1, 1, -2)$. 6

***