2024

(CBCS)

(2nd Semester)

PHYSICS

SECOND PAPER

(Thermodynamics and Mathematical Physics—I)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. Energy associated with 1 gram molecule of a diatomic gas is

(a) $U = \frac{7}{2}RT$ ( )

(b) $U = \frac{3}{2}RT$ ( )

(c) $U = \frac{5}{2}RT$ ( )

(d) $U = \frac{9}{2}RT$ ( )

2. The density of hydrogen at NTP is 0·000089 g/cc. Then r.m.s. velocity of hydrogen is

(a) $4 \cdot 6 \times 10^2$ m/s ( )

(b) $1 \cdot 84 \times 10^3$ m/s ( )

(c) $1 \cdot 6 \times 10^3$ m/s ( )

(d) $4 \cdot 85 \times 10^2$ m/s ( )

3. In adiabatic process, change in entropy is

(a) zero ( )

(b) unity ( )

(c) infinite ( )

(d) unpredictable ( )

4. If dQ is the heat given to thermodynamic system and dU is its change in internal energy and dW is the work done by the system, then according to the first law of thermodynamics

(a) $dQ = dW$ ( )

(b) $dQ = dU - dW$ ( )

(c) $dQ = dU + dW$ ( )

(d) $dQ = dU$ ( )

5. Unit vectors in orthogonal curvilinear coordinate system

(a) are parallel to each other ( )

(b) are perpendicular to each other ( )

(c) can be at any arbitrary angles ( )

(d) have no definite relations ( )

6. In four dimensions, the value of δ_{ii} is

(a) 1 ( )

(b) 2 ( )

(c) 3 ( )

(d) 4 ( )

7. The rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

is

(a) 3 ( )

(b) 2 ( )

(c) 1 ( )

(d) 0 ( )

8. The adjoint of a symmetric matrix is

(a) a diagonal matrix ( )

(b) a scalar matrix ( )

(c) a symmetric matrix ( )

(d) an identity matrix ( )

9. The value of $\beta(1, 2)$ is

(a) $\frac{1}{2}$ ( )

(b) $\sqrt{\pi}$ ( )

(c) $\frac{2}{3}$ ( )

(d) $\frac{1}{2}\sqrt{\pi}$ ( )

10. The value of $\Gamma\left(-\frac{3}{2}\right)$ is

(a) $\frac{4}{3}\sqrt{\pi}$ ( )

(b) $-\frac{4}{3}\sqrt{\pi}$ ( )

(c) $2\sqrt{\pi}$ ( )

(d) $-2\sqrt{\pi}$ ( )

(SECTION : B—SHORT ANSWERS)

(*Marks* : 15)

Answer the following : 3×5=15

UNIT—I

1. Explain kinetic interpretation of temperature.

OR

2. Show that for diatomic gas, the value of $\gamma = \frac{C_P}{C_V} = 1 \cdot 4$, where the symbols have their usual meanings.

UNIT—II

3. State and explain the second law of thermodynamics.

OR

4. "It is not possible to attain absolute zero." Explain in brief.

UNIT—III

5. Find the grad of $r^2 = x^2 + y^2 + z^2$.

OR

6. State the Gauss' divergence theorem. Express it in mathematical form and define all the terms in the expression.

UNIT—IV

7. What do you mean by row, column and symmetric matrices? Give one example each.

OR

8. What do you mean by Hermitian and skew-Hermitian matrices? If H is a Hermitian matrix, show that e^{iH} is a unitary matrix.

UNIT—V

9. Prove that $\Gamma(-n) = \infty$, where n is a positive number.

OR

10. For a β-function, show that $\beta(m, n) = \beta(n, m)$.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer the following : 10×5=50

UNIT—I

1. *(a)* Calculate van der Waals' constants for dry air, given that $T_C = 132$ K, $P_C = 37 \cdot 2$ atm, R per mole $= 82 \cdot 07$ cm^3 atm K^{-1}. 3

(b) Derive the expression for critical constants in terms of van der Waals' constants. Explain corresponding state in connection with critical constants. 5+2=7

OR

2. *(a)* State and explain the principle of equipartition of energy. What is the average energy per degree of freedom? 4

(b) Describe in detail van der Waals' equation of state explaining correction for volume and pressure. 6

UNIT—II

3. *(a)* What is thermal conductivity? Deduce the expression for coefficient of thermal conductivity and show that it is independent of pressure. 1+4+1=6

(b) Mention and explain the four thermodynamic potentials. 4

OR

4. *(a)* State and prove Gibbs' phase rule. 4

(b) From Maxwell's thermodynamic relations, discuss the following : 3×2=6

(i) Variation of internal (intrinsic) energy with volume

(ii) $TdS = C_P dT - T\left(\frac{\partial V}{\partial T}\right)_P dP$

UNIT—III

5. *(a)* Write the transformation relation for $A_{\lambda}^{\alpha\beta\gamma}$. 1

(b) Show that the Kronecker delta is a mixed tensor of rank two. 3

(c) What are covariant and contravariant tensors? Show that the gradient of a scalar function and velocity are respectively a covariant and a contravariant tensors both of rank 1. 2+2+2=6

OR

6. *(a)* Derive the conditions under which a generalized curvilinear coordinate system is orthogonal. 3

(b) Show that cylindrical coordinate system is orthogonal. 3

(c) Apply Green's theorem to evaluate $\oint_C [(2x^2 - y^2)\,dx + (x^2 + y^2)\,dy]$, where C is the boundary of the area enclosed by the X-axis and the upper-half of the circle $x^2 + y^2 = 1$. 4

UNIT—IV

7. *(a)* Find the inverse of the matrix

$$A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$$ 3

(b) If A and B be symmetric matrices, then show that AB is symmetric if and only if A and B commute. 3

(c) Solve by the matrix method the following simultaneous equations : 4

$$x + y + z = 3$$
$$x + 2y + 3z = 4$$
$$x + 4y + 9z = 6$$

OR

8. (a) Show that if A is an n-square non-singular matrix, then $(A^T)^{-1} = (A^{-1})^T$. 3

(b) Find the normalized eigenvectors of the matrix

$$\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$$ 3

(c) Show that any matrix A can be uniquely written as $H_1 + iH_2$, where H_1 and H_2 are both Hermitian matrices. What are the expressions for H_1 and H_2? 4

UNIT—V

9. (a) Show that

$$\beta(m, n) = \frac{(m-1)!\,(n-1)!}{(m+n-1)!}$$ 5

(b) Evaluate the integral

$$\int_0^\infty e^{-ax} x^{m-1} \cos bx \, dx$$ 5

OR

10. (a) From the expression of $\Gamma(n)$, prove that

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$ 5

(b) Show that

$$\Gamma(m)\Gamma\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}} \Gamma(2m)$$ 5

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