MATH/VI/CC/12b

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Student's Copy

2024

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(CBCS)

(6th Semester)

MATHEMATICS

TWELFTH (B) PAPER

(Elementary Number Theory)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A-OBJECTIVE)

(Marks: 10)

Tick (✓) the correct answer in the brackets provided :

 $1 \times 10 = 10$

1. The greatest common divisor of 12378 and 3054 is

- (a) 6 ()
- (b) 12 ()
- (c) 16 ()
- (d) 3 ()

Contd.

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- 2. The least common multiple of 1687 and 482 is
 - (a) 1687 ()
 (b) 2374 ()
 (c) 3374 ()
 (d) 6748 ()

3. The solution of the linear congruence $16x = 25 \pmod{19}$, where $t \in \mathbb{Z}$, is

- (a) x = 16 + 19t () (b) x = 17 + 19t () (c) x = 18 + 28t () (d) x = 20 + 28t ()
- 4. The reduced residue system modulo 12 from the following is
 - (a) {1, 2, 3, 5, 7, 11} ()
 (b) {1, 3, 5, 7, 11} ()
 (c) {1, 5, 7, 11} ()
 (d) {5, 7, 11} ()

5. The remainder when $3^{12} + 5^{12}$ is divisible by 13 is

- (a) 0 ()
- (b) 1 ()
- (c) 2 ()
- (d) 3 ()

6. For any integer n > 2, Euler's function $\phi(n)$ is

- (a) zero ()
- (b) odd ()
- (c) even ()
- (d) None of the above ()

7. If p is an odd prime and g.c.d. (a, p) = 1, then

(a)
$$\left(\frac{a}{p}\right) \equiv a^{\frac{p-1}{2}} \pmod{p}$$
 ()
(b) $\left(\frac{a}{p}\right) \equiv a^{\frac{p-1}{2}} \pmod{p}$ ()
(c) $\left(\frac{a}{p}\right) \equiv a^{\frac{p}{2}} \pmod{p}$ ()
(d) $\left(\frac{a}{p}\right) \equiv a^{p-1} \pmod{p}$ ()

8. If p is an odd prime and if g is a primitive root, then

(a)
$$\left(\frac{g}{p}\right) = 0$$
 ()
(b) $\left(\frac{g}{p}\right) = 1$ ()
(c) $\left(\frac{g}{p}\right) = -1$ ()
(d) $\left(\frac{g}{p}\right) = p$ ()

9. If f is multiplicative and $f(n) \neq z(n)$, then

 $\begin{array}{ll} (a) & f(1) = -1 & (&) \\ (b) & f(1) = 0 & (&) \\ (c) & f(1) = 3 & (&) \\ (d) & f(1) = 1 & (&) \end{array}$

10. The value of σ (20) is

 (a)
 42
 ()

 (b)
 20
 ()

 (c)
 40
 ()

 (d)
 35
 ()

.

(SECTION : B-SHORT ANSWERS)

(Marks: 15)

Answer the following :

Unit—I

1. Prove that if a|b and b|c, then a|c.

OR

2. Find the g.c.d. of 275 and 200, and express it in the form $m \cdot 275 + n \cdot 200$.

Unit—II

3. Show that $3 \times 5^{2n+1} + 2^{3n+1} \equiv 0 \pmod{17}$.

OR

4. If $a \equiv b \pmod{m}$ and d is a positive divisor of m, then show that $a \equiv b \pmod{d}$.

Unit—III

5. Prove that $n^5 - n$ is divisible by 30.

OR

6. Show that if n > 1 and $p_1, p_2, ..., p_m$ are the distinct prime factor of n, then

$$\phi(n) = n \left(1 - \frac{1}{p_1}\right) \left(1 - \frac{1}{p_2}\right) \dots \left(1 - \frac{1}{p_m}\right)$$

Unit—IV

7. Evaluate $\left(\frac{30}{43}\right)$.

OR

8. If the prime p > 3, $p \equiv 3 \pmod{4}$, and $q \equiv 2p + 1$ is prime, then prove that $2^{p} - 1$ is composite.

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3×5=15

9. Prove that for a Dirichlet product of arithmetic functions f and g, we have f * g = g * f.

OR

Evaluate \$\$\phi\$(735).

(SECTION : C-DESCRIPTIVE)

(Marks: 50)

Answer the following :

UNIT-I

1. (a) State and prove division algorithm.

(b) If p is a prime and a, b are any integers, prove that $p|ab \Rightarrow p|a$ or p|b.

OR

2. (a) State and prove the fundamental theorem of arithmetic. 6

(b) For positive integers a and b, prove that g.c.d. $(a, b) \cdot 1.c.m. (a, b) = ab.$ 4

UNIT-II

- 3. (a) Prove that there are arbitrary large gaps in the series of primes, that is, given any positive integer k, there exists k consecutive composite integers.
 - (b) If a and b be two arbitrary integers, prove that $a \equiv b \pmod{m}$ if and only if a and b have the same remainder when divided by m. 5

OR

- 4. (a) Prove that the congruence $ax \equiv b \pmod{m}$ has a solution if and only if the greatest common divisor of a and m divides b. 6
 - (b) Prove that the number of positive primes is infinite. 4

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| Contd.

10×5=50

6

UNIT—III

- 5. (a) State and prove Euler's theorem.
 - (b) If n > 1, then show that the sum of all positive integers which are less than n and prime to n is $\frac{1}{2}n\phi(n)$.

OR

- 6. (a) If p is prime, then prove that $(p-1)! \equiv -1 \pmod{p}$.
 - (b) Show that 16+86 is divisible by 323 using Wilson's theorem.

UNIT-IV

7. (a) Let p and q be distinct odd primes. If p ≡ q ≡ 3 (mod 4), then prove that (^q/_p) = -(^p/_q). Otherwise, if p ≡ 1 (mod 4) or q ≡ 1 (mod 4), then show that (^q/_p) = (^p/_q).
(b) Evaluate (⁵/₁₃).

OR

8. (a) Prove that if $2^n - 15 = x^2$, then n = 4 or n = 6. (b) Evaluate $\left(\frac{5}{11}\right)$.

UNIT-V

9. (a) If f and g are both multiplicative functions, then prove that f * g is also multiplicative.

(b) Prove that if mn > 1, then $\sigma(mn) > m\sigma(n)$.

OR

- 10. (a) State and prove Mobius inversion formula.
 - (b) Find the general solution of 10x 8y = 42.

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(b) $\left(\frac{a}{p}\right) \equiv a^{\frac{p-1}{2}} \pmod{a}$ ()
(c) $\left(\frac{a}{p}\right) \equiv a^{\frac{p}{2}} \pmod{p}$ ()
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3. Show that $3 \times 5^{2n+1} + 2^{3n+1} \equiv 0 \pmod{17}$.

OR

4. If $a \equiv b \pmod{m}$ and d is a positive divisor of m, then show that $a \equiv b \pmod{d}$.

UNIT--III

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6. Show that if n > 1 and p_1, p_2, \dots, p_m are the distinct prime factor of n, then $\phi(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)\dots\left(1 - \frac{1}{p_m}\right)$

UNIT-IV

Evaluate 30. 1

0 B

If the prime p > 3, $p \equiv 3 \pmod{4}$, and q = 2p + 1 is prime, then prove that 2^p -1 is composite. 80

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3×5=15

UNIT-V

9. Prove that for a Dirichlet product of arithmetic functions f and g, we have f * g = g * f.

g

10. Evaluate φ(735).

(SECTION : C-DESCRIPTIVE)

(Marks : 50)

Answer the following :

UNIT-I

9

 $10 \times 5 = 50$

- 1. (a) State and prove division algorithm.
- 4 and a, b are any integers, prove that $p|ab \Rightarrow p|a$ If p is a prime or p|b. (q)

8 B

- 9 State and prove the fundamental theorem of arithmetic. (a) ci,
- For positive integers a and b, prove that g.c.d. $(a, b) \cdot 1.c.m. (a, b) = ab$. (q)

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UNIT--II

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If a and b be two arbitrary integers, prove that $a \equiv b \pmod{m}$ if and only if a and b have the same remainder when divided by m. (q)

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- 9 Prove that the congruence $ax \equiv b \pmod{m}$ has a solution if and only if the greatest common divisor of a and m divides b. **4**. (a)
 - Prove that the number of positive primes is infinite. (q)

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State and If $n > 1$, the second state is a second state of the s	than <i>n</i> ar		If p is pr	Show tha		Let p and	that $\left(\frac{q}{p}\right)$ =	that $\left(\frac{q}{p}\right)$	Evaluate		Prove tha	Evaluate		If f and g	Prove that		State and	Find the g		
(a)			(a)	(q)		(a)			(q)		(a)	(q)		(a)	(q)		(a)	(q)		
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