 Student's Copy

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(CBCS)

(6th Semester)

MATHEMATICS

ELEVENTH PAPER

(Mechanics)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : $1\times10=10$

1. The branch of mechanics that deals with forces acting on a body that is at rest is called

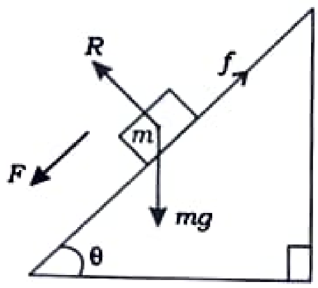
(a) statics ( )

(b) dynamics ( )

(c) kinetics ( )

(d) kinematics ( )

2. For a body of mass m on a rough inclined plane with a constant but arbitrary coefficient of friction as shown in the figure below, what is the acceleration on the inclined plane if the net downward force is denoted as F? Let g be the acceleration due to gravity and μ be the coefficient of friction.



(a) $g \sin \theta$ ( )

(b) $g \cos \theta$ ( )

(c) $g (\sin \theta - \mu \cos \theta)$ ( )

(d) $g (\cos \theta - \mu \sin \theta)$ ( )

3. When does the moment of inertia of a body come into the picture?

(a) When the motion is linear ( )

(b) When the motion is rotational ( )

(c) When the motion is along a curved path ( )

(d) None of the above ( )

4. The centre of gravity of a circular arc of radius 4 cm subtending at an angle 90 degrees lies on the axis of symmetry at a distance of

(a) $\frac{2\sqrt{2}}{\pi}$ from the centre ( )

(b) $\frac{4\sqrt{2}}{\pi}$ from the centre ( )

(c) $\frac{6\sqrt{2}}{\pi}$ from the centre ( )

(d) $\frac{8\sqrt{2}}{\pi}$ from the centre ( )

5. If a particle moves so that its normal acceleration is always zero, then its path is

(a) a circle ( )

(b) a parabola ( )

(c) a straight line ( )

(d) None of the above ( )

6. The equation of SHM of period T of a particle is

(a) $\ddot{x} = -T^2 x$ ( )

(b) $\ddot{x} = -\frac{4\pi^2}{T^2} x$ ( )

(c) $\ddot{x} = -\frac{4T^2}{\pi^2} x$ ( )

(d) $\ddot{x} = -\frac{1}{T^2} x$ ( )

7. An aeroplane flying 490 m above ground level at 100 m/s, releases a block. How far on the ground will it strike?

(a) 2 km ( )

(b) 1 km ( )

(c) 0·1 km ( )

(d) 10 km ( )

8. A bomb is dropped from an aeroplane moving horizontally at a constant speed. When air resistance is taken into consideration, the bomb

(a) flies with the aeroplane ( )

(b) falls to the earth exactly below the aeroplane ( )

(c) falls to the earth ahead of the plane ( )

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9. What is the type of collision in which both the linear momentum and the kinetic energy of the system remain conserved?

(a) Inelastic collision ( )

(b) Elastic collision ( )

(c) Destructive collision ( )

(d) None of the above ( )

10. If the earth's attraction on a particle varies inversely as the square of its distance from the earth's centre and if the radius of the earth is a km, then the work done by the earth's attraction on a particle of weight 6 kg on the surface of the earth is

(a) $3a$ ( )

(b) $4a$ ( )

(c) $5a$ ( )

(d) $6a$ ( )

(SECTION : B—SHORT ANSWERS)

(*Marks* : 15)

Answer the following :

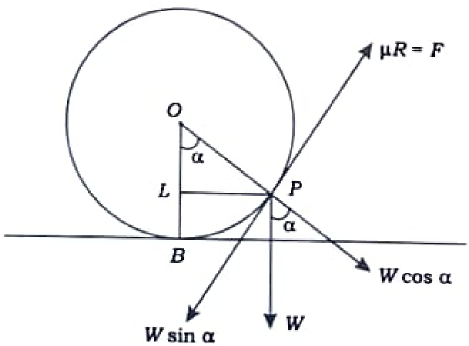
3×5=15

UNIT—I

1. Forces proportional to 1, 2, 3 and 4 act along the sides *AB*, *BC*, *AD* and *DC* respectively of a square *ABCD*, the length of whose sides is 2 feet. Find the magnitude and line of action of their resultant.

OR

2. How high can a particle rest inside a hollow sphere of radius r, μ being the coefficient of friction? Given that O is the center and W be the weight of the sphere, the force of friction μR is acting along the tangent at P as shown in the figure below :



UNIT—II

3. Prove that the centre of gravity of a triangular area coincides with that of three equal particles placed at the middle points of its sides.

OR

4. Find the moment of inertia of a rectangular lamina about a line through its centre and parallel to one of its edges.

UNIT—III

5. If t be regarded as a function of velocity v, prove that the rate of decrease of acceleration is given by $f^3 \dfrac{d^2t}{dv^2}$, f being the acceleration.

OR

6. The velocity of a point moving in a plane curve varies as the radius of curvature. Show that the direction of motion revolves with constant angular velocity.

UNIT—IV

7. If R be the maximum horizontal range of a projectile and H be the greatest height attained by the particle, show that $H = \dfrac{R}{4}$.

OR

8. A particle is thrown vertically upwards with velocity u, the resistance being mkv^2, find the time of ascent.

UNIT—V

9. A gun is mounted on a gun-carriage movable on a smooth horizontal plane, and the gun is elevated at an angle α to the horizon. A shot is fired and leave the gun in a direction inclined at an angle θ to the horizon. If the mass of the gun and carriage be n times that of the shot, show that

$$\tan\theta = \left(1 + \frac{1}{n}\right)\tan\alpha$$

OR

10. A sphere impinges directly on an equal sphere at rest. If the coefficient of restitution be e, show that their ratio of velocities after impact are $(1 - e) : (1 + e)$.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer the following : 10×5=50

UNIT—I

1. *(a)* Prove that a system of forces acting in one plane at different points of a rigid body can be reduced to a single-force through a given point, and a couple. 5

   (b) A uniform beam of length $2a$ rests in equilibrium with one end resting against a smooth vertical wall and the beam is placed upon a peg at a distance b from the wall. Show that the inclination of the beam to the horizon is $\cos^{-1}\left(\frac{b}{a}\right)^{1/3}$. 5

OR

2. *(a)* Derive an expression for the least force P required by a body of weight W acting on an inclined plane making an angle α with the horizon for the case when *(i)* the body is at a point to slide down the plane, *(ii)* the body is at a point of sliding up the plane. 2½+2½=5

   (b) A uniform rod rests in limiting equilibrium within a rough hollow sphere. If the rod subtends an angle 2α at the centre of the sphere and if λ be the angle of friction, show that the angle of inclination of the rod to the horizon is

$$\tan^{-1}\left\{\frac{\sin 2\lambda}{2\cos(\alpha+\lambda)\cos(\alpha-\lambda)}\right\}$$

5

UNIT—II

3. *(a)* State and prove the parallel axis theorem on moments of inertia. 5

   (b) A uniform circular lamina of radius $3a$ and centre O has a hole in the form of equilateral triangle of side $2a$ with one vertex at O. Prove that the centre of gravity of the triangle from O is $\frac{2a}{9\pi-\sqrt{3}}$. 5

OR

4. *(a)* Find the centre of gravity of the arc of the parabola $y^2 = 4ax$ included between the vertex and the latus rectum. 5

(b) Let AB and AC are two uniform rods of lengths $2a$ and $2b$ respectively. If $\angle BAC = \theta$, then prove that the distance of the centre of gravity from A of the two rods is

$$\frac{(a^4 + 2a^2b^2 \cos\theta + b^4)^{\frac{1}{2}}}{a+b}$$ 5

UNIT—III

5. *(a)* The velocity of a train increases at a constant rate f_1 from rest to v, then remains constant for an interval and finally decreases to zero at the constant rate f_2. If x be the total distance described, prove that the total time taken is $\frac{x}{v} + \frac{v}{2}\left(\frac{1}{f_1} + \frac{1}{f_2}\right)$. Also find the least value of time when $f_1 = f_2$. 5

(b) Prove that the radial and transverse component of acceleration for a particle moving along a plane curve are $r - r\theta^2$ and $\frac{1}{r}\frac{d}{dt}(r^2\theta)$. 5

OR

6. *(a)* A particle is moving with simple harmonic motion and while making an excursion from one position of rest to the other, its distance from the middle point of its path at three consecutive seconds is observed to be x_1, x_2, x_3. Prove that the time of complete oscillation is

$$\frac{2\pi}{\cos^{-1}\left(\frac{x_1 + x_3}{x_2}\right)}$$ 5

(b) The tangential acceleration of a particle moving along a circle of radius a is λ times the normal acceleration. If its speed at certain time is u, prove that it will return to the same point after a time $\frac{a}{\lambda u}(1 - e^{-2\pi\lambda})$. 5

UNIT—IV

7. (a) If α be the angle between the tangents at the extremities of any arc of the parabolic path, v and v' the velocities at those extremities and u the constant horizontal velocity, then show that the time of describing the arc is $\dfrac{vv' \sin \alpha}{ug}$. 5

(b) A particle is projected vertically upward with a velocity against a resistance proportional to the square of the velocity. If V is the terminal velocity of the body and m is its mass, then show that when the body has fallen back to the point of projection, the loss of KE is

$$\frac{1}{2} mu^2 \left(\frac{u^2}{V^2 + u^2} \right)$$

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OR

8. (a) The range down an inclined plane of inclination is three times the range up the plane. Prove that the range on a horizontal plane down through the same point of projection is double the maximum height reached by the projectile, the initial velocity and elevation being the same in all the three cases. 5

(b) A ball is projected to just clear two walls, the first of height a at a distance b from the point of projection and the second of height b at a distance a from the point of projection. Show that the range on the horizontal plane is $\dfrac{a^2 + ab + b^2}{a + b}$ and the angle of projection exceeds $\tan^{-1}(3)$. 5

UNIT—V

9. (a) A heavy ring of mass m slides on a smooth vertical rod and is attached to a light string which passes over a small smooth pulley at a distance a from the rod and has mass M ($> m$) fastened to its other end. Show that if the ring be dropped from a point in the rod in the same horizontal plane as the pulley, it will descend a distance $\dfrac{2Mma}{M^2 - m^2}$ before coming to rest. 5

(b) A man of weight W hangs at the end of a light extensible rope, whose modulus of elasticity is nW, the other end being fastened to a fixed point. He proceeds to climb up the rope. Prove that when he reaches the fixed point, he has done $\frac{2n+1}{2n+2}$ times the work he would have done in climbing the same distance up an inextensible rope. 5

OR

10. (a) A sphere impinges obliquely on another sphere at rest. If the two spheres are smooth elastic and equal in mass, then prove that they move at right angle to each other after impact. 5

(b) A ball impinges on another equal ball moving with the same speed in a direction perpendicular to its own, the line joining the centres of the balls at the instant of the impact being perpendicular to the direction of motion of the second ball. If e be the coefficient of restitution, show that the direction of motion of the second ball is turned through an angle $\tan^{-1}\left(\frac{1+e}{2}\right)$. 5

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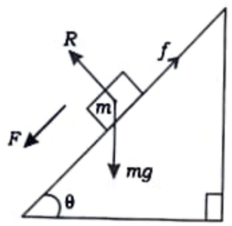

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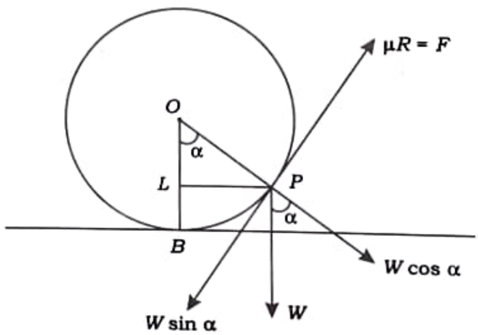

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