MATH/V/CC/05

Student's Copy

2024

(CBCS)

(5th Semester)

MATHEMATICS

FIFTH PAPER

(Computer Oriented Numerical Analysis)

Full Marks: 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A-OBJECTIVE)

(Marks: 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

- 1. Which of the following identities is true?
 - (a) $E = \Delta 1$ (b) $E = 1 \nabla$ (c) (c) $E = 1 + \Delta$ (c) (d) $E = 1 + \nabla$ (c)

2. By definition of forward difference operator, $\Delta^2 f(x)$ equals

(a)
$$f(x+h) - f(x)$$
 ()
(b) $f(x+2h) + f(x+h) + f(x)$ ()
(c) $f(x+2h) - f(x+h) - f(x)$ ()

(d) f(x+2h)-2f(x+h)+f(x) ()

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ы.		If a tabulated function is a polynomial, then
	(a)	value of interpolation = value of extrapolation ()
	(q)	value of interpolation > value of extrapolation ()
	(c)	value of interpolation < value of extrapolation ()
	(q)	None of the above ()
4		The technique of estimating the value of a function for any intermediate value of the independent variable is called
	(a)	extrapolation ()
	(q)	graphical method ()
	(c)	interpolation ()
	(q)	None of the above ()
<u>о</u> .		The coefficient matrix obtained from the simultaneous equations
	ŧ	$a_{11}x + a_{12}y + a_{13}z = d_1; \ b_{21}x + b_{22}y + b_{23}z = d_2; \ c_{31}x + c_{32}y + c_{33}z = d_3$
	IIIw	will be a diagonally dominant matrix, if
	(a)	$(a) a_{11} \ge a_{12} + a_{13} , b_{21} \ge b_{22} + b_{23} , c_{31} \ge c_{32} + c_{33} , (a)$
	(q)	$(b) a_{11} \ge a_{12} + a_{13} , b_{22} \ge b_{21} + b_{23} , c_{33} \ge c_{32} + c_{31} $
	(c)	$ a_{11} \le a_{12} + a_{13} , b_{21} \le b_{22} + b_{23} , c_{31} \le c_{32} + c_{32} , c_{32} + c_{32} , c_{33} $
	(q)	$(d) a_{11} + a_{12} + a_{13} \ge d_1 , b_{21} + b_{22} + b_{23} \ge d_2 ,$
		$ c_{31} + c_{32} + c_{33} \ge d_3 $ ()
é.	In C mat	In Gauss elimination method for solving system of equation $AX = B$, the matrix A is reduced to
	(a)	upper triangular matrix ()
	(q)	lower triangular matrix ()
	(c)	diagonal matrix ()
	(q)	None of the above ()
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7. In the general quadrature formula, trapezoidal rule is obtained by putting

(a) n=2 () (b) n=4 () (c) n=2 and 4 () (d) n=1 ()

- for Which one of the following is the most commonly used method numerical integration? ø
- Newton-Cotes quadrature formula (a)
- (b) Trapezoidal rule (
- (c) Simpson's one-third rule (
- (d) None of the above (
- 9. In numerical solution of differential equations, Euler's method can be considerd as
- (a) Runge-Kutta method of first order
- Runge-Kutta method of second order (q)
- (c) Runge-Kutta method of third order
- Runge-Kutta method of fourth order (q)
- The second-order Runge-Kutta formula for the differential equation 0

$$\frac{dy}{dx} = f(x, y)$$

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(a)
$$y_1 = y_0 + (k_1, k_2)$$
 ()
(b) $y_1 = y_0 - (k_1, k_2)$ ()

(c)
$$y_1 = y_0 + \frac{h}{2}(k_1, k_2)$$
 (

$$(d) \quad y_1 = y_0 - \frac{h}{2}(k_1, k_2) \quad ($$

 $y_0 + k_1$). where $k_1 = h f(x_0, y_0)$ and $k_2 = h f(x_0 + h_1)$

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(SECTION : B-SHORT ANSWERS)

(Marks: 15)

Answer the following questions :

UNIT--I

Express $y = 2x^3 - 3x^2 + 3x - 10$ in factorial notation and hence show that $\Delta^3 y = 12.$ ÷

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Find the third divided difference with arguments 2, 4, 9, 10 of the function $f(x) = x^2 - 2x.$ a

UNIT--II

Use Newton's divided difference formula, find the value of f(2) from the following table : 3. (a)

f(x) : 4 5 7 f(x) : 48 100 294 **OR** Use Lagrange's interpolation formula to find the value of y, when x = 10, if the following x and y are given : 4

x : 5 6 9y : 12 13 14

UNIT-III

Apply Gauss elimination method to solve the equations ŝ.

x+4y=7x+y=1

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3×5=15

g

Solve the following equations by Crout's method ý.

$$x + 2y = 14$$
$$2x - 5y = 10$$

UNIT-IV

7. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using trapezoidal rule.

OR

8. Integrate numerically $\int_0^{\pi/2} \sqrt{\cos\theta}$ by Simpson's one-third rule.

UNIT-V

9. Find by Taylor's series method, the value of y at x = 0.1 and x = 0.2 to three decimal places from $\frac{dy}{dx} = x^2 y - 1$, y(0) = 1.

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10. Apply Runge-Kutta method of second order to find an approximate value of y when x = 0.2 given that

$$\frac{dy}{dx} = x + y$$
 and $y = 1$

when x = 0.

(SECTION : C-DESCRIPTIVE)

(Marks : 50)

Answer the following questions :

 $10 \times 5 = 50$

UNIT-I

Find a real root of the equation $x^3 - 2x - 5 = 0$ by the method of false position correct to three-decimal places. 1. (a)

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Write an algorithm for bisection method. (q)

	Find a real root of the equation $\cos x = 3x - 1$ correct to three decimal places using iteration method.	Find the positive root of $x^{4} - x = 10$ correct to three decimal places using Newton-Raphson method. 5		Derive Newton's forward interpolation formula. 5 The table gives the distance in nautical miles of the visible horizon for the given height in feet above the earth's surface :	300 350 400 18-42 19-90 21-27 54 6ii y = 410 ft		estimate the number of students who 3 and 45 : 5	70 80 159 190	Find the polynomial $f(x)$ by using Lagrange's formula and hence find $f(3)$ from	5 147		equations : 5		lve the equations : 5	
OR	he equation $\cos x = 3x$ on method.	ot of $x^4 - x = 10$ correson method.	UNITII	Derive Newton's forward interpolation formula. The table gives the distance in nautical miles of the the given height in feet above the earth's surface :	ight) : 100 150 200 250 300 350 itance) : 10.63 13.03 15.04 16.81 18.42 19.9 Find the values of u when h $x = 218$ ft and h h $x = 410$ ft	OR	table, estimate the ween 40 and 45 :	40 50 60 31 73 124	f(x) by using Lagrange	0 1 2 2 3 12	UNITIII	Apply Gauss-Jordan method to solve the equations	x + y + z = 9 2x - 3y + 4z = 13 3x + 4y + 5z = 40	(b) Apply Gauss-Seidel iteration method to solve the equations 20x + y - 2z = 17 3x + 20y - z = -18 2x - 3y + 20z = 25	
					X (height) : 100 Y (distance) : 10.63 Find the values of <i>i</i>	2	From the following table, estimate obtained marks between 40 and 45 :	Marks No. of Students :	Find the polynomial $f(3)$ from	X : F(x) :				Apply Gauss-Seidel i	
	5 G	(a)		3. (a) (b)	Y (d		4. (a)		(q)			5. (a)		(q)	

equations
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solve
method,
Crout's
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(a)
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$$2x + 3y + z = -1$$

 $5x + y + z = 9$
 $3x + 2y + 4z = 11$

algorithm for Crout's method for solving simultaneous an equations. Write (q)

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UNIT-IV

- S Derive numerical differentiation formula of second-order by using forward difference formula. 7. (a)
- (b) Given that

$$X : 1.0 \quad 1.1 \quad 1.2 \quad 1.3 \quad 1.4 \quad 1.5 \quad 1.6$$

$$Y : 7.989 \quad 8.403 \quad 8.781 \quad 9.129 \quad 9.451 \quad 9.750 \quad 10.031$$

find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1 \cdot 1$.

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- 8. (a) Derive Newton-Cotes quadrature formula.
- S Use Simpson's one-third rule to find $\int_0^{0.6} e^{-x^2} dx$ by taking seven ordinates. (q)

- ŝ Use Picard's method to solve $\frac{dy}{dx} = x^2 - y$, y(0) = 1 for $x = 0 \cdot 2$. 9. (a)
- Using Euler's method, find the approximate value of y corresponding to x = 1, given that $\frac{dy}{dx} = x + y$ and y = 1 when x = 0. (q)

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OR

10. (a) Using Runge-Kutta method of fourth-order, solve

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$$

with y(0) = 1 at $x = 0 \cdot 2$.

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(b) Using Milne's Predictor Corrector method to find $y(4 \cdot 4)$ given

 $5x\frac{dy}{dx} = y^2 - 2 = 0, \ y(4) = 1, \ y(4 \cdot 1) = 1 \cdot 0049, \ y(4 \cdot 2) = 1 \cdot 0097$ and $y(4 \cdot 3) = 1 \cdot 0143$.

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MATH/V/CC/05	2024	(CBCS)	(5th Semester)	MATHEMATICS	FIFTH PAPER	(Computer Oriented Numerical Analysis)	Full Marks : 75	Time: 3 hours	The figures in the margin indicate full marks for the questions	(SECTION : A-OBJECTIVE)	(Marks : 10)	Tick (\prime) the correct answer in the brackets provided :	1. Which of the following identities is true? (a) $E = \Delta - 1$ () (b) $E = 1 - \nabla$ ((c) $E = 1 + \Delta$ () (d) $E = 1 + \nabla$ (2. By definition of forward difference operator, $\Delta^2 f(x)$ equals (a) $f(x + h) - f(x)$ () (b) $f(x + 2h) + f(x + h) + f(x)$ () (c) $f(x + 2h) - f(x + h) - f(x)$ () (d) $f(x + 2h) - 2f(x + h) + f(x)$ () (d) $f(x + 2h) - 2f(x + h) + f(x)$ ()

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The technique of estimating the value of a function for any intermediate value of interpolation = value of extrapolation value of interpolation > value of extrapolation value of interpolation < value of extrapolation 3. If a tabulated function is a polynomial, then value of the independent variable is called None of the above extrapolation Ø (q) (q <u></u> Ø

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- graphical method (q)
- interpolation 0
- (d) None of the above
- The coefficient matrix obtained from the simultaneous equations ທ່

 $a_{11}x + a_{12}y + a_{13}z = d_1; \ b_{21}x + b_{22}y + b_{23}z = d_2; \ c_{31}x + c_{32}y + c_{33}z = d_3$ will be a diagonally dominant matrix, if

- (a) $|a_{11}| \ge |a_{12}| + |a_{13}|, |b_{21}| \ge |b_{22}| + |b_{23}|, |c_{31}| \ge |c_{32}| + |c_{33}|$
- (b) $|a_{11}| \ge |a_{12}| + |a_{13}|, |b_{22}| \ge |b_{21}| + |b_{23}|, |c_{33}| \ge |c_{32}| + |c_{31}|$
 - $|a_{11}| \le |a_{12}| + |a_{13}|, |b_{21}| \le |b_{22}| + |b_{23}|, |c_{31}| \le |c_{32}| + |c_{33}|$ (c)
 - (d) $|a_{11}|+|a_{12}|+|a_{13}|\geq |d_1|, |b_{21}|+|b_{22}|+|b_{23}|\geq |d_2|,$
- $|c_{31}|+|c_{32}|+|c_{33}|\geq |d_3|$
- In Gauss elimination method for solving system of equation AX = B, the matrix A is reduced to ø.
- upper triangular matrix (a)
- lower triangular matrix (q)
- diagonal matrix 0
- None of the above (q)

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7. In the general quadrature formula, trapezoidal rule is obtained by putting

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- 10. The second-order Runge-Kutta formula for the differential equation

$$\frac{dy}{dx} = f(x, y)$$

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(a)
$$y_1 = y_0 + (k_1, k_2)$$
 ()
(b) $y_1 = y_0 - (k_1, k_2)$ ()
(c) $y_1 = y_0 + \frac{h}{2}(k_1, k_2)$ ()

(d)
$$y_1 = y_0 - \frac{1}{2}(k_1, k_2)$$
 (-)

where $k_1 = h f(x_0, y_0)$ and $k_2 = h f(x_0 + h, y_0 + k_1)$.

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(SECTION : B-SHORT ANSWERS

(Marks: 15)

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UNIT--I

3×5=]

Express $y = 2x^3 - 3x^2 + 3x - 10$ in factorial notation and hence show that $\Delta^3 y = 12.$ ÷

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UNIT-II

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f(x) : 4 5 7f(x) : 48 100 294**OR** Use Lagrange's interpolation formula to find the value of y, when x = 10, if the following x and y are given : 4

x : 5 6 9 y : 12 13 14

UNIT-III

5. Apply Gauss elimination method to solve the equations

x + 4y = 7x + y = 1

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OR

••• 6. Solve the following equations by Crout's method

$$x + 2y = 14$$
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UNIT-IV

by using trapezoidal rule. 7. Evaluate $\int_0^6 \frac{dx}{1+x^2}$

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SECTION : C-DESCRIPTIVE)

 $10 \times 5 = 50$

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(b) Write an algorithm for bisection method.

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	 1 correct to three decimal 5 	ct to three decimal places 5		Nula. 5 of the visible horizon for surface :	350 400 21-27 210 6	C ''' 11 O 11'	umber of students who 5		ດ	147	uations : 5			the equations ; 5			[Contd.
OR	2. (a) Find a real root of the equation $\cos x = 3x - 1$ correct to three decimal places using iteration method.	(b) Find the positive root of $x^4 - x = 10$ correct to three decimal places using Newton-Raphson method.	UNIT-II	 (a) Derive Newton's forward interpolation formula. (b) The table gives the distance in nautical miles of the visible horizon for the given height in feet above the earth's surface : 	X (height) : 100 150 200 250 300 350 Y (distance) : 10.63 13.03 15.04 16.81 18.42 19.9 Find the values of u when $h(t) = 2.18$ ft and $h(t) = 2.10$ ft.		 (a) From the following table, estimate the number of obtained marks between 40 and 45: 	Marks : 40 50 60 70 80 No. of Students : 31 73 124 159 190 (b) Find the polynomial $f(x)$ by using Lagrange's formula and hence find	X : 0 1 2	$F(\mathbf{x})$: 2 3 12	UNIT—III (a) Apply Gauss-Jordan method to solve the equations ;	x+y+z=9 $2x-3y+4z=13$	3x + 4y + 5z = 40	(b) Apply Gauss-Seidel iteration method to solve the equations $20x + u - 2z = 17$	3x + 20y - z = -18	2x - 3y + 20z = 25	y
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By Crout's method, solve the system of equations : (a) <u>ن</u>

2x + 3y + z = -13x + 2y + 4z = 115x + y + z = 9

algorithm for Crout's method for solving simultaneous an equations. Write (q)

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UNIT-IV

Derive numerical differentiation formula of second-order by using forward difference formula. 7. (a)

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Given that (q)

10.031 1.6 9.750 1:2 9-451 1.4 9.129 1:3 8.781 1.2 at $x = 1 \cdot 1$. 8-403 Ŀ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ a 7-989 10 •• XX

NOR

- Derive Newton-Cotes quadrature formula. 8. (a)
- S Use Simpson's one-third rule to find $\int_0^{0.6} e^{-x^2} dx$ by taking seven ordinates. (q)

UNIT-V

- S 9. (a) Use Picard's method to solve $\frac{dy}{dx} = x^2 - y$, y(0) = 1 for $x = 0 \cdot 2$.
- Using Euler's method, find the approximate value of y corresponding to x = 1, given that $\frac{dy}{dx} = x + y$ and y = 1 when x = 0. 9

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$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$$

with y(0) = 1 at x = 0.2.

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and $y(4 \cdot 3) = 1 \cdot 0143$.

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