2024

(CBCS)

(5th Semester)

MATHEMATICS

FIFTH PAPER

(Computer Oriented Numerical Analysis)

Full Marks : 75

Time : 3 hours

The figures in the margin indicate full marks for the questions

(SECTION : A—OBJECTIVE)

(*Marks* : 10)

Tick (✓) the correct answer in the brackets provided : 1×10=10

1. Which of the following identities is true?

   (a) $E = \Delta - 1$ ( ) (b) $E = 1 - \nabla$ ( )

   (c) $E = 1 + \Delta$ ( ) (d) $E = 1 + \nabla$ ( )

2. By definition of forward difference operator, $\Delta^2 f(x)$ equals

   (a) $f(x+h) - f(x)$ ( )

   (b) $f(x+2h) + f(x+h) + f(x)$ ( )

   (c) $f(x+2h) - f(x+h) - f(x)$ ( )

   (d) $f(x+2h) - 2f(x+h) + f(x)$ ( )

3. If a tabulated function is a polynomial, then

(a) value of interpolation = value of extrapolation ( )

(b) value of interpolation > value of extrapolation ( )

(c) value of interpolation < value of extrapolation ( )

(d) None of the above ( )

4. The technique of estimating the value of a function for any intermediate value of the independent variable is called

(a) extrapolation ( )

(b) graphical method ( )

(c) interpolation ( )

(d) None of the above ( )

5. The coefficient matrix obtained from the simultaneous equations

$$a_{11}x + a_{12}y + a_{13}z = d_1;\ b_{21}x + b_{22}y + b_{23}z = d_2;\ c_{31}x + c_{32}y + c_{33}z = d_3$$

will be a diagonally dominant matrix, if

(a) $|a_{11}| \geq |a_{12}| + |a_{13}|$, $|b_{21}| \geq |b_{22}| + |b_{23}|$, $|c_{31}| \geq |c_{32}| + |c_{33}|$ ( )

(b) $|a_{11}| \geq |a_{12}| + |a_{13}|$, $|b_{22}| \geq |b_{21}| + |b_{23}|$, $|c_{33}| \geq |c_{32}| + |c_{31}|$ ( )

(c) $|a_{11}| \leq |a_{12}| + |a_{13}|$, $|b_{21}| \leq |b_{22}| + |b_{23}|$, $|c_{31}| \leq |c_{32}| + |c_{33}|$ ( )

(d) $|a_{11}| + |a_{12}| + |a_{13}| \geq |d_1|$, $|b_{21}| + |b_{22}| + |b_{23}| \geq |d_2|$, $|c_{31}| + |c_{32}| + |c_{33}| \geq |d_3|$ ( )

6. In Gauss elimination method for solving system of equation $AX = B$, the matrix A is reduced to

(a) upper triangular matrix ( )

(b) lower triangular matrix ( )

(c) diagonal matrix ( )

(d) None of the above ( )

7. In the general quadrature formula, trapezoidal rule is obtained by putting

(a) $n = 2$ ( )

(b) $n = 4$ ( )

(c) $n = 2$ and 4 ( )

(d) $n = 1$ ( )

8. Which one of the following is the most commonly used method for numerical integration?

(a) Newton-Cotes quadrature formula ( )

(b) Trapezoidal rule ( )

(c) Simpson's one-third rule ( )

(d) None of the above ( )

9. In numerical solution of differential equations, Euler's method can be considerd as

(a) Runge-Kutta method of first order ( )

(b) Runge-Kutta method of second order ( )

(c) Runge-Kutta method of third order ( )

(d) Runge-Kutta method of fourth order ( )

10. The second-order Runge-Kutta formula for the differential equation

$$\frac{dy}{dx} = f(x, y)$$

is

(a) $y_1 = y_0 + (k_1, k_2)$ ( )

(b) $y_1 = y_0 - (k_1, k_2)$ ( )

(c) $y_1 = y_0 + \frac{h}{2}(k_1, k_2)$ ( )

(d) $y_1 = y_0 - \frac{h}{2}(k_1, k_2)$ ( )

where $k_1 = hf(x_0, y_0)$ and $k_2 = hf(x_0 + h, y_0 + k_1)$.

(SECTION : B—SHORT ANSWERS)

(*Marks* : 15)

Answer the following questions : 3×5=15

UNIT—I

1. Express $y = 2x^3 - 3x^2 + 3x - 10$ in factorial notation and hence show that $\Delta^3 y = 12$.

OR

2. Find the third divided difference with arguments 2, 4, 9, 10 of the function $f(x) = x^2 - 2x$.

UNIT—II

3. *(a)* Use Newton's divided difference formula, find the value of $f(2)$ from the following table :

x	:	4	5	7
$f(x)$	:	48	100	294

OR

4. Use Lagrange's interpolation formula to find the value of y, when $x = 10$, if the following x and y are given :

x	:	5	6	9
y	:	12	13	14

UNIT—III

5. Apply Gauss elimination method to solve the equations

$$x + 4y = 7$$
$$x + y = 1$$

OR

6. Solve the following equations by Crout's method :

$$x + 2y = 14$$
$$2x - 5y = 10$$

UNIT—IV

7. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using trapezoidal rule.

OR

8. Integrate numerically $\int_0^{\pi/2} \sqrt{\cos\theta}$ by Simpson's one-third rule.

UNIT—V

9. Find by Taylor's series method, the value of y at $x = 0 \cdot 1$ and $x = 0 \cdot 2$ to three decimal places from $\frac{dy}{dx} = x^2 y - 1$, $y(0) = 1$.

OR

10. Apply Runge-Kutta method of second order to find an approximate value of y when $x = 0 \cdot 2$ given that

$$\frac{dy}{dx} = x + y \text{ and } y = 1$$

when $x = 0$.

(SECTION : C—DESCRIPTIVE)

(*Marks* : 50)

Answer the following questions : 10×5=50

UNIT—I

1. (a) Find a real root of the equation $x^3 - 2x - 5 = 0$ by the method of false position correct to three-decimal places. 5

(b) Write an algorithm for bisection method. 5

OR

2. *(a)* Find a real root of the equation $\cos x = 3x - 1$ correct to three decimal places using iteration method. 5

(b) Find the positive root of $x^4 - x = 10$ correct to three decimal places using Newton-Raphson method. 5

UNIT—II

3. *(a)* Derive Newton's forward interpolation formula. 5

(b) The table gives the distance in nautical miles of the visible horizon for the given height in feet above the earth's surface :

X (height)	:	100	150	200	250	300	350	400
Y (distance)	:	10·63	13·03	15·04	16·81	18·42	19·90	21·27

Find the values of y when *(i)* $x = 218$ ft and *(ii)* $x = 410$ ft. 5

OR

4. *(a)* From the following table, estimate the number of students who obtained marks between 40 and 45 : 5

Marks	:	40	50	60	70	80
No. of Students	:	31	73	124	159	190

(b) Find the polynomial $f(x)$ by using Lagrange's formula and hence find $f(3)$ from 5

X	:	0	1	2	5
$F(x)$	:	2	3	12	147

UNIT—III

5. *(a)* Apply Gauss-Jordan method to solve the equations : 5

$$x + y + z = 9$$
$$2x - 3y + 4z = 13$$
$$3x + 4y + 5z = 40$$

(b) Apply Gauss-Seidel iteration method to solve the equations : 5

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

OR

6. *(a)* By Crout's method, solve the system of equations : 5

$$2x+3y+z=-1$$
$$5x+y+z=9$$
$$3x+2y+4z=11$$

(b) Write an algorithm for Crout's method for solving simultaneous equations. 5

UNIT—IV

7. *(a)* Derive numerical differentiation formula of second-order by using forward difference formula. 5

(b) Given that

X :	1·0	1·1	1·2	1·3	1·4	1·5	1·6
Y :	7·989	8·403	8·781	9·129	9·451	9·750	10·031

find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x=1\cdot1$. 5

OR

8. *(a)* Derive Newton-Cotes quadrature formula. 5

(b) Use Simpson's one-third rule to find $\int_0^{0\cdot6} e^{-x^2} dx$ by taking seven ordinates. 5

UNIT—V

9. *(a)* Use Picard's method to solve $\frac{dy}{dx}=x^2-y$, $y(0)=1$ for $x=0\cdot2$. 5

(b) Using Euler's method, find the approximate value of y corresponding to $x=1$, given that $\frac{dy}{dx}=x+y$ and $y=1$ when $x=0$. 5

OR

10. *(a)* Using Runge-Kutta method of fourth-order, solve

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$$

with $y(0) = 1$ at $x = 0 \cdot 2$. 5

(b) Using Milne's Predictor Corrector method to find $y(4 \cdot 4)$ given

$$5x\frac{dy}{dx} = y^2 - 2 = 0,\ y(4) = 1,\ y(4 \cdot 1) = 1 \cdot 0049,\ y(4 \cdot 2) = 1 \cdot 0097$$

and $y(4 \cdot 3) = 1 \cdot 0143$. 5

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Student's Copy

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